COMBINED MODELLING OF LONG, SHORT INTERRUPTIONS AND VOLTAGE DIPS: A MARKOVIAN SOLUTION

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SUMMARY

A model of an electrical system node is presented. It allows predicting the effects of long and short interruptions as well as of voltage dips on final users starting from statistical input data. The model is developed using an extension of the homogeneous multi-state Markov process to take into account the non exponential nature of the failure-repair process. Case studies demonstrate the usefulness of the model for practical applications.

I. INTRODUCTION

In the last years, with the liberalization of energy markets, regulation of electricity supply quality has taken on growing interest. Utilities have to assure continuity of supply and voltage quality, in order to meet Authorities rules and customer satisfaction [1-2].

The detrimental effects of long interruptions affect all customers. Moreover, short interruptions and voltage dips may cause functional problems to many kinds of customers because of the sensitivity of electrical and electronic equipments, such as process controllers, adjustable speed drives, programmable logic controllers, personal computers, etc.

The quantification of the damages caused by these disturbances is very important because it allows evaluating the opportunity of adopting local actions to reduce totally or partially the users’ sensitivity [3-5]. Unfortunately, these damages are difficult to estimate because of: i) the random nature of the abovementioned phenomena and ii) the fact that disturbance effects depend on the final user [6-7]. The use of Power Quality monitors and statistic indicators by itself does not solve the problem.

In this paper, a model of an electrical system node is developed extending the homogeneous multi-state Markov process [8-9]. This extension allows considering the non exponential nature of the recovery time consequent to long and short interruptions as well as to voltage dips. The resulting model, even if apparently complex, allows taking into account the user’ sensitivity simply drawing border lines between the states whose effects are relevant and the others.

In the following sections, after a brief description of the standard disturbance characterization, the proposed model is presented. Finally, with reference to representative case-studies, the parameter setting is discussed in order to demonstrate the flexibility and the powerfulness of the methodology.

II. DISTURBANCE CHARACTERIZATION

An electrical node behaviour is usually characterized separately in terms of Long Interruptions 1 (LI), Short Interruptions 1 (SI) and Voltage Dips 2 (VD). This characterization is made according to the indexes reported in the Standards [2].

Typical data structures are reported in Table 1, 2 and 3 with reference to LI, SI and VD, respectively. The sources of the table data are the Italian Authority for Energy (2002) and the results of a measurement campaign on an Italian bus bar performed over 3416 days.

The interruption indexes used are the well known:
- CAIDI (Customer Average Interruption Duration Index): average duration of LI per customer per year.
- SAIFI (System Average Interruption Frequency Index): average number of LI per customer per year.
- MAIFI (Momentary Average Interruption Frequency Index): average number of short (momentary) interruptions per customer per year.

Voltage dips are characterized each by a pair of data, duration and either retained voltage or depth; for this reason, their average number per year is given in a duration-depth table.

TABLE 1 - LONG INTERRUPTIONS

<table>
<thead>
<tr>
<th>CAIDI</th>
<th>Class</th>
<th>SAIFI</th>
</tr>
</thead>
<tbody>
<tr>
<td>44.38</td>
<td>L</td>
<td>2.93</td>
</tr>
</tbody>
</table>

TABLE 2 - SHORT INTERRUPTIONS

<table>
<thead>
<tr>
<th>DURATION</th>
<th>Class</th>
<th>MAIFI</th>
<th>Freq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;0.5 s</td>
<td>S1</td>
<td>0.41</td>
<td>6.10</td>
</tr>
<tr>
<td>0.5-1 s</td>
<td>S2</td>
<td>1.02</td>
<td>15.15</td>
</tr>
<tr>
<td>1-3 s</td>
<td>S3</td>
<td>1.02</td>
<td>15.15</td>
</tr>
<tr>
<td>3-60 s</td>
<td>S4</td>
<td>2.45</td>
<td>36.40</td>
</tr>
<tr>
<td>60-180 s</td>
<td>S5</td>
<td>1.83</td>
<td>27.20</td>
</tr>
<tr>
<td>TOT S</td>
<td></td>
<td>6.73</td>
<td>100</td>
</tr>
</tbody>
</table>

TABLE 3 - VOLTAGE DIPS

<table>
<thead>
<tr>
<th>DURATION</th>
<th>VD&lt;30%</th>
<th>VD&gt;30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class</td>
<td>Num.</td>
<td>Freq.</td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>10-100 ms</td>
<td>DL1</td>
<td>12.84</td>
</tr>
<tr>
<td>100-500 ms</td>
<td>DL2</td>
<td>63</td>
</tr>
<tr>
<td>0.5-1 s</td>
<td>DL3</td>
<td>17.34</td>
</tr>
<tr>
<td>1-3 s</td>
<td>DL4</td>
<td>8.24</td>
</tr>
<tr>
<td>3-20 s</td>
<td>DL5</td>
<td>17.34</td>
</tr>
<tr>
<td>20-60 s</td>
<td>DL6</td>
<td>8.24</td>
</tr>
<tr>
<td>TOT</td>
<td>DL</td>
<td>127</td>
</tr>
</tbody>
</table>

1 Interruption: condition in which the voltage on the delivery point of electrical energy for end-user is less than 1% of declared voltage. The interruptions are classified into long interruptions (duration > 3 min) and short (and momentary) interruptions (duration ≤ 3 min).

2 Voltage dips: variation of nominal voltage > 10% with duration included into the interval 10ms-180s.
It is worthwhile to note that each row of the tables individuates the disturbances in terms of type (Class symbol) and level (Class number) that will be used as one of the possible states of the system \((L(.), S(.), DL(.), DH(.))\). In Table 2 and 3 also the relative level frequency is given. In other words, the electrical node is either in a normal state (Node OK) or in one of the fourteen states that together with the normal state constitute a set of fifteen exhaustive and mutually exclusive states.

III. MODELLING

In Fig. 1 a first general Markovian model that can be used to include, at the same time, long and short interruptions and voltage dips is represented. According to this model, no importance to the duration of SI and VD and to the depth of VD is given. The time spent in each of the states is exponentially distributed, so the failure-repair process can be viewed as an homogeneous Markov process (see Appendix). \(\lambda_i\) and \(\mu_i\) represent the transition rates between the state “Node OK” and the state in which the \(i\)-th disturbance is present. Initial values for these transition rates (see the next section for detail about parameter setting) can be calculated from the data contained in Tables 1-3; for example, with reference to LI, \(\lambda_3=SAIFI\) [transition per year] and \(\mu_3=1/CAIDI\) [transition per min] are obtained. The other four transition rates can be evaluated in a similar way, with some little mathematical complications.

This simplified model does not provide the opportunity of distinguishing among the levels of the different type of disturbances. So, the different customer sensitivities can not be properly modelled.

In order to solve the aforementioned problem, a fifteen \((14+1)\) states model according to the classes defined in the rows of Tables 1, 2 and 3 has been firstly used. But, in spite of the increased number of states, this model is only able to consider exponential distributed disturbance duration (time to recovery).

Unfortunately, the disturbance duration, for each state, can not realistically be considered an exponential random variable.

To solve this problem, the method of stages has been used (see Appendix). It has allowed authors developing different complex, but easy to handle, models.

For the sake of brevity, in the following part of the paper, reference is made only to the model consisting of a pure parallel of series stages, represented in Fig. 2.

According to this model, the “OK” state is connected with fourteen states/columns (see rows of Tables 1-3). The parallel columns 1-6 are used to model VD of depth \(\leq 30\%\); the columns 7-8 are used to model VD of depth \(>30\%\); the columns 9-13 are used to model SI and, finally, the last column is used to model LI.

Each column consisting of a series of stages. For example, \(d_{1,1}\) is the number of stages of the first state, DL1 (first column) and \(d_{6,6}\) is the number of stages of the 6-th state, DL6. For the sake of clarity, only the first and the last stage of each state are represented.

In Fig. 2, it is possible to observe two dashed border lines: “A” is representative of users sensitive only to short and long interruptions and “B” of users sensitive to short interruptions with duration greater than 0.5 s and to long interruptions.

Each border line separates, for the users they represent, the states that do not cause damage to the users (i.e. those on the left of the border line) from the states that cause damage to the users (i.e. those on the right of the border line).

Given the user, the probability to be “up” (“down”) can be calculated as the sum of the state probabilities of the states that are on the left (right) of the border line.

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**III.A. Structure**

Fig. 1 - Simplified model of the electrical system node.

Fig. 2 - Proposed model of the electrical system node constituted by a pure parallel of series stages.
### III.B. Parameter Setting

The experimental data reported in Section II have been used to set all the transition rates, $\lambda_{ij}$ and $\mu_{ij}$ as well as the number of stages for each of the fourteen states (columns) of the model in Fig. 2 (here, $\mu_{ij}$ is an exit transition rate equal for each of the stages appertaining to the given state $(i,j)$).

Initial values of the transition rates $\lambda_{ij}$ and of the MTTR (Mean Time To Repair) can be obtained using directly the number of events per year reported in Tables 1-3. For example, with reference to SI (Tab.2), it is possible to set $\lambda_{S5}=MAI_{S5}=1.83$ [transitions per year]. Similarly, an initial value of the exit transition rate, $\mu_{S5}$, can be calculated as: $(s5-s4)/MTTR_{S5}$, being $(s5-s4)$ and $MTTR_{S5}$ the number of stages and the mean “traversing” time associated to state S5, respectively. The MTTR$_{S5}$ can be set to 120 s, that is the centre of the interval duration of the corresponding class.

To complete the “initial” parameter setting, it is necessary to assign the number of stages for each of the fourteen states.

Given the initial values of the transition rates $\lambda_{ij}$ and $\mu_{ij}$, the number of stages for the different columns have been heuristically chosen with the aim of obtaining a time-to-repair (or recovering time) distribution with characteristics as close as possible to those of the real distributions reported in Tables 1, 2 and 3.

The index used to evaluate this closeness is the Root Mean Square Error (RMSE):

$$RMSE = \sqrt{\frac{1}{k} \sum_{i=1}^{k} \left( \frac{mf_i - rf_i}{rf_i} \right)^2}$$

(1)

where $rf_i$ (i=1,2,…,k) are the real relative frequencies reported in Tables 1, 2 and 3 and $mf_i$ the corresponding frequencies calculated on the basis of the model. Here and in the following of this section, the subscript “$r$” is used to make reference to the i-th state on the right side of the border line.

Given the structure of the model in Fig 2, the pdf of the time-to-repair (or recovering time) distribution with characteristics as close as possible to those of the real distributions reported in Tables 1, 2 and 3.

The mixture can be obtained by:

$$f_{mix,i} = \sum_{i=1}^{k} w_{ii} \cdot f_i .$$

(2)

where $k$ is the number of states on the right side of the border line and $w_{ii}$ is a set of weights so that $\sum_{i=1}^{k} w_{ii}=1$.

The $k$ value depends on the kind of final user under consideration and can reach the maximum value of 14.

The weight $w_{ii}$ is the ratio between the transition rate from the State OK to the i-th column and the summation of the transition rates extended to the k states under consideration.

The authors’ experience has demonstrated that (given the initial setting of the rates $\lambda_{ij}$ and $\mu_{ij}$) just changing the number of stages, it is not possible to obtain optimal results. Sensible improvements of the solution can be produced introducing some little adjustments of the transition rates $\lambda_{ij}$ and $\mu_{ij}$ previously calculated. Obviously, these adjustments must be done assuming a constant value for the real “global” MTTR of all the DOWN states and for the MTTF (Mean Time To Failure), which the steady state solution depends on. These MTTR and MTTF can be calculated using the data in Tables 1, 2 and 3.

This calibration can be accomplished adopting the following relations:

$$\dot{\lambda}_{ij} = \dot{\lambda}_{i} = \dot{\lambda}_{j} = \dot{\lambda}_{ii} \sum_{i=1}^{k} \lambda_i$$

(3)

and

$$\mu_i = \mu_i \frac{MTTR_i^*}{MTTR_i}$$

(4)

where the “asterisk” denotes the transition rates obtained after the adjustment, and MTTR and the MTTR* can be calculated by the following formulas:

$$MTTR = \sum_{i=1}^{k} w_{ii} \cdot MTTR_i; \quad MTTR^* = \sum_{i=1}^{k} w_{ii}^* \cdot MTTR_i$$

(5)

being MTTR the mean traversing time of the i-th state on the right side of the border line.

Equation (3) allows changing the weights without changing the “global” MTTF of the node (it is sufficient to use the initial $\lambda_{ij}$ and to respect the condition $\sum_{i=1}^{k} w_{ii}=1$).

Equation (4) allows restoring the MTTR of the model, changed due to the weights adjustment, to the initial real “global” MTTR value.

### IV. APPLICATIONS

Two case studies corresponding to the two kinds of users whose sensitivities are represented by the borderlines of Fig. 2 are developed.

#### IV.A. Case-Study A

Reference is made to users sensitive only to short and long interruptions.

First of all, using initial values of the transition rates, a starting set of number of stages for each class has been found. Then, for several adjusted sets of weights, the relative errors between the pdf experimentally obtained and the pdf reproduced by the model have been calculated for each class of interruptions.

The resulting best combination of weights and transition rates, heuristically found, are reported in Table 4. The RMSE is equal to 0.065.

<table>
<thead>
<tr>
<th>CLASS</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>N(1)</td>
<td>5</td>
<td>10</td>
<td>7</td>
<td>10</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>$w_i$</td>
<td>0.0280</td>
<td>0.1385</td>
<td>0.1010</td>
<td>0.2360</td>
<td>0.2180</td>
<td>0.2785</td>
</tr>
<tr>
<td>$\mu_i$</td>
<td>18.4293</td>
<td>12.5056</td>
<td>3.2910</td>
<td>0.3009</td>
<td>0.0780</td>
<td>0.0017</td>
</tr>
<tr>
<td>Error %</td>
<td>-3.5</td>
<td>-2.8</td>
<td>4.0</td>
<td>-2.1</td>
<td>0.75</td>
<td>1.2</td>
</tr>
</tbody>
</table>

| TABLE 4 – Case Study A: Best combination of weights and transition rates heuristically found and characterized by an RMSE=0.065 for the starting set of stages. |
Figure 3 reports a comparison between the real pdf of the time-to-repair obtained from Tables 1, 2 and 3 and the pdf reproduced by the model using the values indicated in Table 4.

![Figure 3](image)

Fig. 3 - Case Study A: Comparison of the real pdf of the time-to-repair obtained and the pdf reproduced by the model (Table 4).

Figure 4 reports the absolute value of the relative errors, class by class, for three different combinations of weights and transition rates all characterized by the number of stages of Table 4. The RMSE value corresponding to each combination is also reported.

![Figure 4](image)

Fig. 4 - Case Study A: Relative errors, class by class, for three different combinations of weights and transition rates all characterized by the number of stages of Table 4.

After the first calibration, the solution found (TABLE 4) has been compared to other solutions, obtained varying only the number of stages (TABLE 5). A better solution with RMSE=0.060 has been found. The corresponding absolute values of the relative errors are drawn in Fig.5.

![Figure 5](image)

Fig. 5 - Case Study A: Relative errors, class by class, for three different combinations of weights and transition rates all characterized by the number of stages of Table 6.

### IV.B. Case Study B

As done in the previous Sub-section, some simulations have been run for users sensitive to short interruptions with duration greater than 0.5 s and to long interruptions. The resulting best combination of weights and transition rates, heuristically found, are reported in Table 6. The RMSE is equal to 0.027.

<table>
<thead>
<tr>
<th>CLASS</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>L</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ni(1)</td>
<td>5</td>
<td>10</td>
<td>7</td>
<td>10</td>
<td>5</td>
<td>0.065</td>
</tr>
<tr>
<td>Ni(2)</td>
<td>7</td>
<td>20</td>
<td>6</td>
<td>5</td>
<td>8</td>
<td>0.314</td>
</tr>
<tr>
<td>Ni(3)</td>
<td>5</td>
<td>7</td>
<td>10</td>
<td>7</td>
<td>8</td>
<td>0.250</td>
</tr>
<tr>
<td>Ni(4)</td>
<td>25</td>
<td>11</td>
<td>7</td>
<td>10</td>
<td>6</td>
<td>0.060</td>
</tr>
</tbody>
</table>

TABLE 5 – Case Study B: Best combination of weights and transition rates heuristically found and characterized by an RMSE=0.027.

Figure 6 reports a comparison between the real pdf of the time-to-repair obtained from Tables 1, 2 and 3 and the pdf reproduced by the model using the values reported in Table 6.

![Figure 6](image)

Fig. 6 - Case Study B: Comparison of the real pdf of the time-to-repair obtained and the pdf reproduced by the model (Table 6).
VI. CONCLUSIONS

In this paper, a model of an electrical system node has been presented. The model has been developed extending the homogeneous multi-state Markov process to take into account the non-exponential nature of the recovery time process consequent to long and short interruptions as well as to voltage dips. The resulting model, even if apparently complex, has allowed taking into account the user’s sensitivity simply drawing border lines between states whose effects are relevant and the others.

By means of the introduced models it is possible:
- assessing the economical damages due to interruptions and voltage dips;
- evaluating the convenience of local actions to improve reliability and Power Quality.

VII. APPENDIX - Remarks on the Markov Approach and its extension

The Markov approach is based on the following hypothesis: the prediction of the future states of the system, based on the present state alone, does not differ from that formulated on the basis of the whole history of the system (Markov property). Whether this transition probability does not depend on the age of the system (time) the Markov process is called homogeneous. In a homogeneous Markov process the time between successive transactions has an exponential distribution.

In the applications to single two state components in which both time to failure and time-to-repair are exponentially distributed, the failure-repair process can be viewed as a two state homogeneous Markov process.

When the random variables time to failure and/or time-to-repair can not be assumed exponentially distributed, extensions of the homogeneous Markov process have to be adopted.

A simple, but powerful solution consists in dividing a state into sub-states, each being defined as a stage. If two or more exponentially distributed stages are combined, the time spent in the resulting state is non-exponentially distributed.

In particular:
- if the N stages (see Fig.7) are traversed in a sequential order (series) and have constant and equal transition rate, μ, the time spent to pass through the state is a “Special Erlangian” random variable of parameters μ and N having pdf:

\[ f(t) = \frac{\mu^{N-1}}{(N-1)!} e^{-\mu t} \]  

(6)

- if the N (see Fig.8) stages are in parallel (i.e.: the state can be traversed by traversing one of its stages) given the transition rates μi (i=1,2,...,N) of the i-th stage and the probability, wiλ, of traversing the state passing across the i-th stage, the probability density function of the time spent in the state is:

\[ f(t) = \sum_{i=1}^{N} w_i \mu_i e^{-\mu_i t} \]  

(7)

VIII. BIBLIOGRAPHY