THE QUALITY OF SUPPLY AND THE ASSESSMENT OF THE TECHNICAL LIMITS OF A DISTRIBUTION NETWORK

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INTRODUCTION

The planners of the distribution network development are inevitably confronted with the problems regarding the uncertainty of various circumstances affecting the decisions, which are to be made. A useful tool in such an analysis offers the fuzzy mathematics [1,2,3] if properly applied and interpreted. This paper outlines a fuzzy mathematics based method for assessing the technical limits of distribution networks throughout a long-term period making it possible to predict the weak points and plan the remedy actions.

APPLICATION OF FUZZY MODELING

Motivation

In distribution network planning there are many variables and parameters, which can not be predicted with certainty. This is particularly the case with the consumers load growth and various data being of importance for the reliability and associated load supply interruption costs evaluation. Fuzzy mathematics provides an adequate means for encompassing the said uncertainties and for drawing the necessary engineering conclusions with regard to the actions which have to be undertaken.

An engineering interpretation

Consider a variety q (variable, parameter) which values are not known with certainty. This variety may be modeled as a normalized unimodal fuzzy number (FN) as depicted in Fig.1. FN models of guessed quantities are further on denoted by capital letters.

Parameter $\alpha \in$ (0,1] (α- cut) is introduced which may be interpreted as the level of uncertainty of the guess made at q. Each α yields an interval of guessed values of q with lower bound $Q_{\alpha l}$ and upper bound $Q_{\alpha u}$. For increasing α these bounds become closer to one another tending to a single value as α approaches to 1. This value is the *kernel* of Q, denoted as Q_K. If q is modeled by a triangular FN, then this FN is completely defined by the triple (Q_{ol}, Q_K, Q_{0u}).

The relative weighted uncertainty in q may be determined as, for Q_K>0,

$$UN = \frac{1}{Q_K}\int_0^1 (Q_{\alpha u} - Q_{\alpha l})(1-\alpha)d\alpha \qquad (1)$$

The benefits offered by the application of the fuzzy mathematics are the following:

- The result obtained includes the conventional crisp result as this coincides with the kernel Q_K.
- The information on the possible values of the output variety of interest due to the uncertainty of some input varieties is obtained. This information is substantially more comprehensive than that obtainable by conventional sensitivity analysis. The latter considers possible deviations of inputs taken one by one and only in close proximity to a fixed system operating state. The intervals of possible values managed by the FN approach may be of any size and all uncertain inputs are simultaneously considered.
- The output obtained can be assessed with regard to the grade of the uncertainty by applying (1). If this grade is considered to be too high, the uncertain inputs should be reexamined for a more precise quantification.

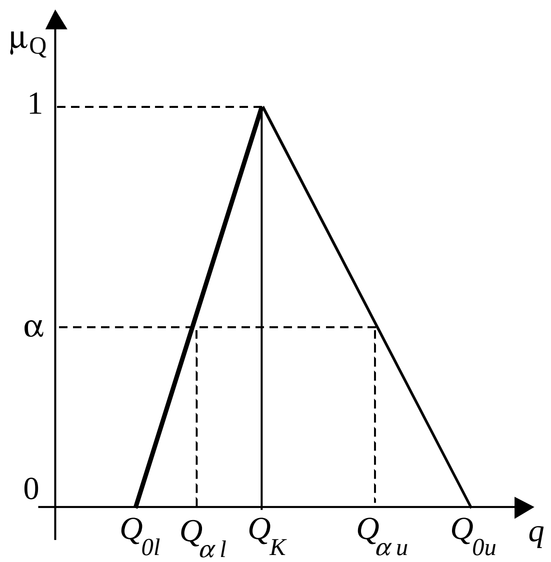

Fig.1 Characteristic function of a FN

Calculation flow

Presume that q is a function $F(\ )$ of inputs q_i, i=1,...,n. If inputs are modeled as FNs to encompass their uncertainty, then q is also a FN which may be formally expressed as

$$Q = F(Q_1,...,Q_n) \qquad (2)$$

To define Q, its characteristic function μ_Q has to be constructed. This is done by generating a series of α values from the whole interval (0,1). For each α the lower and upper bounds of Q should be determined as

$$\begin{aligned} Q_{\alpha l} &= \min\{\, F(q_1,...,q_n)\,\} \\ Q_{\alpha u} &= \max\{\, F(q_1,...,q_n)\,\} \end{aligned} \qquad (3)$$

for

$$Q_{i\alpha l} \le q_i \le Q_{i\alpha u}\,, \quad i = 1,...,n \qquad (4)$$

which defines μ_Q.

The calculation of $Q_{\alpha l}$ and $Q_{\alpha u}$ is trivial if $F(\,)$ is a monotonic increasing or decreasing function with regard to all arguments being within the intervals in (4). In the first case, $Q_{\alpha l}$ is obtained from $F(\,)$ for $q_i = Q_{i_{\alpha}l}$, and $Q_{\alpha u}$ for $q_i = Q_{i_{\alpha}u}$, i=1,...,n. In the latter case, $q_i = Q_{i_{\alpha}u}$ should be inserted to obtain $Q_{\alpha l}$, and $q_i = Q_{i_{\alpha}l}$ to obtain $Q_{\alpha l}$.

NETWORK MODEL

Consider a radial feeder of a MV distribution network (Fig.2). Each load point, consumer loads and the branch supplying the load point are marked by the same index. It is supposed that there is no back up facility in the initial network state. The capability of the network to preserve the quality of supply throughout a 10 year planning period is to be examined. The possibility of violating thermal and voltage drop limits as well as the acceptable supply interruption costs is considered allowing for uncertainties in load prediction and reliability associated data.

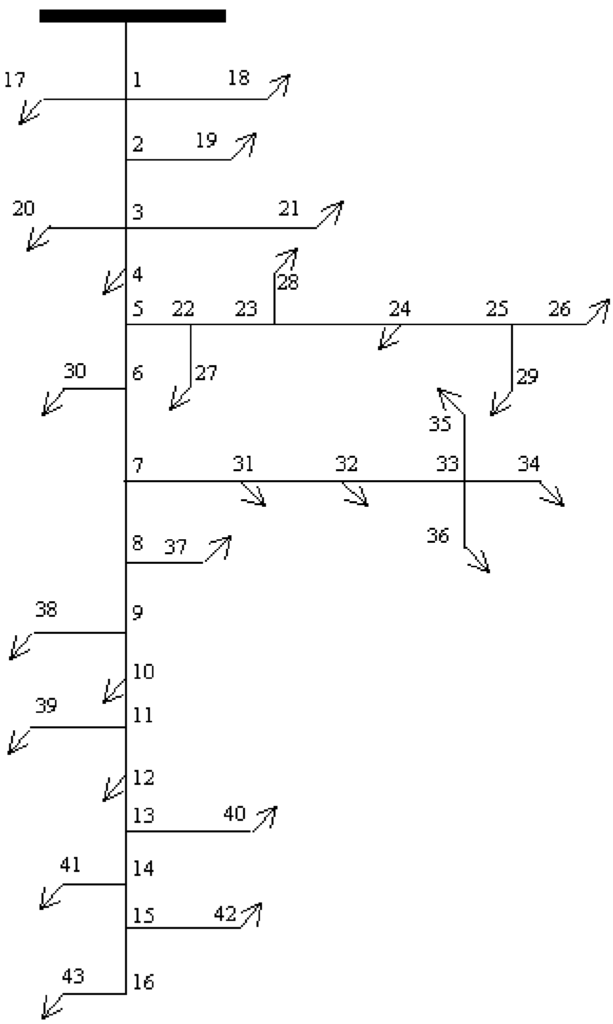

Fig.2 Network sample under consideration

Consumer loads

Both real and imaginary parts of all load currents peaks are presumed to be fuzzy and rising in time by obeying the polynomial law

$$J_i(t) = J_{Ri}(t) - j\,J_{Ii}(t) = (1+R)^{t-t_{0i}}[\,J_{Ri}(t_{0i}) - j\,J_{Ii}(t_{0i})\,] \qquad (5)$$

Indices R and I in (5) denote the real and imaginary parts, t_{0i} is the initial year of supplying consumer i while R is the annual increment rate, which is also modeled as a FN. For brevity, arguments t will be omitted further on.

Average yearly real power demand of consumers is taken to be

$$[P] = \sqrt{3}\,U_r\,B\,[J_R] \qquad (6)$$

with $[P]$ and $[J_R]$ being n by 1 column vectors of consumers average real power demands and load current peaks real parts. B is the load factor represented as a FN and U_r is the network rated voltage.

Maximum branch currents and voltage drops

Real and imaginary parts of maximum branch currents are correlated as [4]

$$[I_R] = [m][J_R]\,,\ \ [I_I] = [m][J_I] \qquad (7)$$

where $[I_R]$ and $[I_I]$ are n by 1 column vectors of real and imaginary parts of branch currents. The elements of the *n by n* incidence matrix $[m]$ are

$$m_{ik} = \begin{cases} 1 & \text{if branch } i \text{ supplies branch } k \\ 0 & \text{otherwise} \end{cases} \qquad (8)$$

From (7), it follows that the maximum branch currents also are fuzzy quantities. The rms of the current in branch k equals

$$|\,I_k\,| = \left\{([m_k][J_R])^2 + ([m_k][J_I])^2\right\}^{\frac{1}{2}} \qquad (9)$$

where $[m_k]$ is a *1 by n* row vector built of the row k elements of $[m]$.

Denote by [l] the diagonal n *by* n matrix of the lengths of feeder branches. Then the voltage drops at load points are

$$[\Delta U] = [l_z]\, z\, [J] \tag{10}$$

where

$$[l_z] = [m]^t\, [l]\, [m] \tag{11}$$

It is presumed that the feeder branches have the same impedance per unit length $z=r+jx$. From (10), it is clear that the elements of the n *by* 1 column vector [ΔU] are fuzzy quantities as they depend on [J].

Bearing in mind (10) the following expressions hold

$$\begin{aligned}[\Delta U_R] &= [l_z]\,(r[J_r] + x[J_R]) \\ [\Delta U_I] &= [l_z]\,(x[J_R] - r[J_I])\end{aligned} \tag{12}$$

The rms values of voltage drops at feeder load points are

$$|\Delta U_k| = (\Delta U_{Rk}^2 + \Delta U_{Ik}^2)^{\frac{1}{2}} \quad k = 1,...,n \tag{13}$$

with ΔU_{Rk} and ΔU_{Ik} being elements of vectors [ΔU_R] and [ΔU_I], respectively.

Supply interruption costs

If there is no back feed available, each failure of a branch, say k, interrupts the supply to all branches and associated loads fed by this branch until its repair is terminated. The sound feeder portion is separated from the faulted one by opening the disconnectors, if available, or by removing the phase conductors ties at the corresponding tower. However, each feeder failure causes a temporary complete supply interruption needed for fault tracking and separation of feeder portions. Bearing the aforementioned in mind, the expected cost due to the energy not delivered equals

$$C_w = c_w \left[\sum_{k=1}^{n} (l_k\, \Lambda D[m_k][P]) + l\Lambda D' \sum_{k=1}^{n} P_k \right] \tag{14}$$

The failure transition rate per unit line length Λ has been modeled as a FN as well as the repair duration D and the fault tracking and localization time D'. Symbol l is the total line length and c_w is the cost per unit of energy not delivered.

The annual cost caused by load interruption is

$$C_p = c_p \left[\sum_{k=1}^{n} (l_k\, \Lambda[m_k][P]) + l\Lambda \sum_{k=1}^{n} P_k \right] \tag{15}$$

where c_p is the cost per interrupted unit of load.

The total supply interruption cost in year t referred to the first year is

$$C = (C_w + C_p)(1 + p)^{-(t-1)} \tag{16}$$

with p denoting the discount rate.

Network capability evaluation

Network capability in providing service quality in uncertain environment is assessed using the characteristic function calculated for the relevant varieties. Consider Fig.3 displaying the characteristic function of a performance index g with FN G. The maximum acceptable value of g is g_{max}. The grade of satisfying the required condition $g \le g_{max}$, say network grade of goodness concerning g, is assessed as

$$e_g = \frac{a_l}{a_l + a_r} \tag{17}$$

with a_l and a_r being the areas under the characteristic function left and right to the value g_{max}.

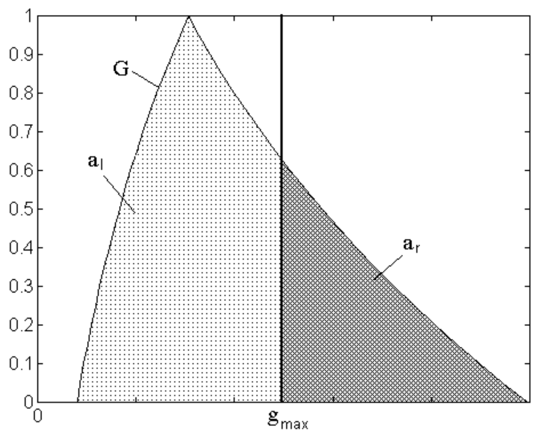

Fig.3 Assessment of the grade of goodness

The overall assessment of the grade of nonviolating the limit for the network as a whole may be of the form

$$e_{Ng} = \frac{\sum a_{lk}}{\sum a_{lk} + \sum a_{rk}} \tag{18}$$

Index k runs over all corresponding network elements . Areas a_{lk} and a_{rk} are determinable from the characteristic functions of the FN G_k calculated for element k. In our case, the grade of goodness for branch currents is calculated with regard to their thermal limits as well as the corresponding grade for voltage drops, both for the most critical element and for the network as a whole.

APPLICATION EXAMPLE

The proposed method was applied to the feeder depicted in Fig.2. Tables 1 to 3 quote the adopted system data. It was presumed that all load points are present at the beginning of the first year. The exception are loads at nodes #4 and #6 which are presumed to be connected to the feeder at the beginning of the third and sixth year, respectively. The maximum tolerable supply interruption cost is taken to be equal to the cost of providing a feedback supply.

Table 1 Maximum load current in the first year of supply

Branch, *k*	J_{k0l}, A	J_{kK} , A	J_{k0u}, A
17, 19-21, 24, 26-27, 29, 40, 42, 43	0.90+j0.43	1.17+j0.57	1.71+j0.83
6	0	0	1.80+j0.87
18, 28, 30, 35, 36	1, 71+j0.83	2.34+j1.13	3.5+j1.70
4, 12, 31-32, 34, 37-39, 41	2.79+j1.35	3.69+j1.79	5.58+j2.70
10	4.32+j2.09	5.85+j2.83	8.73+j4.23

Table 2 Branch lengths

k	l_k, m	*k*	l_k, m	*k*	l_k, m	*k*	l_k, m	*k*	l_k, m
1	3080	10	450	19	410	28	430	37	700
2	1300	11	1020	20	1350	29	550	38	300
3	560	12	1820	21	650	30	330	39	50
4	460	13	600	22	150	31	630	40	600
5	350	14	1340	23	1030	32	1070	41	150
6	1200	15	540	24	1120	33	830	42	40
7	480	16	1500	25	900	34	50	43	240
8	360	17	840	26	750	35	330		
9	350	18	1260	27	180	36	280		

Table 3 Other sample data

R	*B*	Λ, fl/(km, yr)
(0.0, 0.05, 0.08)	(0.2; 0.3; 0.4)	(0.07, 0.10, 0.13)
U_r (V)	c_p, DEM/kW	c_w, DEM/kWh
10000	3	3,7
I_{max}, A	ΔU_{max}, *V*	c_{max}, DEM
170	1000	50000
r, Ω/km	x, Ω/km	*p*
0.6	0.36	0.05
D, h	*D'*, h	
(4, 6, 8)	(1, 1.5, 3)	

Fig.4 depicts the grades of goodness with respect to the thermal rating of the critical branch (e_i) and of the feeder as a whole (e_{Ni}) as well as with regard to the acceptable supply interruption costs (e_c).

Fig.5 displays the feeder grade of goodness with regard to the maximum voltage drop of most critical load point (e_u) and such overall grade for the feeder (e_{Nu}).

As may be seen, the network is most critical regarding the voltage conditions. If we take 0.9 to be an acceptably high grade of goodness, the voltage drop at the critical load point is evaluated to be unsatisfactory at the end of the third year and the overall voltage conditions at the end of the fifth year. As to the thermal limits and the supply interruption costs, the feeder may be considered to be adequate throughout almost the entire 10 year planning period.

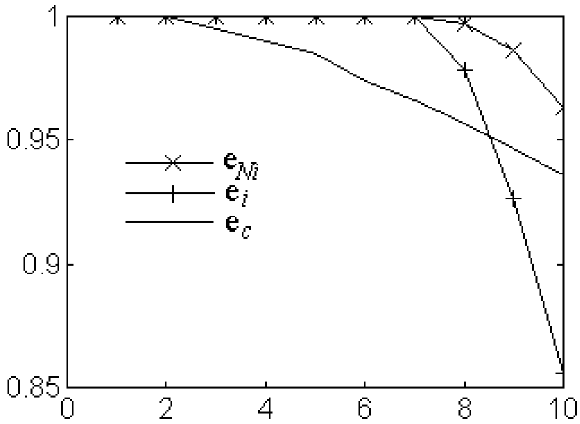

Fig.4 Grade of goodness with respect to thermal rating and interruption costs during the planning period

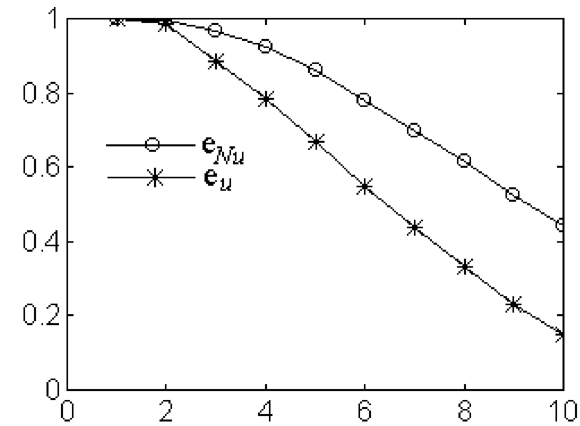

Fig.5 Grade of goodness with respect to voltage drops during the planning period

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