INTRODUCTION

This paper deals with the subband coding (wavelet multiresolution analysis) signal processing tool on corrected PSCAD/EMTDC® simulations and some results of a full scale earth fault experiment carried out on the Petersen-Coil compensated 10 kV research/laboratory distribution network at Køndbyvaerket, Denmark. The PSCAD/EMTDC® simulation model is equivalent to the research/laboratory network at Køndbyvaerket, build on 25 corrected cable models and 4 line models in a radial network of approximately 7.4 km length connected to four distribution transformers. The purpose is to show that the transient sequence of currents and voltages not exceed a bandwidth of approximately 10 kHz, for which signal measurements on conventional current transformers (CT’s) and voltage transformers (VT’s) is sufficient. Further analysis with the Prony signal processing tool will show the connexion of the earth fault transients and the Prony estimated exponential damped sinusoids.

BACKGROUND

Grounding systems in power systems play an essential role in personal safety and equipment protection. For Medium-Voltage systems the grounding method is somewhat sophisticated [4] since it for this voltage level is possible to connect the power system isolated, high or low resistance grounded or Petersen-Coil protected. Since nothing can be done perfectly the Petersen-Coil [8,9] (ground fault neutralizer, arc-suppression coil) compensated systems have some disadvantages, under which it can be listed that the residual current for an earth fault is very small and this results in poor possibilities to localize the fault location.

The traditional power system engineering discipline has, caused the great extend and complexity of electric power systems control, dynamics, stability and monitoring, through the last twenty years expanded to include even more disciplines. Some of these disciplines are more advanced mathematics, signal analysis, power electronics and economics.

The most expanding disciplines are mathematics and signal analysis since control and monitoring of electric power systems to a considerable extent will secure power quality. Understanding more advanced signal analysis techniques includes higher mathematics, which involved that several signal analysis tools for pre- and post processing have been improved and new developed through the last 20 years [7].

Traditional Fourier signal analysis methods (FT and FFT) [11] consider that a signal is a sum of sinusoids, which is a problem since a sinusoid never begins and never stops. Transient temporal sequences and signals contain several different frequencies occurring at different times and are often also damped. By using the FT analyzing technique on a transient signal one would obtain an frequency domain representation that would be almost identical to the FT of a signal that was composed of the same several frequencies, but yet occurring simultaneously. One solution to this problem is the short-time Fourier transform (STFT) [7,10], which gives both time and frequency resolution so that the informative content not only tells the user what frequencies exists, but also the temporal location. The STFT technique is almost identical to the FT technique except for adding a window function. By using the STFT it is now possible to consider a signal stationary by using a narrow enough window function, which gives a good time resolution at the expense of the frequency resolution. I other words with the STFT it is impossible knowing the exact frequency at an exact time. Some improvements of this uncertainty is done by using other developed methods under which the the Wigner-Ville analysis [10] can be mentioned, these other methods will caused their great extensive not be examined.

Relatively new signal analysis techniques in electric power systems is the Wavelet transform (WT), which makes use of multiresolution analysis (MRA) [5] and Wavelet Packet [2] decompositions, and Prony analysis [6], which considers that a signal is a sum of damped sinusoids. As mentioned the time-frequency resolution problem is a problem accounted to physical phenomena (Heisenberg uncertainty/inequality principle) and will exists in any transform method. The MRA analyzes a signal at different frequencies with different resolutions, i.e. the spectral components are not resolved equally as in the case with the STFT. Therefore the continuous Wavelet transform (CWT) [1,5,7] was developed as an alternative approach to the STFT. A more or less detailed list of references covering all the above mentioned signal analysis methods is listed in [14].

This paper will therefore deal with this two signal analyzing techniques; Wavelet transform [2,7] using the software package UviWave [15] and Prony analysis [6] using the software developed by [6] on simulated and measured earth faults on a compensated 10 kV power distribution network. The aim with these analyzing techniques is an approach geographically to localize earth faults on compensated distribution networks, which caused its great extend not is included in this paper.

The purpose with this paper is with the above mentioned methods to show that the transient sequence of currents and voltages not exceed a bandwidth of approximately 10 kHz, for which signal measurements on conventional current transformers (CT’s) and voltage transformers (VT’s) is sufficient.
MEASURED AND SIMULATED EARTH FAULTS

The presented results of the earth fault experiment are described in [3,12,14], where also a description for the laboratory network and measurement setup can be found. The initial phase-angle for the earth fault is 90° with a fault resistance of 0 kΩ.

From now on the measurements on the CT's and VT's [12,14] are called measA, and respectively another set of measurements with high bandwidth equipment from [3] called measB. PSCDAD/EMTDC® simulations are called simC. All measurements and simulations are re-sampled to a sampling frequency of 125 kHz (8 μs).

It should at this point be mentioned, that the load in measB is somewhat different which results in a different phase angle and transient current amplitude and that the simulation results are different to that in [12] since an PSCDAD/EMTDC® cable series impedance calculation error was discovered and corrected by the method described in [13].

SIGNAL ANALYSIS METHODS

Now by using the two signal analyzing techniques; Wavelet transform (WT) [2,7] using the software package UviWave [15] and Prony analysis [6] using the software developed by [6] on the measured and simulated earth faults voltages and currents, it can be shown that the transient sequence not exceed a bandwidth of approximately 10 kHz. The general Wavelet and Prony theory will not be examined.

Wavelet transform

Sinusoid waves, which are smooth and periodical have unlimited duration (they will never end), are used in FT where a signal is decomposed into sine waves of various frequencies. WT decomposes a signal into shifted and scaled versions of the mother Wavelet, which has some special characteristics (limited duration, compact support, average of zero). There are many families of Wavelets for many different applications; they are irregular, asymmetric, and short oscillating waveforms. The Wavelet transform Ψ(β,α) of a continuous time signal x(t) (CWT) is given as [7]

\[ CWT_x(\beta,\alpha) = \left|\alpha\right|^{-1/2} \int_{-\infty}^{\infty} x(t) \Psi^{*}\left(\frac{t-\beta}{\alpha}\right) dt \]  \hspace{1cm} (1)

which means that the Wavelet transform is computed as the inner product of x(t), and a translated by β and scaled by α versions of a single complex conjugated function Ψ(t), also called mother Wavelet. The pre-factor \(|\alpha|^{-1/2}\) ensures normalization, i.e. energy conservation of all scaled functions Ψ. Assuming that \(\beta = 2^{j+1}\) and \(\alpha = 2^j\), i.e dyadic sampling of x(t) when \(k\) and \(n\) are integers, the discrete Wavelet transform (DWT) can be derived to

\[ DWT_x(j,k) = \left|\alpha\right|^{-1/2} \sum_{n=-\infty}^{\infty} x[n] \Psi\left(2^{-j} n - kT\right) \]  \hspace{1cm} (2)

where a scaling parameter of 2 gives an octave band analysis.

The DWT gave the idea [5] to the multiresolution analysis (MRA), which decomposes a signal from a subspace \(V_0\) into a high \(W_1\) and a low frequency \(V_1\) part. If this is done on \(V_1\) gain and getting \(W_2\) and \(V_2\), the DWT is working as a filter bank [7]. Also the decomposition of a function space in sequences of subspaces (scales). For the MRA there exists a generator \(\phi(x)\) which is a refinable function (father Wavelet), i.e. there exists coefficients \(a_k\) with

\[ \phi(x) = \sum_k a_k \phi(2x-k) \]  \hspace{1cm} (3)

Equation (3) is the dilation equation (2-scale equation) with the filter taps \(a_k\) translates \(\phi(x-k)\) to span \(V_0\), \(\phi(2x-k)\) to span \(V_1\) ... \(\phi(2^n x-k)\) to span \(V_n\). The complement spaces \(W_n\) \(W_{n+1}\) \(V_{n+1}\) added to \(V_f\) forms the subspace \(V_j\). The construction of a wavelet basis with

\[ \Psi(x) = \sum_k b_k \phi(2x-k) \]  \hspace{1cm} (4)

where the filter taps \(b_k\) forms the mother Wavelet

\[ \Psi_{j,k}(x) = 2^{-j/2} \Psi\left(2^{-j} x-k\right) \]  \hspace{1cm} (5)

Now a signal \(x_m(t)\) can be decomposed to

\[ x_m(t) = x_{m+1}(t) + y_{m+1}(t) \]  \hspace{1cm} (6)

where \(x_{m+1}(t)\) is the low frequency part and \(y_{m+1}(t)\) is the high frequency part, also multiresolution analysis

\[ \begin{array}{cccc}
  y_1(t) & y_2(t) & y_3(t) & \\
  \vdots & \vdots & \vdots & \\
  x_0(t) & x_1(t) & x_2(t) & x_3(t) & \ldots \end{array} \]  \hspace{1cm} (7)

Prony analysis

Pronyms method is a technique for modeling sampled data as a linear combination of exponential damped sinusoids, where the Prony method has a close relationship to the least squares linear prediction algorithms used for AR (autoregressive) and ARMA (autoregressive-moving average) parameter estimation [6].

Prony method seeks to a deterministic exponential model to the data, where AR and ARMA methods seek to fit a random model to the second order statistics.

The Prony analyzing method is because of the complex data analysis not easy to understand, and will therefore not be described in more details. In [6] the method is well documented and will therefore be used as the only reference, except for some papers where the method is used in the power engineering discipline [14].

Many functions used to model physical systems in
engineering, arise as the solution of a homogeneous differential equation. By letting \( y(t) \) be the concentration or intensity of the physical process at time \( t \), it is often the case that there are physical reasons for supposing that \( y(t) \) satisfies a differential equation, where a typical solution is a sum of exponential functions. Such a solution is appropriate for a transient signal, which dies to zero as time goes on.

Another typical solution is a sum of sinusoids. Such a solution is persistent.

A third common solution is the damped sinusoid, which combines the transient and periodic behaviour. In this case the third solution is the most interesting, which in discrete form is defined as a \( p \)-term complex exponential model

\[
y[n] = \sum_{k=1}^{p/2} A_k \exp^{\alpha_k (n-1)T} \cos(2\pi f_k (n-1)T + \theta_k) \tag{8}
\]

where \( T \) is the sample interval, \( A_k \) is the amplitude of the complex exponential, \( \alpha_k \) is the damping factor, \( f_k \) is the sinusoidal frequency and \( \theta_k \) is the sinusoidal initial phase. The parameters are completely arbitrary.

In the case of real data samples the complex exponentials must occur in complex conjugate pairs of equal amplitude. If the number of complex exponentials \( p \) is even, then there are \( p/2 \) damped cosines. If \( p \) is odd, then there are \( (p-1)/2 \) damped cosines plus a single purely damped exponential. From this follows that there are two methods of Prony parameter estimation; method 1 for damped exponentials and method 2 for undamped sinusoids.

**ANALYSIS OF EARTH FAULT SIGNALS**

With a sampling rate of 125 kHz and a scaling by 2, see equation (2), the MRA gives an octave band analysis as shown on figure 1. Figure 1 shows also what the frequency octave bands are for a certain sampling rate.

Using the MRA on the measured voltage \( u_{L2} \) from measA and measB and the simulated voltage \( u_{L2} \) from simC we can decompose the signal into the following high frequency parts as shown on figure 2.

From figure 2 it can be seen that the bulk of energy in the measured transient signal measA and measB appears from scale 4 through scale 11. Scale 11 contains the 50 Hz power frequency. The most dominant transient bulks are at scale 4 (between 7812.5 and 3906.3 kHz) and scale 6 (between 976.56 and 1953.1 kHz) for both the measurements, but not for the simulated transient voltage \( u_{L2} \).

This deviation is ascribed to the perfect risetime of the transient signal simC at the time where the earth fault occurs, which also is shown that the bulk energy of scale 1 through scale 3 is much higher than in the measurements.
FIGURE 2: MRA of voltage $u_{L2}$ from measA, measB and simC (1.graph=original signal. 2.graph=scale 1, 3.graph=scale 2 …).

FIGURE 3: MRA of current $i_{L2}$ from measA, measB and simC (1.graph=original signal. 2.graph=scale 1, 3.graph=scale 2 …).

FIGURE 4: MRA of voltage $u_{L2}$, $u_{L2}$ and $u_{L2}$ from measA (1.graph=original signal. 2.graph=scale 1, 3.graph=scale 2 …).
FIGURE 5: MRA of current $i_{L1}$, $i_{L2}$ and $i_{L3}$ from measA (1.graph=original signal. 2.graph=scale 1, 3.graph=scale 2 …).

FIGURE 6: Upper; Prony estimated and reconstructed with 160 exponential damped sinusoids of the transient voltage $u_{L2}$ from measA. Lower; Error deviation of reconstructed Prony estimated transient voltage $u_{L2}$ from measA.

FIGURE 7: Prony estimated parameters (one sided only) of reconstructed signal of the transient voltage $u_{L2}$ from measA.

FIGURE 8: Prony energy spectrum density of estimated signal of the transient voltage $u_{L2}$ from measA.

Figure 6 shows the result of using Pronys method 1 on the transient voltage $u_{L2}$ from measA and estimating 160 exponential damped sinusoids. From this it can be seen that the Prony estimated/reconstructed signal not differs much from the original. The deviation of the Prony estimated signal to the original signal is also shown on figure 6.

On figure 7 are the Prony parameters for the reconstructed signal shown, and on figure 8 the Prony energy spectrum density. One can also with these two informations select the most significant exponential damped sinusoids and reconstruct the original signal with less than 160 exponential damped sinusoids.

Further can it from figure 8 be seen, that the energy in the frequency content above 6 kHz is damped approximately 70 dB compared to the 50 Hz signal. Also from this signal analysis method it can be concluded, that the transient signal of an earth fault on a compensated network not exceeds approximately 10 kHz.

Only selecting exponential damped sinusoids with the most significant amplitudes and a relatively small damping factor, and a purely exponential will result in a reconstructed transient voltage $u_{L2}$ from measA as shown on Table 1. Similar Pronys method is used on the transient voltage $u_{L2}$ from measB, where the coefficients are shown in table 2.
measuring methods it can be concluded that the necessity of 10 kHz. From this and also from the comparison of the two bandwidth of the transient signal not exceeds approximately of earth faults on a compensated network have shown that the Signal analysis with these methods on the transient signal is somewhat useful for fast distance relaying. The MRA is extracted with the use of either WT or the Prony method. This CONCLUSIONS

The MRA signal processing tool combined with the Prony signal processing tool appears to be a promising method for power system transient distinguishing and discrimination, i.e. inrush current of a transformer bank, capacitor switching, fault discrimination etc.

The 50 Hz component of a transient signal can easy be extracted with the use of either WT or the Prony method. This is somewhat useful for fast distance relaying. The MRA is useful for power system transient distinguishing and discrimination, i.e. inrush current of a transformer bank, capacitor switching, fault discrimination etc.

The Prony parameter estimation is not easy to use, and one needs more insight to fully understand this method. The Prony method requires as the MRA that the to be analyzed signal starts at the initial point, i.e. where the earth fault occurs. Comparison of the coefficients listed in table 1 and 2, shows also that the Prony method is very sensitive to a noisy signal.

REMARKS

The MRA on transient signal is quiet easy to use with the software package UviWave [8]. The Wavelet Packet, i.e. CWT, requires more knowledge on signal analysis methods, which not can be overgone in details in this paper.

REFERENCES