CALCULATION OF DISTRIBUTION TRANSFORMER LEAKAGE REACTANCE USING ENERGY TECHNIQUE

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Abstract- Energy technique procedure for computing the leakage reactance in distribution transformers is presented. This method is very efficient compared with the use of flux element and image technique and is also remarkably accurate. Examples of calculated leakage inductance and the short circuit impedance are given for illustration. For validation, the results are compared with the results obtained using test.

Keywords: Leakage reactance of Transformers, Electromagnetic energy

List of symbols

\( \lambda \)  Leakage fluxes
\( N \)  No. Of turn of winding
\( I \)  Current
\( L_{\text{mt}} \)  Mean length of one turn of winding
\( L_c \)  Length of window of core
\( d \)  Width of winding
\( s_1 \)  Distance between core and LV windings
\( s_1, s_2 \)  Distance between LV and HV windings
\( r_{\text{ave}1}, r_{\text{ave}2} \)  Mean radius of HV and LV winding respectively
\( X \)  Reactance of transformer
\( f \)  Frequency of current of windings
\( W \)  Electromagnetic energy stored in active part
\( L_{\text{eq}} \)  Equivalent inductance of transformer
\( H_x \)  Magnetic field intensity in a distance \( x \) of first layer of primary winding

I. INTRODUCTION

Determination of transformer leakage reactance using magnetic cores has long been an area of interest to engineers involved in the design of power and distribution transformers. This is required for predicting the performance of transformers before actual assembly of the transformers. A method has been presented [1] for estimating the leakage reactance by flux tube in order to include in an electric circuit model of transformer. Computer-based numerical solution techniques using finite elements analysis, boundary element method and boundary-integral method are accurate and form an important part of the design procedures but require rather elaborate computer resources and a somewhat lengthy setup before a solution can be obtained. Also a closed-form solution often provides more insight about critical physical parameters than a computer-based numerical solution.

In this paper a closed form solution technique applicable to the leakage reactance calculations for transformers is presented. An emphasis is on the development of a simple method to characterize the leakage reactance of the transformers.

Leakage reactance calculations play an important role in designing geometry of transformers. The design parameters may be varied as such that the required short circuit impedance is determined. A 2D representation proves to be satisfactory in determining the leakage reactance. Final expressions are developed on a per-unit-of-length basis for the third dimension. Certain assumptions have been made in this calculation. End effects introduced by the terminations in 3D configurations are not evaluated here.

There are different techniques for the leakage reactance evaluation in transformer. The most common technique is the use of the flux leakage elements and estimation of the flux in different parts of the transformer [1-6]. The images technique can be also used. The basis of this method is considering the image of every turn of the winding where the magnetic potential vector [7,8] is employed to compute the mutual and leakage inductance. Although the technique is effective, the computation result depends on the current of the image conductor [7].

This paper presents a novel technique for calculation of the leakage inductance in different parts of the transformer using the electromagnetic stored energy.

2. COMPUTATION USING FLUX ELEMENT TECHNIQUE

In order to compute the leakage reactance analytically, some approximation is required to achieve a closed-loop solution [1]. These assumptions are:

1. The leakage flux distribution in the winding and the space between the windings must be in the direction of the winding axial.
2. The leakage flux is uniformly distributed along the length of the windings.
3. The leakage flux in the space of two windings is divided equally between them.

The leakage flux for each winding for a two-winding transformer, based on the above assumptions is[1]:

\[
\lambda = \mu_0 N^2 I_{\text{ave}} \left( \frac{d}{3} + \frac{s}{2} \right) / L_c 
\]  

(1)

Using the following equation:

\[
X = \frac{2\pi f \lambda}{I}
\]  

(2)
and reflecting leakage reactance between HV windings to the primary side yields:

\[ X = X_1 + \left( \frac{N_1}{N_2} \right)^2 X_2 \]

Eqn. 3 will be as follows:

\[ X = 2\pi \frac{N_1^2}{L_c} \left[ L_{m1} \left( \frac{d_1}{3} + \frac{s}{2} \right) + L_{m2} \left( \frac{d_2}{3} + \frac{s}{2} \right) \right] \]

If it is assumed \( L_{m1} = L_{m2} \) (it means the length of each turn of HV and LV windings are equal), Eqn. 3 can be simplified as follows:

\[ X = 2\pi \frac{N_1^2}{L_c} \left( \frac{d_1 + d_2}{3} + s \right) \]

This is the conventional equation used in the references [3, 4, 5].

3. COMPUTATION USING IMAGE METHOD

This method is based on considering an Image conductor to the core for each loop, because the surface of core is an equipotential for the magnetic scalar potential so the core surface will be a mirror for magnetic field [7].

Using the magnetic vector potential for a circular filament for two conductors, the leakage inductance is [7]:

\[ L_{leakage} = f(r_1, r_2, A(r_1), A_{image}(r_1), A(r_1-r_2), A_{image}(r_1-r_2), i_1, i_2, i_1_{image}, i_2_{image}) \]

Where \( r_1 \) and \( r_2 \) are the radiiuses of conductors, \( A \) and \( A_{image} \) are the magnetic vector potential respect to actual and image conductors that are dependent of elliptic integral of first and second kinds [7, 8].

The current of image conductor has a value different from real conductor and it should be adjusted for each image conductor. So the result of leakage inductance changes according to default of image conductor currents.

There are several recommendations for this parameter to evaluate the best result in Image Method but there is not a constant rule for it and the error of calculation and test result may be not optimized.

4. COMPUTATION USING ENERGY TECHNIQUE

The electromagnetic energy stored in the windings and the space between them can be used to calculate the inductance between the windings and the leakage inductance.

The previous assumptions are considered here in order to obtain a closed-form solution. Consider the path \( F_1 \) in Fig. 1, mmf for the path having distance \( x \) from the beginning of LV winding is [2]:

\[ \text{mmf}_{l} = N_1 I_1 x / d_1 \]

Increasing \( x \) from 0 to \( d_1 \) increases the magnetic field intensity and approaches its peak value \( N_1 I_1 / L_c \). A volume differential from LV winding as shown in Fig. 1, with height \( L_c \), thickness \( dx \) and radius \( r_1 + s_1 + x \) is considered. The electromagnetic energy stored in this element is [9]:

\[ dv = \frac{1}{2} \frac{dH}{dx} \]

and the total energy is:

\[ W = \frac{1}{2} \mu_0 \mu_n N_1^2 I_1^2 \left[ \frac{r + s_1}{3} + \frac{d_1}{4} \right] \]

Similarly the stored energy in HV winding can be determined:

\[ W = \frac{1}{2} \mu_0 \mu_n N_2^2 I_2^2 \left[ \frac{r + s_1 + d_1 + d_2}{3} - \frac{d_1}{4} \right] \]

With a constant magnetic field intensity between the windings, \( W_a \) the electromagnetic energy stored between them is:

\[ W_a = \frac{1}{2} H_a V_a \]

Hence:

\[ W_a = \frac{1}{2} \mu_0 \mu_n N_2^2 I_2^2 \left[ \frac{r + s_1 + d_1 + s_2}{2} \right] \]

The stored energy for this two-winding transformer is:

\[ W = W_1 + W_2 + W_a = L_{eq} I^2 / 2 \]

Using Eqns. 9-13, the inductance will be as follows:

\[ L_{eq} = 2\pi \mu_0 \mu_n N_2^2 \left[ \left( \frac{r_1}{3} + \frac{d_1}{4} \right) d_1 + \left( \frac{r_2}{3} + \frac{d_2}{4} \right) d_2 + r_a s_1 \right] \frac{1}{L_c} \]

\( r_1 \) and \( r_2 \) are defined as follows:
\[ r_1 = r + s_1 \]  
\[ r_2 = r + s_1 + d_1 + s_2 + d_2 \] (15) (16)

A notable point is that if term \( (r_1/3 + d_1/4) \) is substituted by \( r_{ave1}/3 \) and term \( (r_2/3 - d_2/4) \) in replaced by \( r_{ave2}/3 \), \( L_{eq} \) will be:

\[ L_{eq} = 2\mu_0 N_1^2 \left[ (r_{ave1}d_1 + r_{ave2}d_2) / 3 + r_x t_x \right] / L_x \] (17)

If the following simplification is also applied:

\[ (L_{mt1} + L_{mt2})/2 = 2\pi r_a \] (18)

Because we know:

\[ L_{mt1} = 2\pi r_{ave1} \] (19)

\[ L_{mt2} = 2\pi r_{ave2} \] (20)

Eqn. 17 is converted into Eqn. 5. It means that the flux element method is an approximation of the energy method.

### 3. SIMULATION

**Case 1: A single-phase transformer**

A small single-phase transformer with specifications given in Table 1 is simulated [7]. Image method result of this reference is used to compare the accuracy of different methods. As shown in Table 2, the error using the energy method is lower than that of the image method. Also the flux element method has larger error compared to two other methods.

<table>
<thead>
<tr>
<th>Method</th>
<th>Inductance (mH)</th>
<th>Error with respect to the test result (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test results</td>
<td>0.45</td>
<td>***</td>
</tr>
<tr>
<td>Energy method</td>
<td>0.430</td>
<td>2.42</td>
</tr>
<tr>
<td>Image method</td>
<td>0.4609</td>
<td>4.44</td>
</tr>
<tr>
<td>Flux method</td>
<td>0.4673</td>
<td>10.51</td>
</tr>
<tr>
<td>Data sheet</td>
<td>0.5475</td>
<td>21.68</td>
</tr>
</tbody>
</table>

**Table 1. Specifications of the Proposed Single-Phase Transformer**

<table>
<thead>
<tr>
<th>Power kVA</th>
<th>N1/N2</th>
<th>Voltage (V)</th>
<th>Lx (mm)</th>
<th>S1 (mm)</th>
<th>S2 (mm)</th>
<th>d1 (mm)</th>
<th>d2 (mm)</th>
<th>R (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>118/118</td>
<td>110</td>
<td>198</td>
<td>5</td>
<td>17</td>
<td>5</td>
<td>5</td>
<td>60</td>
</tr>
</tbody>
</table>

**Table 2. Computations Results Using Different Methods**

**Case 2: Three-phase distribution transformers**

3 three-phase distribution transformers with voltages 20/0.4 kV and connection group YZn5 with specification given in Table 3 is considered. Fig. 2 shows the required dimensions of the transformer for 25, 50, and 100 kVA transformers. The last column of Table 3 shows the result of short-circuit test of transformers.

**Table 3. Specifications of the Proposed Transformers**

<table>
<thead>
<tr>
<th>Power kVA</th>
<th>N1 (mm)</th>
<th>N2 (mm)</th>
<th>D1 (mm)</th>
<th>D2 (mm)</th>
<th>D3 (mm)</th>
<th>V/N</th>
<th>Zsc</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>116</td>
<td>5224</td>
<td>105</td>
<td>110</td>
<td>135</td>
<td>2.30</td>
<td>4.13</td>
</tr>
<tr>
<td>50</td>
<td>88</td>
<td>3962</td>
<td>111</td>
<td>116</td>
<td>161</td>
<td>3.03</td>
<td>4.09</td>
</tr>
<tr>
<td>100</td>
<td>68</td>
<td>3062</td>
<td>122</td>
<td>127</td>
<td>177</td>
<td>3.92</td>
<td>4.25</td>
</tr>
</tbody>
</table>

**Table 4. Computation Results for Transformers**

<table>
<thead>
<tr>
<th>Power kVA</th>
<th>Method</th>
<th>Impedance (%)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>Eq. (5)</td>
<td>6.56</td>
<td>58.7</td>
</tr>
<tr>
<td></td>
<td>Flux Method</td>
<td>4.37</td>
<td>5.7</td>
</tr>
<tr>
<td></td>
<td>Image Method</td>
<td>4.30</td>
<td>4.03</td>
</tr>
<tr>
<td></td>
<td>Energy Method</td>
<td>4.27</td>
<td>3.45</td>
</tr>
<tr>
<td></td>
<td>Test result</td>
<td>4.13</td>
<td>****</td>
</tr>
</tbody>
</table>
The result shows that energy method has the best accuracy comparing other methods for all of these distribution transformers.

**Case 3: A Single-phase High Voltage Test Transformer**

The result of simulation of a 500 kVA, 250kV test transformer illustrated in figure 3, which was designed by the authors and manufactured in Iran is shown in table 4. The schematic of windings are in figure 4. It is noticeable that the transformer has 3 windings: low voltage, high voltage and coupling (for energy transmission to upper step). The reactance between windings calculated two by two and the detail of calculation is in the appendix. The result shows that the energy method is also reliable in this case.

<table>
<thead>
<tr>
<th></th>
<th>Eq. (5)</th>
<th>LV – HV</th>
<th>HV- Coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>Flux Method</td>
<td>4.33</td>
<td>5.97</td>
</tr>
<tr>
<td></td>
<td>Image Method</td>
<td>4.23</td>
<td>3.47</td>
</tr>
<tr>
<td></td>
<td>Energy Method</td>
<td>4.20</td>
<td>2.81</td>
</tr>
<tr>
<td></td>
<td>Test result</td>
<td>4.09</td>
<td>****</td>
</tr>
<tr>
<td>100</td>
<td>Eq. (5)</td>
<td>5.086</td>
<td>19.71</td>
</tr>
<tr>
<td></td>
<td>Flux Method</td>
<td>4.416</td>
<td>3.94</td>
</tr>
<tr>
<td></td>
<td>Image Method</td>
<td>4.358</td>
<td>2.57</td>
</tr>
<tr>
<td></td>
<td>Energy Method</td>
<td>4.340</td>
<td>2.16</td>
</tr>
<tr>
<td></td>
<td>Test result</td>
<td>4.248</td>
<td>****</td>
</tr>
</tbody>
</table>

Fig 3- A 250kV, 500 kVA test transformer designed and manufactured in IRAN

Fig 4- Schematic of windings of the test transformer

<table>
<thead>
<tr>
<th>Impedance %</th>
<th>LV – HV</th>
<th>HV - Coupling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux Method</td>
<td>6.12</td>
<td>22.5</td>
</tr>
<tr>
<td>Image Met</td>
<td>6.05</td>
<td>22.6</td>
</tr>
<tr>
<td>Energy Met</td>
<td>5.83</td>
<td>22.81</td>
</tr>
<tr>
<td>Test result</td>
<td>5.85</td>
<td>22.74</td>
</tr>
</tbody>
</table>

4. CONCLUSIONS

Different analytical methods for the leakage inductance of transformer calculation have been compared. It has been shown that the energy method is the most accurate one. Although the image method is accurate and convenient.
method, it depends on the current of the image conductors. So by calculating stored electromagnetic energy in windings and distance between them leakage reactance will be calculated simply and accurately comparing to test result.

5. APPENDIX:
CALCULATION OF LEAKAGE REACTANCE OF TEST TRANSFORMER

Because of trapezoid shape of HV winding of test transformer according to Fig. 5, the formula of 10 will be changed:

\[ W_2 = \pi \mu_n \frac{N_1^2 I_1^2}{2} \left[ (r_2 + 3kLC_2)(LC_1 + LC_2) + \frac{2LC_1}{3} (r_2 + 1.5kLC_1) \right] \]

\[ = \frac{k}{3} (LC_1^2 + LC_2^2 + LC_1LC_2) + \frac{(r_2 + kLC_2)LC_2^2}{LC_1 - LC_2} - \frac{LC_1}{LC_2} \]

(A1)

![Figure 5- Dimensions of windings of the test transformer](image)

Considering Leakage reactance between LV & HV:

\[ X_{eq1} = \frac{4\pi \mu_n N_1^2}{L_1} \left[ \frac{2}{3} \frac{dw_1}{4} (r_2 + 3kLC_1)(LC_1 + LC_2) + \frac{1}{2} \frac{dw_2}{4} (r_2 + 1.5kLC_1) \right] \]

\[ - 2LC_1 (r_2 + 1.5kLC_1) - \frac{k}{3} (LC_1^2 + LC_2^2 + LC_1LC_2) + \frac{(r_2 + kLC_2)LC_2^2}{LC_1 - LC_2} - \frac{LC_1}{LC_2} \]

(A2)

6. REFERENCES