USING SMALL SIGNAL FREQUENCY DOMAIN MODELLING FOR PREDICTING HARMONIC AND INTERHARMONIC DISTORTIONS GENERATED BY TWO BACK-TO-BACK PWM CONVERTERS

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1. ABSTRACT

This paper describes an approach to modelling power electronic devices and power distribution networks in the frequency domain using frequency coupling matrices. The benefits of this modelling approach will be discussed by focussing on the small signal frequency domain model of two back-to-back PWM converter feeding a simple L/R load. The set-up is presented schematically in Fig. 1.1. Additionally, the effect of AC side impedance unbalance is addressed briefly. The time domain simulation program SIMPLORER was used to validate the results of this modelling approach.

2. INTRODUCTION

Understanding the generation and transfers of harmonics is essential for predicting the impact and interactions of numerous power electronic devices, such as variable speed AC drives, active filters or switched mode power supplies and for meeting the statutory demands [1]. Consequently, accurate modelling of these interactions is required. The modelling approach presented in this paper is based on linearisation of the system transfers around a nominal operating point, which results in a linear time periodic (LTP) model. The modelling takes place in the frequency domain and uses Frequency Coupling Matrices which incorporate the non-linear and time variant behaviour and the inherent frequency coupling of most power electronic devices. The visualisation of the transfers and the reduction to the most important frequencies provides a deep insight into the generation and transfer of distortions and is in excellent agreement with time domain simulations. Furthermore, this simulation technique is extremely fast compared to the well-established time domain simulations such as SIMPLORER and avoids difficulties that arise from time domain data analysis.

The modelling approach was first mentioned by Larsen et al. for thyristor controlled AC/DC systems [2]. In [3] occurring resonances in a railway system were investigated by means of frequency coupling matrices. A model in the frequency domain of a synchronous generator is presented in [4]. Smith describes a harmonic domain model for predicting the interactions of HVDC converters with AC and DC systems [5]. A model for a fully controlled PWM converter is derived in [6].

The parameters used for modelling are somehow arbitrary and are summarised in table I. However, the switching frequencies, base frequencies and R/L values are the same on the load and grid side to allow easier understanding of the modelling technique.

<table>
<thead>
<tr>
<th>TABLE I Electrical parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>load side</td>
</tr>
<tr>
<td>modulation index              ( m_{sl} = 0.5 )</td>
</tr>
<tr>
<td>switching frequency           ( f_{sl} = 2200,\text{Hz} )</td>
</tr>
<tr>
<td>frequency modulation ratio    ( m_{f1} = \frac{f_{f1}}{f_{f1}} = 44 )</td>
</tr>
<tr>
<td>base frequency                ( f_{base1} = 50,\text{Hz} )</td>
</tr>
<tr>
<td>Resistance                    ( R_L = 1,\Omega )</td>
</tr>
<tr>
<td>Inductance                    ( L_1 = 1,\text{mH} )</td>
</tr>
<tr>
<td>DC bus</td>
</tr>
<tr>
<td>nominal voltage               ( V_{dc} = 650,\text{V} )</td>
</tr>
<tr>
<td>Resistance                    ( R_{dc} = 100,\text{k}\Omega )</td>
</tr>
<tr>
<td>Capacitance                   ( C_{dc} = 2.2,\text{\mu F} )</td>
</tr>
<tr>
<td>grid side</td>
</tr>
<tr>
<td>modulation index              ( m_{s2} = 0.5 )</td>
</tr>
<tr>
<td>switching frequency           ( f_{s2} = 2200,\text{Hz} )</td>
</tr>
<tr>
<td>base frequency                ( f_{base2} = 50,\text{Hz} )</td>
</tr>
<tr>
<td>frequency modulation ratio    ( m_{f2} = \frac{f_{f2}}{f_{base}} = 44 )</td>
</tr>
<tr>
<td>Resistance                    ( R_2 = 1,\Omega )</td>
</tr>
<tr>
<td>Inductance                    ( L_2 = 1,\text{mH} )</td>
</tr>
</tbody>
</table>

Due to the absence of zero sequence components in a three phase three wire system, the modelling will take place with sequence components instead of the three phase components. Details to this transfer can be found in [7], and it is given in

\[
\begin{bmatrix}
  v_0 \\
  v_p \\
  v_n
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
1 & 1 & 1 \\
1 & a & a^2 \\
1 & a^2 & a
\end{bmatrix} \begin{bmatrix}
v_a \\
v_b \\
v_c
\end{bmatrix}
\]

(2.1)
A schematic representation of the circuit in sequence components is shown in Fig. 2.1 and is used for modelling.

Fig. 2.1 Circuit schematic in sequence components

The modelling approach followed in this paper is twofold and requires some iterations. The set-up is parted along the dashed line in Fig. 2.1. Firstly the base case quantities of the load and its converter are analysed yielding characteristic harmonics on the load side. Once their currents and voltage quantities are established the base case quantities of the grid side converter are analysed yielding characteristic harmonics on the grid side. Once all the base case quantities and characteristic harmonics are established the non-characteristic harmonics can be found. Another approach using one matrix to describe the whole system was proposed by Saniter et al. [8].

3. FREQUENCY COUPLING MATRICES FCM

For each component the model will be derived for one operating point and then small changes (e.g. perturbations) are considered. A sparse linear equation set in the form

\[ \mathbf{b} = [\mathbf{A}] \mathbf{x} \]  

(3.1)
is used in this approach to modelling and describes the transfers from input quantities to output quantities. This linear equation set consists of an input vector \( \mathbf{x} \) of \( n \) knowns, an output vector \( \mathbf{b} \) of \( n \) unknowns and an \( n \times n \) coefficient matrix \([\mathbf{A}]\) describing the linearised time variant and frequency cross coupling nature of the components around an operating point. The size of the matrix \([\mathbf{A}]\) depends on the frequency range and the incremental step required. The matrix \([\mathbf{A}]\) is called Frequency Coupling Matrix (FCM) and contains the (complex) Fourier-coefficients of the transfers. Further details can be found in [6].

4. THE FCMS OF THE COMPONENTS

A detailed analysis of the fully controlled PWM converter can be found in [6]. The FCM of the converter is built up from individual transfers, which relate a distortion in the DC voltage and the AC current to the AC voltage and the DC current respectively. Thus, an input/output model as in Fig. 4.1 is obtained. The individual FCMs are obtained by linearising the converter around a base operating point.

\[
\begin{bmatrix}
 v_p \\
 v_n \\
 i_{dc}
\end{bmatrix}
=
\begin{bmatrix}
 0 & 0 & T_{13} \\
 0 & 0 & T_{23} \\
 T_{31} & T_{32} & 0
\end{bmatrix}
\begin{bmatrix}
 i_p \\
 i_n \\
 v_{dc}
\end{bmatrix}
\]  

(4.1)

The FCM and how the base case quantities and characteristic harmonics can be found is visualised in Fig. 4.2 where a dot stands for a non-zero element resulting in the bold diagonals.

For a symmetrical, balanced L/R load the transfer is described by

\[
\begin{bmatrix}
 v_p \\
 v_n \\
 i_{dc}
\end{bmatrix}
=
\begin{bmatrix}
 0 & 0 & T_{13} \\
 0 & 0 & T_{23} \\
 T_{31} & T_{32} & 0
\end{bmatrix}
\begin{bmatrix}
 i_p \\
 i_n \\
 v_{dc}
\end{bmatrix}
\]  

(4.2)

The effects of ac system impedance unbalance are addressed briefly in section 6.

The dc bus is described by

\[
v_{dc} = Z_{dc} \cdot i_{dc}
\]  

(4.4)
with
\[ Z_{dc} = \frac{R_{dc}}{1 + j \omega C_{dc} R_{dc}}. \]  \hspace{1cm} (4.5)

The FCM of the load is visualised in Fig. 4.3.

Fig. 4.3 FCM of the L/R load

5. Harmonics of the system

Under normal operating conditions, i.e. a balanced ac impedance and a perfect dc voltage the characteristic harmonics can be found. They are shown for the ac side currents in sequence components in Fig. 5.1. The comparison with the SIMPLORER simulations show a high degree of conformity.

Fig. 5.1 Returned spectrum of the ac side current (amplitudes), 1st figure: SIMPLORER, 2nd and 3rd figures: positive and negative sequence currents as a result of the proposed modelling approach

Once the ac side currents are found the dc side currents are obtained by solving (4.1) yielding the spectrum in Fig. 5.2.

Fig. 5.2 Returned spectrum of the dc side current, 1st figure: as a result of the proposed modelling approach, 2nd figure: SIMPLORER

At this stage it is possible to analyse the grid side converter and its circuit. A suitable representation is shown in Fig. 5.3 where \( v'_{p2} \) and \( v'_{n2} \) represent the grid voltages in sequence components, \( i'_n \) represents a current source, that is supplying the power to the load, \( Z_{dc} \) is the dc bus capacitor including a large shunt resistance and \( Z_{p2} \) and \( Z_{n2} \) are the grid impedances in sequence components. The dc current that is resulting from the load side is the current source and only a 50Hz positive sequence grid voltage is present to get the characteristic harmonics generated by this converter.

Fig. 5.3 Grid side converter circuit, schematic

The system can be solved using the equation
\[
\begin{bmatrix}
    v'_{p2} \\
    0 \\
    i'_n \\
    v'_{dc}
\end{bmatrix}
= 
\begin{bmatrix}
    -Z_{p2} & 0 & T_{13} \\
    0 & -Z_{n2} & T_{23} \\
    T_{31} & T_{32} & -Y_{dc}
\end{bmatrix}
\begin{bmatrix}
    i_{p2} \\
    i_{n2} \\
    v_{dc}
\end{bmatrix}
\] \hspace{1cm} (5.1)

The independent sources (knowns) are on the left hand side and the dependent variables (unknowns in the circuit) are on the right hand side. When the matrix is inverted,
\[
\begin{bmatrix}
    i_{p2} \\
    i_{n2} \\
    v'_{dc}
\end{bmatrix}
= 
\begin{bmatrix}
    v'_{p2} \\
    0 \\
    i'_n \\
    v'_{dc}
\end{bmatrix}
\begin{bmatrix}
    -Z_{p2} & 0 & T_{13} \\
    0 & -Z_{n2} & T_{23} \\
    T_{31} & T_{32} & -Y_{dc}
\end{bmatrix}^{-1}
\] \hspace{1cm} (5.2)
the ac side currents and dc side voltage are calculated for a given control signal embodied in the transfers $T_{13}, T_{23}, T_{31}$ and $T_{32}$ and a given dc side current. However, to establish the characteristic harmonics, the harmonics in the dc side current are neglected in a first step. This yields the spectrum for the grid side ac currents in Fig. 1.1.

The additional non-characteristic distortions on the load side can be calculated by including additional harmonic components in the input vector of $v_{dc}$ in (4.1) yielding additional non-characteristic harmonics in the dc current. Including the characteristic and non-characteristic harmonic into the dc current and solving

$$\begin{bmatrix}
\Delta i_{p2} \\
\Delta i_{n2} \\
\Delta v_{dc}
\end{bmatrix} = [H]^{-1} \begin{bmatrix}
0 \\
0 \\
\Delta v_{dc}
\end{bmatrix}$$

yields the spectrum in Fig. 5.5 with the non-characteristic harmonics for the grid side ac current.

The visualisation of $[H]^{-1}$ helps again the understanding of the generation of harmonics. However, the gain of a certain harmonic can not be predicted by just looking at the visualisation. This can be demonstrated, by visualising the matrix $[H]^{-1}$ with all elements (or transfers) whose absolute value is larger than $10^{-20}$ in Fig. 5.6, or with elements only, with an absolute value larger than $10^{-5}$ in Fig. 5.7. The visualisation only aids the understanding of “where to expect harmonics”.

The effect that (characteristic and non-characteristic) harmonics will cause additional harmonics will continue infinitely. However, due to damping effects of the dc bus capacitance and the ac side inductances, the effect will cease rather quickly. This can already be seen in the very small contributions of the additional dc bus harmonics.
compared to the base case quantities in the grid side ac current.  
By considering the absolute values of the currents in Fig. 5.2, Fig. 5.5 or Fig. 5.5 one is easily led to wrong assumptions. It is important to look not only at the absolute values of the currents and voltages but at the correct phase angle as well as it is demonstrated with the following rough calculations for the real power.

1. The real power on the load side is
   \[ P_{\text{load}} = 3 \cdot I_{ac}^2 \cdot R \approx 3 \left( \frac{1}{\sqrt{2}} \cdot 155,0 A \right)^2 \cdot 1 \Omega = 36,0 k\text{W} \]

2. Neglecting the large shunt resistor the power in the dc bus is given by
   \[ P_{dc} \approx I_{dc} \cdot V_{dc} \approx 55,5 A \cdot 650 V = 36,1 k\text{W} \]

3. The power on the grid side is given by
   \[ P_{\text{grid}} = \left( 3 \cdot I_{a} + 3 \cdot I_{b} + 3 \cdot I_{c} \right) \cdot R \]
   \[ \approx 3 \cdot R \left( \frac{1}{\sqrt{2}} \cdot 325 V e^{90^\circ} + \frac{1}{\sqrt{2}} \cdot 174,5 V e^{72,5^\circ} \right) + 3 \cdot I_{ac}^2 \cdot R \]
   \[ \approx 9R \left( 85,1 kVA e^{162,5^\circ} \right) + 45,7 k\text{W} \]
   \[ \approx -81,3 k\text{W} + 45,7 k\text{W} \]
   \[ = -35,6 k\text{W} \]

It is important to notice, that the whole simulation is carried out in a cosine reference frame resulting in the phase angle of 90° for the grid voltage.

6. AC SIDE IMPEDANCE UNBALANCE

Taking the effect of ac side impedance unbalance into account (4.2) has to be evaluated again. Equation (4.2) was obtained by transforming

\[
\begin{bmatrix}
  v_a \\
  v_b \\
  v_c \\
  i_a \\
  i_b \\
  i_c \\
\end{bmatrix} = \begin{bmatrix} Z_a & 0 & 0 \\
  0 & Z_b & 0 \\
  0 & 0 & Z_c \\
\end{bmatrix} \begin{bmatrix} i_a \\
  i_b \\
  i_c \\
\end{bmatrix}
\]

(6.1)

into sequence components assuming balanced loads. Without this assumption (6.1) becomes

\[
\begin{bmatrix}
  v_p \\
  v_n \\
\end{bmatrix} = \begin{bmatrix} M \end{bmatrix} \begin{bmatrix} i_p \\
  i_n \\
\end{bmatrix}
\]

\[
[M] = \frac{1}{3} \begin{bmatrix} Z_a + Z_b + Z_c & Z_a + aZ_b + aZ_c & Z_a + a^2Z_b + aZ_c \\
  Z_a + aZ_b + aZ_c & Z_a + Z_b + Z_c & Z_a + aZ_b + aZ_c \\
  Z_a + a^2Z_b + aZ_c & Z_a + aZ_b + aZ_c & Z_a + Z_b + Z_c \\
\end{bmatrix}
\]

(6.2)

for positive and negative sequence components.

The matrix \([M]\) can be reduced to

\[
[M] = \frac{1}{3} \begin{bmatrix} Z_{p2p} & Z_{n2p} \\
  Z_{p2n} & Z_{n2n} \\
\end{bmatrix}
\]

(6.3)

To investigate the effects of ac side impedance unbalance in more detail the following relationship between the phase impedances is assumed

\[ Z_b = Z_c = x \cdot Z_a \]

(6.4)

where \(x\) is any complex number with its absolute value between 0 and 1 (i.e. 100%) and a phase angle between \(\phi = 90^\circ\) leading and \(\phi = 90^\circ\) lagging. To appreciate the effect of ac side impedance unbalance the two transfers \(Z_{p2p}\) and \(Z_{p2n}\) are shown in Fig. 6.1 and Fig. 6.2.

From (6.2), (6.3) and Fig. 6.2, Fig. 6.1 can be seen, that characteristic and non-characteristic harmonics are not only a result of the switching action of the two converters but can be a result of ac side impedance unbalance, too. The additional harmonics result mainly from cross-modulation of the base case quantities. This is based on the effect, that the base case fundamental current cross-modulates through the unbalanced impedance to a negative sequence fundamental.

7. CONCLUSION

In this paper some mechanisms that lead to the generation of characteristic and non-characteristic harmonics of two back-to-back converters are analysed. It is shown, that the characteristic and non-characteristic harmonics and interharmonics result from three main mechanisms:
1. Harmonics generated by the switching action of one converter only resulting in characteristic harmonics. This mechanism leads to harmonics that may have significant amplitudes, depending on the modulation index.

2. Non-characteristic harmonics and interharmonics generated by one converter as a result of the switching action of the second converter. This mechanism leads to non-characteristic harmonics if both converters operate at the same switching frequency and to interharmonics if they operate at different switching frequencies. This mechanism leads generally to distortions with rather small amplitudes.

3. Non-characteristic harmonics and interharmonics due to AC side impedance unbalance. Depending on the degree of unbalance this mechanism leads to distortions which may have significant amplitudes.

Further mechanisms of harmonic generation is not investigated in this paper. However, it is obvious that any additional disturbing source (e.g. negative sequence grid voltage in (5.2)) could be included.

Additionally, the proposed modelling approach is extremely fast due to the sparse matrices.

8. REFERENCES

[1] EN 61000-2-4, DIN VDE 0839 Teil 2-4, IEC1000-2-
4, Verträglichkeitspegel für niederfrequente leitungsgeführte Störgrößen in Industrieanlagen, 1994


