TRANSFORMER MODELLING FOR SIMULATION OF LOW FREQUENCY TRANSIENTS IN POWER SYSTEMS

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SUMMARY

This paper presents a review of models proposed to date for representing transformers in low- and mid-frequency transients. This task is part of an ongoing project aimed at developing more reliable transformer models. The document presents a classification of the most popular models and discusses guidelines for representation of non-linear and frequency dependent phenomena associated with transients below the first winding resonance.

INTRODUCTION

The development of an accurate transformer model can be very complex due to the large number of core designs and to the fact that several transformer parameters are both non-linear and frequency dependent. Physical attributes whose behaviour may need to be correctly represented are core and coil configurations, self- and mutual inductances between coils, leakage fluxes, skin effect and proximity effect in coils, magnetic core saturation, hysteresis and eddy current losses in core, and capacitive effects. Models of varying complexity have been developed and implemented in simulation tools to duplicate the transient behaviour of transformers.

This paper is aimed at presenting the state-of-the-art on transformer models for simulation of low frequency transients, that is, phenomena well below the first winding resonance (several kHz). They include ferroresonance, most switching transients, and harmonic interactions. The first section presents a summary of the main models proposed for representation of power transformers in low- and mid-frequency power system transients. The representation of non-linear and frequency-dependent parameters, which can have a significant influence on transformer behaviour, is analysed in the subsequent section. Several issues related to the development of a transformer model are discussed in the last section.

TRANSFORMER MODELS

This section presents a summary of the main principles and the assembly equations of transformer models. They have been classified into three groups. The first group uses either a branch impedance or admittance matrix. The second group is an extension of the Saturable Transformer Component model to multi-phase transformers. Both types of models have been implemented in the EMTP and both of them have important limitations for simulating some core designs. Topology-based models form a larger group for which many approaches have been proposed. Their derivation is performed from the core topology and can represent very accurately any type of core design in low- and mid-frequency transients if parameters are properly determined.

This classification does not cover all transformer models, but the most accurate models and those already implemented in a transients program.

Matrix representation

The steady state equations of a multi-phase multi-winding transformer can be expressed using the branch impedance matrix, \([V]=[Z][I]\). \([Z]\) is symmetric and its elements can be derived from excitation tests. There could be some accuracy problems with the above calculations since the branch impedance matrix can become ill-conditioned for very small exciting currents or when they are totally ignored. In addition, the short-circuit impedances get lost in such excitation measurements. To solve these problems an admittance matrix representation should be used \([I]=[Y][V]\). \([Y]\) elements can be obtained directly from standard short-circuit tests.

In transient calculations, the transformer equation must be rewritten as

\[
[v] = [R] \cdot [i] + [L] \cdot \frac{di}{dt}
\]

(1)

where \([R]\) and \(j\omega[L]\) are respectively the real and the imaginary part of \([Z]\). In case of a very low excitation current, the transformer should be described by the following equation

\[
\frac{di}{dt} = [L]^{-1}[v] - [L]^{-1}[R][I]
\]

(2)

Both approaches have been implemented in the EMTP and are derived by using the supporting routine BCTRAN [1]. The first model, equation (1), is known as \([R]−\omega[L]\) option, while the second one, equation (2), is known as \([A]−[R]\) option.

Phase-to-phase couplings, as well as the terminal characteristics, are included with these approaches, but they do not consider differences in core or winding topology, since all core designs get the same mathematical treatment. All of these models are linear; however, for many transient studies it is necessary to include saturation and hysteresis effects. Exciting current effects can be linearized and left in the matrix description, which can lead to simulation errors when the core saturates. Alternately, excitation may be omitted from the matrix description and attached externally at the models terminals in the form of non-linear elements, see
The matrix product leg of the core, interphase magnetic coupling, and leakage models that include the effects of saturation in each individual using the principle of duality [3], [4]. This approach results in effects. In the equivalent magnetic circuit, windings appear as Duality-based models.

Topologically correct equivalent is symmetric, which is not true in the general case. Saturation and hysteresis effects can be modelled by adding an extra non-linear inductor at the star point. This model can be extended to three-phase units through the addition of a zero-sequence reluctance parameter, but its usefulness is limited. The input data consists of the R-L values of each star branch, the turn ratios and the information for the magnetising branch. This model has some limitations since it cannot be used for more than 3 windings, the linear/non-linear magnetising inductance in parallel, is connected to the star point, which is not always the correct topological connecting point, and numerical instability has been reported for the 3-winding case.

Topology-based models

Duality-based models. Topologically correct equivalent circuit models can be derived from a magnetic circuit model using the principle of duality [3], [4]. This approach results in models that include the effects of saturation in each individual leg of the core, interphase magnetic coupling, and leakage effects. In the equivalent magnetic circuit, windings appear as MMF sources, leakage paths appear as linear reluctances, and magnetic cores appear as saturable reluctances. The mesh and node equations of the magnetic circuit are duals of the electrical equivalent node and mesh equations respectively. To make models practically useful, the current sources resulting from the transformation are replaced with ideal transformers to provide primary-to-secondary isolation and coupling to the core, while preserving the overall primary to secondary turns ratios. Turns ratios are chosen so that core parameters are referenced to the low voltage winding. The portion of a model inside the coupling transformers represents the core and leakages. Winding resistances and interconnection of the windings appears external to the coupling transformers. The advantage of this is that the derived core equivalent functions independently of winding configuration. Winding resistance, core losses, and capacitive coupling effects are not obtained directly from the transformation, but can be added to this equivalent electrical circuit. Fig. 3 shows the equivalent circuit of a single-phase shell form transformer, with concentric windings, derived using this methodology. Keywork performed during the last years is listed below.

- In 1981 Dick and Watson presented the derivation of the model of a three-legged stacked core transformer [5]. The main contributions of their work was the proposal of a new hysteresis model and the determination of transformer parameters from measurements.
- In 1991 Arturi applied this technique for representing a five-legged step-up transformer working in highly saturated conditions [6].
- In 1994 De León and Semlyen proposed a very complete transformer model that was derived from a hybrid approach, a combination of duality, which was used to obtain the iron core model, and their own technique for calculation of leakage inductances [7].
- In 1994 Narang and Brierley used duality to derive the equivalent circuit of the magnetic core, which is interfaced by means of a three-phase fictitious winding to an admittance matrix that represents the correct magnetic coupling among windings [8].
- In 1999 Mork presented the derivation of a five-legged wound core transformer model, which was validated by duplicating ferroresonant phenomena [9].

Geometric models. This concept is used to name those models based on a mathematical formulation for which the magnetic equations and their coupling to the electrical equations is made taking into account the core topology. The general mathematical form for all of them can be expressed as follows

\[ [v] = [R] [i] + [di / dt] \]

A short summary of the main models is presented below.

- The coupled magnetic model was developed by Yacamini and Bronzeado [10]. Because the permeability of the ferromagnetic elements varies with flux density, the magnetic material is divided into sections, each of which has a substantially uniform flux density. The link between magnetic equations, \( F = \Phi \), and (4) is the Ampere’s law, \( F = Ni \).
- The Unified Magnetic Equivalent Circuit model, proposed by Arrillaga et al. [11], uses the normalised core concept for derivation of the inductance matrix. Leakage permeances can be obtained from open and short-circuit tests; the effective lengths and cross-sectional areas of their leakage paths are not required.
- GMTRAN was developed by Hatziargyriou, Prousalidis and Papadias [12]. The magnetic equations were included in (4) by means of the inductance matrix, \( \lambda = [L][i] \).
- SEATTLE XFORMER was developed by Chen [13]. Flux linkages were chosen in this model as state variables. That is, the magnetic equations in (4) are included by means of the relationship \( [i] = [T][\lambda] \).

Table I presents a summary of equations and characteristics of the models detailed in this section.
Winding Leakages
BCTRAN Model

Transformer Core Equivalent

Fig. 1. BCTRAN model for a three-phase three-legged stacked core transformer (iron core is attached externally).

Fig. 2. Star-circuit representation of single-phase N-winding transformers.

Fig. 3. Duality-derived model for a single-phase shell-form transformer.

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<th>MODEL</th>
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| Matrix Representation (BCTRAN model) | • $[v] = [R][i] + [L][d i / d t]$ | • These models include all phase-to-phase coupling and terminal characteristics.  
• Only linear models can be represented.  
• Excitation may be attached externally at the terminals in the form of non-linear elements.  
• They are reasonable accurate for frequencies below 1 kHz. |
| Saturable Transformer Component (STC model) | • $[L]^{-1}[v] = [L]^{-1}[R][i] + [d i / d t]$ | • It cannot be used for more than 3 windings.  
• The magnetising inductance is connected to the star point.  
• Numerical instability can be produced with 3-winding models. |
| Topology-based models | • Duality-based models: They are derived using a circuit-based approach without a mathematical description  
• Geometric models | • Duality-based models include the effects of saturation in each individual leg of the core, interphase magnetic coupling, and leakage effects.  
• The mathematical formulation of geometric models is based on the magnetic equations and their coupling to the electrical equations, which is made taking into account the core topology. Models differ from each other in the way in which the magnetic equations are derived. |
NONLINEAR AND FREQUENCY-DEPENDENT PARAMETERS

Some transformer parameters are non-linear and/or frequency dependent due to three major effects: saturation, hysteresis and eddy currents. Saturation and hysteresis introduce distortion in waveforms; hysteresis and eddy currents originate losses. Saturation is the predominant effect in power transformers, but eddy current and hysteresis effects can play an important role in some transients.

Modelling of iron cores

Iron core behaviour is usually represented by a relationship between the magnetic flux density $B$ and the magnetic field intensity $H$. A general difficulty in modelling the magnetisation curves is the fact that each of the magnetic field values is related to an infinity of possible magnetisations depending on the history of the sample. To characterise the material behaviour fully, a model has to be able to plot numerous associated curves, see Fig. 4. A major hysteresis loop is the largest possible loop whose ends enter into technical saturation. Any other closed loop is called a minor loop with a distinction also being made between symmetric and asymmetric minor loops.

Hysteresis can be caused by several types of phenomena being the dominant cause dependent on the material. Detailed models of hysteresis loops based on a rigorous physical basis are too time-consuming, so most practical approaches are curve fits that ignore the underlying physics of the material behaviour [14]. For a review of hysteresis models see [14] and [15].

Hysteresis loops usually have a negligible influence on the magnitude of the magnetising current, although hysteresis losses can have some influence on some transients. The residual flux has a major influence on the magnitude of inrush currents.

Magnetic saturation of an iron core can be represented by the anhysteretic curve, the $B–H$ relationship that would be obtained if there was no hysteresis effect in the material. The saturation characteristic can be modelled by a piecewise linear inductance with two slopes. Increasing the number of slopes does not improve significantly the accuracy. The slope in the saturated region above the knee is the air-core inductance, which is almost linear and very low compared with the slope in the unsaturated region. The specification of this inductor requires a curve relating the flux linkage, $\lambda$, to the current, $i$. The information usually available is the rms voltage as a function of the rms current.

In transients simulations, an iron core, with or without hysteresis, can be represented by the equivalent circuit shown in Fig. 5 [16]. It is similar to the equivalent circuit of a linear inductor; however, the resistance depends on the segment slope and needs to be updated with changes of the operating segment. This requires partial retriangularization of the nodal conductance matrix. The current source consists of the past recorded history and must be updated every time step, as for a linear inductor.

Eddy current effects

Several physical phenomena occur simultaneously in a loaded transformer that result in a nonuniform distribution of current in the conductors, and manifest themselves as an increase in the effective resistance and winding losses with respect to those for direct current.

The magnetisation curves presented above are only valid for slowly varying phenomena, as it has been assumed that the magnetic field can penetrate the core completely. In general this is not always true. A change in the magnetic field induces eddy currents in the iron. As a consequence of this, the flux density will be lower than that given by the normal magnetisation curve. As frequency changes, flux distribution in the iron core lamination changes. For high frequencies the flux will be confined to a thin layer close to the lamination surface, whose thickness decreases as the frequency increases. This indicates that inductances representing iron path magnetisation and resistances representing eddy current losses are frequency dependent [17]. The circulation of these eddy currents introduces additional losses. To limit their influence, a transformer core is built up from a large number of parallel laminations.

Excitation losses are mostly iron-core losses. These losses consist of hysteresis and eddy current losses. They cannot be separated, although in modern transformers hysteresis losses are much smaller than eddy current losses.

Eddy current models for transformer windings. Foster equivalent circuits have been selected to represent
frequency dependence of the windings, see Fig. 6. These circuits must be of infinite order to exactly reproduce the impedance at all frequencies; however, a computationally efficient circuit can be derived by fitting only at certain pre-established frequencies. Several fitting procedures to determine circuit parameters have been developed, see for instance [18]. For practical studies a series model of order 2 or 3 is adequate. Such a model neglects displacement currents, so it is valid at frequencies below the first winding resonance, that is, up to tens of kHz.

**Eddy current models for iron laminated cores.** Efficient models intended for simulation of the frequency dependence of the magnetising inductance as well as losses have been derived by synthesising Cauer equivalent circuits to match the equivalent impedance of either a single lamination or a coil wound around a laminated iron core limb.

a) The accuracy of the standard Cauer representation over a defined frequency range depends on the number of sections, see Fig. 7a. To represent the frequency range up to 200 kHz with error less than 5%, only four terms are required [19]. The first section governs its characteristics at frequencies up to a few kHz; each subsequent section comes into play as the frequency increases.

b) In the dual Cauer model, see Fig. 7b, the inductances represent the flux paths and the resistances are in the path of eddy currents. The high frequency response is defined by the blocks near the terminals. The blocks of this model can be thought as being a discretization of the lamination. Errors smaller than 1% up to a frequency of 200 kHz can be obtained with model order of 4.

Inductive components of the models representing the magnetising reactances have to be made non-linear to account for the hysteresis and saturation effects. Since the inductances and resistances in this model do not represent any physical part of the iron lamination it is not obvious how to incorporate the effects. However, since the high frequency components do not contribute appreciably to the flux in the transformer core, it can be assumed that only low frequency components are responsible for driving the core into saturation. It may, therefore, be justifiable to represent as non-linear only the first section of the model.

**DISCUSSION**

a) A model for representing transformers in low- and mid-frequency transients has to incorporate accurate representation of the transformer core, leakage inductances, eddy current effects in windings and core, saturation and/or hysteresis effects. Models for three-phase core designs must be based on the core topology. Terminal capacitances can also have an important effect in some transients [20]. The development of a model that could include all these effects and could be used in any transient simulations for frequencies below 10 kHz is not an easy task. The computational burden for the most complex core design would be considerable, and the accurate determination of all parameters a very complex task. To date the most complete model was developed by De León and Semlyen [7].

b) An important issue for any transformer model is the nodes to which the core equivalent must be connected. It may be unimportant to which node an unsaturated inductance is connected to in the single-phase model shown in Fig. 2, but it may make a difference when the inductance is saturated, because of its low value. Ideally, the non-linear inductance should be connected to a point where the integrated voltage is equal to the iron-core flux. To identify that point requires design details not normally available.

c) Simplified models could be accurate enough for simulating some transients. There are many low frequency transients in which terminal capacitances will not play any important role. Hysteresis can be reduced to a single-valued saturation curve in those cases for which the residual flux has no effect and the representation of losses is not critical. Eddy current effects in windings and iron core can be represented by low order equivalent circuits for a frequency range below 10 kHz, usually a model of order 2 or 3 will suffice.

d) Data usually available for any power transformer are: power rating, voltage rating, excitation current, excitation voltage, excitation losses, short-circuit current, short-circuit voltage, short-circuit losses, saturation curve, capacitances between terminals and between windings. Excitation and short-circuit currents, voltages and losses must be provided from both direct and homopolar measurements. Procedures for determining transformer parameters from standard tests have been proposed, but depending on the model selected for representing some effects, the calculation of some parameters is not covered, and additional information is usually required. The specification of some parameters can be a bottleneck due

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**Fig. 6.** Series Foster equivalent circuit for windings.

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**Fig. 7.** Cauer equivalent circuits for iron cores.
to the lack of reliable procedures for their determination. Works related to parameter determination are listed below.
• For the calculation of leakage inductances from standard test values see [1]. Other contributions in this field can be found in [21] and [22]. The last reference includes a procedure for calculation of capacitances.
• Contributions on the calculation of parameters to be specified in duality-based models were made by Dick and Watson [5], Stuehm [23] and Narang and Brierley [8]. See also references [6] and [9].
• The influence of eddy current losses and the determination of resistances as a function of frequency have been studied by Fuchs et al. [24].
• For the determination of the saturation characteristic see [25] – [28], as well as the discussion of [26].
• The determination of hysteresis parameters is very dependent on the selected hysteresis model [29].
e) Temperature influence should not be neglected, either in laboratory tests or in field measurements.

CONCLUSIONS

This paper has presented a summary of the most important issues related to transformer modelling for simulation of transients below 10 kHz. Important difficulties are the great variety of three-phase transformer core designs, the non-linear and frequency dependent behaviour of many transformer parameters, as well as the acquisition and determination of some transformer parameters. The development of an accurate transformer model is a sophisticated work; however, several modelling levels could be considered since not all parameters have the same influence on all transient phenomena.

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