INTRODUCTION

Knowledge about bus voltage and power losses in primary distribution feeders is important to system planning and operation. However, to obtain this information is not a simple task because there are a few measurements just ever. Many times, measurements are only taken in the substation exit or in the beginning of feeders. Usually, one applies a load flow program to calculate bus voltages and power losses. As the actual demand is not known, one supposes the load curve in each bus is similar to the load curve of the whole feeder, determined from measurements on the substation [1]. Another usual way to remove this trouble is taken by typical load curve in substitution to the proper load curve that is not given [2]. At any rate, the result of the load flow program can be too unrealistic. This paper presents a different procedure to estimate bus voltage and power losses on radial distribution feeders, which is based on the optimization algorithm so-called simulated annealing. The proposed method has been applied to the IEEE 13, 34, 37 and 123 buses test feeders and the validation is done confronting the results to known values, as it is explained just beyond. The total power losses estimated are close to the actual losses.

SIMULATED ANNEALING

In the beginning of the eighties, Scott Kirkpatrick and collaborators of him, working at the IBM laboratories, found out the process of metal annealing was very much analogous to the solution process of any combinatorial optimization problem. So, the annealing process, in the way that it was modelled by Nicholas Metropolis [3] three decades ago could be applied to solve a great variety of scientific and technological problems.

Metropolis modelled the annealing process in the following way: given a state \( i \), on which the energy is \( E_i \), one generates another state, \( j \), whose energy is \( E_j \), by means of some mechanism of perturbation. If the energy difference, \( \Delta E = E_j - E_i \), is equal to or lesser than zero the new estate, \( j \), is accepted as the current state. Otherwise, if \( \Delta E > 0 \), the new state is not accepted or refuse for sure. The probability of acceptance in this case is

\[
P(\Delta E) = e^{-\Delta E/kT},
\]

where \( k \) is the Boltzmann’s constant and \( T \) is the temperature, the process control variable.

The annealing starts when the metal is fused. As the temperature is high its atoms are disordered and consequently may occupy several positions. In those conditions the probability of going to a state of higher energy is very large. The Metropolis’s model is analogous to a generic process of combinatorial optimization on the following points:

- The several positions occupied by the atoms determine the state of the system and correspond to a solution of the problem that one wants resolve by optimization.
- To each state is associated a determinate energy level which corresponds to a value of the objective function of the optimization problem.
- In begin and while the temperature stays high, all of the states are possible being reached. This works to escape from local minimum, allowing all solution space may be explored.
- The probability of getting a higher energy state is each time smaller when the temperature decreases. In this manner, while the process runs, regions of search space more and more closed around the best solution found until then are explored.
- Just in case of the annealing be carried out carefully the material getting a perfect crystalline structure on which the energy is minimal when thermal equilibrium is reached. This state corresponds to the global optimal solution or a good approach of it.

Kirkpatrick employed as a metaphor, the Metropolis’ model to resolve an optimization problem concerning to project of electronic circuit on large scale [4]. Ever since, the algorithm applied pioneering by him has been used to solve several combinatorial optimization problems and has became well-known how simulated annealing. Basically, the algorithm is the following:

1. Define the energy function \( E \);
2. Choose a final temperature \( T_{\text{final}} > 0 \) and an initial temperature \( T_{\text{initial}} >> T_{\text{final}} \);
3. Take the initial temperature as the current temperature: \( T = T_{\text{initial}} \);
4. Define a cooling schedule, \( \Delta T \), as a \( T \) function;
5. Choose \( n \), a number of search to be done on a same temperature;
6. Create randomly a point \( x \) in state space and let it be the current state: \( x = x_0 \);
7. While the system is not cooled (\( T > T_{\text{final}} \)) do the following:
   7.1 Start a counter: \( i = 1 \);
   7.2 While \( i \leq n \) do
      Create a random state-point \( x_i \) in the surrounds of \( x \);
      Calculate the energy difference between the two states: \( \Delta E = E(x_i) - E(x) \) and
      if \( \Delta E < 0 \) then
         accept the new state as the current state: \( x = x_i \);
   7.3 Increase \( i \) by 1;
   7.4 While \( T > T_{\text{final}} \) do
      Decrease the current temperature by the cooling schedule \( \Delta T \);
5. Output the current state of the system as the best solution found.

The result of the load flow program given \( T = T_{\text{final}} \) is equal to or lesser than zero the new estate, \( j \), on which the energy is \( E_j \), one generates another state, \( i \), on which the energy is \( E_i \), by means of some mechanism of perturbation. If the energy difference, \( \Delta E = E_j - E_i \), is equal to or lesser than zero the new estate, \( j \), is accepted as the current state. Otherwise, if \( \Delta E > 0 \), the new state is not accepted or refuse for sure. The probability of acceptance in this case is

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POWER FLOW ON RADIAL FEEDERS

Usually, every buses of a primary distribution feeder are PQ type. In this manner, the active and reactive demanded at each bus is supposed known. The substation bus, where the feeder origins, is the slack bus. The predominant configuration of primary distribution feeders is radial, like a tree. So, the feeder can be seen as a succession of section like the one shown in the Figure 1. Shunt admittance is negligible.

An efficient computational method of power flow on radial feeder is the power summation method [5]. It is interactive in loss-variable, that is, one starts supposing there is not any loss on the feeder and successively losses values will be being corrected. Once the voltage on the bus \( \phi_i \) is determined (Figure 1), the voltage on the bus \( i \) is calculated by:

\[
V_i = \sqrt{A - \sqrt{A^2 - B}} ,
\]

where

\[
A = \frac{1}{2} r_i^2 x_i^2 - r_i P_i - x_i Q_i
\]

and

\[
B = (r_i^2 + x_i^2) (P_i^2 + Q_i^2) .
\]

The active and reactive losses along the section \( i \) are given as functions of \( V_i \) respectively by:

\[
\Delta P_i = r_i \left( \frac{P_i^2 + Q_i^2}{V_i^2} \right)
\]

and

\[
\Delta Q_i = x_i \left( \frac{P_i^2 + Q_i^2}{V_i^2} \right).
\]

The power flow at the end of section \( i \) is the load at the bus \( i \) plus the summation of power flows at the beginning of every one section starting at the bus \( i \). As the flow at the beginning of each section is the one at the end plus the losses along this same section,

\[
P_i = P_{Li} + \sum_{k \in \Phi_i} P_k + \Delta P_k
\]

and

\[
Q_i = Q_{Li} + \sum_{k \in \Phi_i} Q_k + \Delta Q_k
\]

\( \Phi_i \) is the set of sections which begin at the bus \( i \); that is, \( k \in \Phi_i \) if \( \phi_k = i \).

THE PROBLEM DEFINITION

There would not be any problem to calculate losses and bus voltages if active and reactive power demand by every bus were known. However, it is not like this in practice, because most of the times no more than the nominal power of distribution transformer is given. In such case,

\[
P_{Li} = S_{Li} f_{ui} f_{pi}
\]

and

\[
Q_{Li} = S_{Li} f_{ui} \sqrt{1 - f_{pi}^2}
\]

have to be considered in equations (7) and (8). \( S_{Li} \), \( f_{ui} \) and \( f_{pi} \) is nominal power, utilization factor and power factor of transformer on bus \( i \), respectively.

Once in a while one has transformers supplying special consumer. In such case, power factor is measured. Besides, the active demand can be known, even so that be small in number.

By convenience, equations from (7) to (10) can be re-arranged and therefore resulting:

\[
f_{pi} = \frac{P_i - \sum_{k \in \Phi_i} P_k + \Delta P_k}{\sqrt{\left( P_i - \sum_{k \in \Phi_i} P_k + \Delta P_k \right)^2 + \left( Q_i - \sum_{k \in \Phi_i} Q_k + \Delta Q_k \right)^2}}
\]

and

\[
f_{ui} = \frac{P_i - \sum_{k \in \Phi_i} P_k + \Delta P_k}{S_{Li} f_{pi}} .
\]

The problem is set out in the following mode: on a primary radial feeder, whose physical arrangement and electrical parameters are given, nominal power of transformer installed at each bus are known. The power factors are measured in some of these buses and in a fewers others, power factors and also utilization factors (in fact, active demands) are measured. On the other hand, active and reactive powers, as well voltage magnitude are registered in beginning of the feeder. What can be done to calculate the total losses in the best possible way?

In the way as the problem was formulated above, power and utilization factors are the feeder’s state variables. If all of
them are determined the bus voltages and losses are calculated without problem, being enough to run the load flow program. Therefore, to resolve the state estimation problem it is necessary finding out the unknown values of power and utilization factors.

THE SOLUTION METHOD

To have a method that does not depend strongly of a good quantity of data, one considers that all of the unknown power factors have the same value. A similar hypothesis is done to the unknown utilization factors. Be those common values factors have the same value. A similar hypothesis is done to

1. Take an initial estimative of values \( f_p^* \) and \( f_u^* \).
2. Determine the demands by each bus, applying equations (9) and (10) and considering \( f_p^* \) or \( f_u^* \) whether actual value of \( f_p^* \) or \( f_u^* \) are not known, respectively.
3. Run the load flow program and therefore determine the active and reactive flows \( (P_i^*, Q_i^*) \) and the losses \( (\Delta P_i, \Delta Q_i) \) on each section.
4. Compute the power and utilization factors relative to buses where they are given, applying the results of step 3 in the equations (11) and (12).
5. Get all of given values of power and utilization factors beside the measured values in the substation, as control values, then define the following energy function:

   \[
   E = \left(1 - \frac{P_0}{P_0^*}\right)^2 + \left(1 - \frac{Q_0}{Q_0^*}\right)^2 + \left(1 - \frac{I_0}{I_0^*}\right)^2 + \sum_{k \in \Theta} \left(1 - \frac{f_{p_k}^*}{f_{p_k}^*}\right)^2 + \sum_{k \in \Psi} \left(1 - \frac{f_{u_k}^*}{f_{u_k}^*}\right)^2 \tag{13}
   \]

   \( P_0 \) \( Q_0 \) is the active \( \text{reactive} \) power measured in the beginning of the feeder;
   \( I_0 \) is the magnitude of current measured in the beginning of the feeder;
   \( \Theta \{ \Psi \} \) is the set of buses where power \{ utilization \} factors are given;
   \( f_{p_k}^* \) \( f_{u_k}^* \) is the power \{ utilization \} factors known, being \( k \in \Theta \{ \Psi \} \).
6. Do a simulated annealing to obtain the optimal values of \( f_p^* \) and \( f_u^* \).
7. Run the load flow program considering the optimal values of \( f_p^* \) and \( f_u^* \) got in step 6, thus determine the total losses on the feeder.

The proposed method will work whether the available data are enough or not, because has been supposed unique values of power factor and utilization factor. At worst, when nothing more than the measurements at substation are available, an estimated value of the total losses is gotten even so. However, that value is faithful at all whether the feeder is very long.

Large feeder needs additional measurements to improve the estimation quality. For instance, consider active and reactive power as well current and voltage magnitudes are measured in two points along the feeder besides the substation bus according to the Figure 2. In such case, the feeder can be separated on three smaller others according to Figure 3. Each one of these subsystems has its own common values \( f_p^* \) and \( f_u^* \) and the boundaries of them are the buses where measurements are taken. No modification is necessary to the proposed method be applied to subsystem C. To the subsystems A and B the measured flow is taken as additional load. The measured voltage, \( V_2 \) and \( V_6 \), is taken into account as control variable and therefore equation (13) is modified by including of

\[
\left(1 - \frac{V_m'}{V_m}\right)^2,
\]

where \( m \) is 2 or 6 just in case of the subsystem A or B.

THE METHOD VALIDATION

To check the efficacy of proposed method it has applied to IEEE test feeders of 13, 34 and 123 buses.

First of all, the complete data of these feeders, obtained from [6] and [7], have been used as entry of the power flow program and therefore the total losses have been determined. These values have taken as reference value. Some bus voltage and power flows by sections, also resulting from power flow program, have been taken as pseudo-measurements. Having finished this preliminary stage, load data was discarded. Instead of them, standard nominal capacity of transformer are imagined.

To the 13 buses feeder one supposes that measurements are done just at the substation and none load is special. That is, every power and utilization factors are unknown. The Table 1 presents the results of this case.

The Table 2 presents the results relative to 34 buses feeder. One had supposed that measurements are done at the substation and at the bus 8 and power factors are measured at bus 7, 14, 19 and 28.

![Figure 2 — A radial feeder on which measurements are done at the substation and others two point.](image1)

![Figure 3 — Subsystems associated to the system of the Figure 2.](image2)
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An alternative method to estimate total losses on radial distribution feeder based on simulated annealing and power summation method has been proposed. A lot of different situations of load and measurement relative to IEEE test feeders of 13, 34 and 123 buses were analysed. Three of which have been presented in this paper. Every one of the studied cases has evidenced the method efficacy. According to expected, the estimation quality depends on the number of measurements. Comparative studies of the proposed method with the method based on the hypothesis that every individual loads have same characteristics as the whole of them are carry out at the present time. Next, the prime objective of authors is apply the method to real cases.

REFERENCES


