DETERMINATION OF OPTIMAL DIRECT LOAD CONTROL STRATEGY USING LINEAR PROGRAMMING

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SUMMARY

Direct load control (DLC) is one of the most widely used techniques of load management. This technique is usually applied at residential sector. In most cases DLC is being applied to minimize peak load or production costs in power system. In this paper is exposed a model for determining an optimal DLC strategy of devices in households with the aim of reducing system peak load. The proposed model enables optimal (maximal) reduction of the system peak load in cases when various types of devices in household are controlled at the same time. The presented model is based on linear programming and tested on a real power system where water heaters and air conditioners are controlled at the same time.

INTRODUCTION

Many load management (LM) programs have been applied by the utilities in recent years. Load management objectives for different utilities may be different. In the power systems with a little reserve in power production and transmission load management is primarily applied for reducing system peak load. Thus, the needs for building new capacities are postponed (reduced). Systems with relatively high reserve are applying load management for reducing production costs and dependence of expensive fuels [1,2]. For achieving load management objectives, it is usually used DLC of some devices in households. Commonly controlled devices are the following: water heaters, air conditioners, electric space heaters and storage heaters. In the literature, a considerable attention is dedicated to DLC and especially to determination of optimal DLC strategy. Direct load control strategy, which determines turning on/off diary schedule of controlled devices, is composed of several elements, the most important of which are: (i) type of controlled devices and the way of their control, (ii) beginning and duration of control period during a day, (iii) number of groups of controlled devices and their mutual time shift, i.e. turning on/off moments of some groups and (iv) number of devices according to groups. There is a series of models for determining an optimal DLC strategy [2-13]. Majority of the proposed models is developed with the aim of reducing production costs of the power system [2-9], and a smaller part with the aim of reducing system peak load [10-13]. This paper is dedicated to determination of optimal DLC strategy with the aim of reducing system peak load.

Cohen [10] has developed a model for determining elements of DLC with the aim of reducing system peak load. The model is based on dynamic programming. In that model, total number of controlled devices is in advance divided in determined, beforehand known (assigned), number of groups of the same size (the same number of devices in each group). During optimization procedure, the exposed model treats each group of devices separately. In this way, proper (optimal) coordination of load control (connection and disconnection) between different device groups is missing and maximum reduction of the system peak load is not obtained. Lee [11] has developed a model based on linear programming which enables determination of number of controlled groups and number of devices in each group as a result of optimizing procedure. In the model, turning on/off moments of some groups of devices are determined in advance. A set of these moments forms so-called control shame. Thus, the problem of coordination of load control between some groups of devices is partially surpassed, but maximal reducing of system peak load is not possible because of limits imposed by the method of forming control scheme. We can find similar drawbacks in the model exposed in the literature [12], which is based on the combination of linear and dynamic programming. The model proposed in the literature [13] is based on linear programming. This model enables determination of optimal DLC strategy when devices, which can be turned off for a long period of time (for example, water heaters), are controlled, while it is not possible for devices which can be turned off only for a brief periods of time (periodically) (for example, air conditioners).

In this paper is developed a model, based on linear programming, which enables determination of most important elements of DLC strategy, and consequently realizing an appropriate (optimal) coordination of load control (turning on/off moments) between different groups of controlled devices. In this way, proposed model ensures maximal reduction of system peak load (maximal utilization of available DLC resources) even in case of applying DLC to various types of devices at the same time.

This paper is composed of five parts. In the second part is exposed mathematical model for determining an optimal DLC strategy. Appliance of the proposed model to peak load reduction of the real power system, when water heaters and air conditioners are controlled simultaneously is exposed in the third part. The most important conclusions are given in the fourth and the list of cited works in the fifth part.

MATHEMATICAL MODEL

Direct load control is generally realized in two ways:

with periodical turn off of devices (several times during control period (cycling)) or with long-term turn off of devices (once during control period). The first way is usually applied to devices that can accumulate a little quantity of energy (air conditioners, space heaters, etc.), and the second to devices that are capable of considerable accumulation of energy (water heaters, storage heaters, etc.). For that reason, in the proposed model will be made difference between these two ways of controlling of devices. Mathematical model, based on linear programming, for determining an optimal DLC strategy when are simultaneously controlled different types of devices which can be turned off only for a long period of time and different types of devices which can be turned off only for a brief periods of time (cycling), is the following:

$$\text{Minimize } P_p \qquad (1)$$

Subject to:

$$P_p - \sum_{d=1}^{m} \sum_{j=1}^{n_d} A_{ij}^{d} \bullet x_j^d - \sum_{l=1}^{p} \sum_{j=1}^{n_l} B_{ij}^{l} \bullet y_j^l \geq d_i, \qquad i=1,..,k, \qquad (2)$$

$$\sum_{j=1}^{n_d} x_j^d \leq M^d, \qquad d=1,..,m, \qquad (3)$$

$$[C^l] \bullet [Y^l] \leq [M^l], \qquad l=1,..,p, \qquad (4)$$

$$\sum_{j=1}^{n_l} y_j^l \leq z^l \bullet M^l, \qquad l=1,..,p, \qquad (5)$$

$$x_j^d \geq 0, \qquad j=1,..,n_d, \quad d=1,..,m,$$

$$y_j^l \geq 0, \qquad j=1,..,n_l, \quad l=1,\dots,p,$$

where:

P_p – unknown system peak load (after DLC),

m – number of different types of devices which can be turned off only for a long period of time,

n_d – number of observed 15-minutes lasting periods for d-type devices (devices that are capable of a long-term turn off); due to the fact that peak load is being registered in 15-minutes lasting intervals for the period of 24 hours, it is usually assumed that $n_d \leq 96$,

A^d_{ij} – known value which describes a load control influence of d-type devices on the power system load curve,

$$A^d_{ij} = \begin{cases} -P_u^d(i), & \text{during disconnection periods,} \\ P_{pb}^d(i), & \text{during payback periods,} \\ 0, & \text{in other cases,} \end{cases} \qquad (6)$$

$P_u^d(i)$ – diversified load of d-type device ,

P_{pb}^d (i) – net restore demand of d-type device,

x^d_j – unknown number of d-type devices that should be turned off in j-th period,

p – number of different types of controlled devices which can be turned off only for a brief periods of time (cycling),

n_l – number of observed 15-minutes lasting periods for l-type devices (devices that are capable of periodical turn off); in this case, for the above mentioned reasons, it is usually assumed that $n_l \leq 96$,

B^l_{ij} – known value which describes a load control influence of l-type devices on the power system load curve,

$$B^l_{ij} = \begin{cases} -P_u^l(i), & \text{during disconnection periods,} \\ P_{pb}^l(i), & \text{during payback periods,} \\ 0, & \text{in other cases,} \end{cases} \qquad (7)$$

$P_u^l(i)$ – diversified load of l-type device ,

P_{pb}^l (i) – net restore demand of l-type device,

y^l_j – unknown number of l-type devices that should be turned off in j-th period,

d_i – forecasted 15-minute load of the power system (before DLC),

k – number of 15-minute lasting intervals when is observed DLC influence of all controlled devices on system peak load reduction,

M^d – available number of controlled d-type devices,

M^l – available number of controlled l-type devices,

z^l – maximal number of turning off of l-type devices during control period.

Matrix $[C^l]$ has the following form:

$$[C^l] = \begin{array}{c|cccccccccccc|}
 & 1 & . & . & . & . & . & c_l & c_l+1 & . & . & . & n_l \\
\hline
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & \cdots & 0 \\
2 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & \cdots & 0 \\
\vdots & & & & & & \vdots & & & & & & \\
n_l+1-c_l & 0 & 0 & 0 & \cdots & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{array} \qquad (8)$$

With c_l is marked control "cycle" of l-type devices. It is composed of turning off time length (number of 15-minutes lasting intervals) and minimal turning on time length (number of 15-minutes lasting intervals) of devices. For example, if control "cycle" is 2 hours (30-minutes lasting turning off period and minimum 90-minutes lasting turning on period) c_l=8. Control "cycle" for the same devices can have different lengths (values) during control period, making possible to be combined different cycling scenarios within one control period. Colum vector $[Y^l]$ has dimension n_l x 1 and colum vector $[M^l]$, whose elements are the same (available number of controlled devices), has dimension $(n_l + 1 - c_l)$ x 1.

The aim of the proposed model, exposed with expression (1), is to reduce a system peak load. Constraints given with expression (2) (one for each i-th time interval) ensure that the new peak load is not going to be smaller that new load for each i-th time interval. The new load for each i-th time interval is calculated as a sum of original forecasted load and a total of load changed due to load control of all types of devices. Total of changed load during i-th time interval, due to control of all types of devices, is given by $\sum_{d=1}^{m} \sum_{j=1}^{n_d} A_{ij}^{d} \bullet x_j^d + \sum_{l=1}^{p} \sum_{j=1}^{n_l} B_{ij}^{l} \bullet y_j^l$.

Constraints (3) ensure, for each d-type device, that number of controlled devices in not going to surpass a total number available for control. Constraints (4), for each l-type device, ensure cycling (controlling) according to fixed scenario without disturbing consumer's comfort.

Constraints (5) ensure, for each l-type device that, if there are such demands, turning off number of devices during control period is not going to be more than it is maximally allowed.

Number of examined periods n_d and n_l depends on power system daily load curve shape, that is on a peak load time length. It could be usually assumed that n_d and n_l are less than 96, what reduces number of variables in the model. Also, in such cases, can be observed a period shorter than 24 hours (k < 96), what reduces number of constraints in the model. Interval (k) where is observed DLC influence on system peak load reduction, must include payback influence of all controlled devices on daily load curve shape of the observed power system. In described way, dimensionality of linear programming problem (relations (1)-(5)) is being reduced and settling of the problem is being accelerated.

RESULTS

The proposed model is applied to the peak load reduction of a real power system, where are simultaneously controlled water heaters (devices capable of a long-term turn off) and air conditioners (devices capable of a periodical turn off). Daily load curve shape of the observed power system is taken from the literature [14] and is exposed in Figure 3. Daily load curves of controlled devices, the way of its acquiring as well as effects of control of their work (payback) are described in the literature [6, 11, 14-19]. Daily diversified load curves of uncontrolled and controlled water heaters and air conditioners are shown in Figure 1 and Figure 2, respectively [14]. Shape and values of payback for water heaters and air conditioners are described in the Table 1 [11] and Table 2 [6], respectively, where E stands for energy (in kWh) that devices were deprived during turning off period, and P_{pb} stands for net restore demand. Maximally allowed turning off time length for water heaters is 3 hours [14]. Air conditioners are turned off for 30 minutes and after that they should be turned on for 1 hour and 30 minutes [6]. According to this scenario, they can be turned off several times during control period.

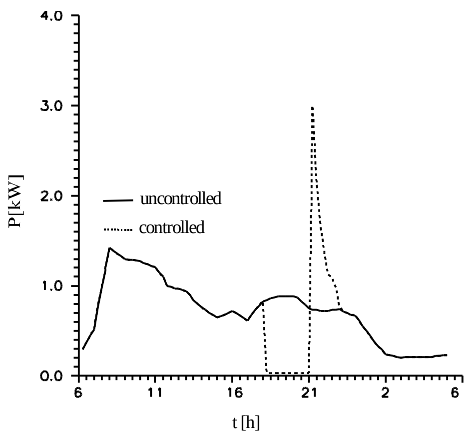

Figure 1. Daily diversified load curve of controlled and uncontrolled water heater

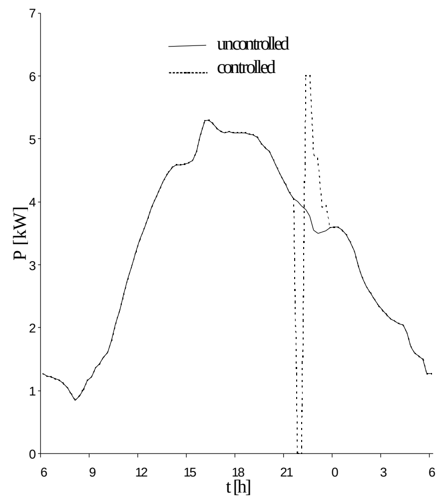

Figure 2. Daily diversified load curve of controlled and uncontrolled air conditioner

Table 1. Relations for net restore demand calculation of water heater

Time elapsed after restoration of service	P_{pb} (kW)	
	for E < 3.16 kWh	for E > 3.16 kWh
0:15	$-0.217\ E^2 + 1.375\ E + 0.266$	2.442
0:30	Max(0, 0.600 E - 0.100)	1.798
0:45	Max(0, 0.486 E - 0.243)	1.295
1:00	Max(0, 0.320 E - 0.160)	0.852
1:15	Max(0, 0.200 E - 0.100)	0.533
1:30	Max(0, 0.207 E - 0.166)	0.489
1:45	Max(0, 0.160 E - 0.160)	0.346
2:00	Max(0, 0.231 E - 0.554)	0.177

Note: Expression Max(0,…) means that for small values of E, for which second part of expression in parenthesis becomes negative, the net restore demand is zero i. e. payback period is finished.

Table 2. Relations for net restore demand calculation of air conditioners

Time elapsed after restoration of service	P_{pb} (kW)
0:15	0.6 E / 0.5
0:30	0.6 E / 0.5
0:45	0.3 E / 0.5
1:00	0.3 E / 0.5
1:15	0.1 E / 0.5
1:30	0.1 E / 0.5

Results of appliance of the model, for different number of controlled devices, are shown in Table 3. In the Table 3 we can see work schedule for each type of device separately, that is for each type of device is given number of groups and number of devices according to the groups, initial time and length of control period as well as time shift between the groups. For example, x_{43}=5252 means that 5252 water heaters should be turned off in the 43rd 15-minutes lasting interval, that is at 16 hours and 45 minutes. Those water heaters will be turned off for the 3 hours. It also can be concluded from the Table 3 that the total number of air conditioners according to the groups is greater than the total number of devices available for control, what means that some air conditioners will be turned off several times during a day, i.e. control period. In the first case, when are controlled 100000 air conditioners and the same number of water heaters, system peak load, which is 8000 MW, is reduced for 263,8 MW (3,3 %). In the second case, when are controlled 200000 air conditioners and water heaters, this reduction is 438 MW (5,5 %), and in the third case, when are controlled 300000 air conditioners and water heaters, the reduction amounts to 557,4 MW (7,2 %). Power system daily load curve shape, when are controlled 300000 air conditioners and water heaters, is shown in Figure 3.

Table 3. Results of DLC appliance for different number of controlled water heaters and air conditioners

Total number of controlled devices	Working schedule	
	Water heaters	Air conditioners
M_{WH} = 100000 M_{AC} = 100000	x_{43}= 5252 x_{44}= 21677 x_{45}= 59253 x_{48}= 13817	y_{48}= 8405 y_{49}= 6667 y_{50}= 16359 y_{51}= 14706 y_{52}= 26196 y_{53}= 13995 y_{54}= 13670 y_{58}= 31431
M_{WH}= 200000 M_{AC} = 200000	x_{44}= 15206 x_{45}= 67441 x_{46}= 70746 x_{47}= 46605	y_{46}= 6292 y_{47}= 5360 y_{48}= 22938 y_{49}= 12777 y_{50}= 33064 y_{51}= 21699 y_{52}= 45444 y_{53}= 21964 y_{54}= 35231 y_{55}= 6878 y_{56}= 22938 y_{57}= 12777 y_{58}= 33064 y_{59}= 12532
M_{WH}= 300000 M_{AC} = 300000	x_{43}= 7483 x_{44}= 16586 x_{45}= 57569 x_{46}= 38087 x_{47}= 41676 x_{48}= 76482 x_{49}= 54614 x_{50}= 7499	y_{43}= 1675 y_{44}= 15239 y_{45}= 11718 y_{46}= 29297 y_{47}= 19938 y_{48}= 38708 y_{49}= 21098 y_{50}= 48887 y_{51}= 30227 y_{52}= 60879 y_{53}= 29826 y_{54}= 50434 y_{55}= 19938 y_{56}= 38708 y_{57}= 21098 y_{58}= 32088 y_{59}= 23287 y_{60}= 57115 y_{61}= 39340 y_{62}= 51678 y_{63}= 36681 y_{64}= 38708 y_{65}= 21098 y_{66}= 32088

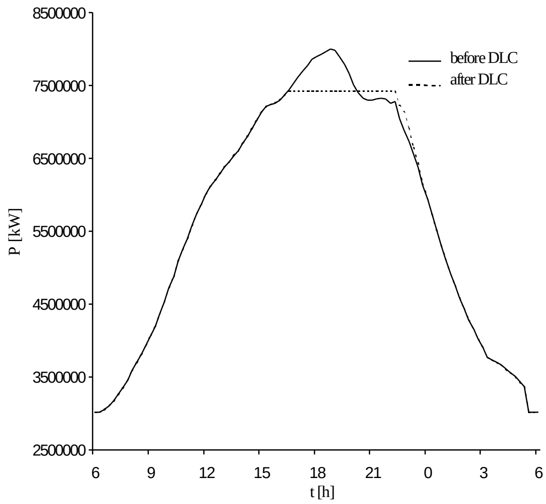

Figure 3. Power system daily load curve when are simultaneously controlled 300.000 water heaters and air conditioners.

CONCLUSION

In this paper is developed a model for determining an optimal direct load control strategy of devices in residential sector with the aim of reducing system peak load. It is based on linear programming. The proposed model enables determination of the most important elements of direct load control strategy, and consequently obtaining appropriate (optimal) coordination of load control (turning off/on moments) between different groups of devices. With this is ensured a maximal (optimal) reduction of power system peak load (maximal utilization of available resources of controlled devices). Developed model ensures determination of optimal direct load control strategy when are observed individual types of devices, which are controlled either by periodical turn off (cycling) or by long-term turn off, as well as different types of devices simultaneously. Moreover, the proposed model can be, in an appropriate manner, applied to other load management techniques that are being used in residential sector.

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