INTRODUCTION

Pulse transformers capable of transmitting substantially rectangular voltage pulses, with durations of less than one millisecond, were developed for radar application, NLC klystron pulse modulator, driving a microwave amplifier, x-rays for medical and industrial use, gas lasers for plasma technology and plasma immersion ion implantation [1-7].

Several application of them, need medium or high voltage pulses (1 up to 700kV) that medium or high voltage pulse transformer increases the output pulse voltage to the value required for the load. The usually large number of turns in the secondary winding (the transformer ratio is frequently 1:10), together with the insulation gap between windings and between winding layers increase the value of the equivalent parasitic elements (leakage inductance and inter-winding capacitance). These elements extend the pulse rise time and cause overshoot and oscillations. Hence, the design of the pulse transformer is critical, not only because all materials must sustain the medium or high voltage across them, but also because the output pulse shape depends heavily on several transformer parasitic parameters that are difficult to master [8].

Pulse transformer modeling is done by two methods: firstly, the well-known lumped parameter theory of transformer is used [3, 8]. Most pulse transformer models treat each winding as a single circuit element. This limits analysis of these types of models to the bulk properties of the model. Details of winding interaction with stray capacitances can only be lumped into a single circuit model element and these values are usually determined by measurement. This can limit a designer to a trial and error approach to the subtleties of pulse transformer design. Secondly, transformer considers as a distributed parameter circuit [9, 10]. However, nonlinear core are not included [11]. The windings are separated into multiple sections and all combinations of mutual inductances are calculated. The distributed capacitance between the core and primary, the primary and secondary, and the secondary to case are included. The individual inductance and coupling coefficients are calculated based on the magnetized inductances and the air mutual. These values are used in a circuit model developed in PSPICE. Especially, this method is used for simulation of air core pulse transformer [12-14].

On the other hand, because of the transformer parasitic elements involved, the transformer is the critical device in shaping the rising characteristics of the output pulse [3, 15-19].

Other related works on pulse transformer are harmonics, thermal and mechanical force using finite element method (FEM), reducing size, and electromagnetic interference (EMI) and so on [4, 7, 20-22].

To contribute to a better understanding of pulse transformer operation considering leakage inductance and inter-winding capacitance, this paper proposes a mathematical model based on the flux linkage as state variable. Our main aim is to identify a critical values causing unsuitable rise time of the output, especially damaged output pulse completely. It is interesting to demonstrate conditions for leakage inductance of windings, pulse frequency, and leakage capacitance that are destroying output pulse that hasn’t been discussed clearly in the literature up to now. Finally results of simulation are able to show rated value of pulse transformer parameters. If parameters of pulse transformer are not allowable value obtaining measurement and calculation, we must choose methods for reduction of them, such as using auxiliary windings or active shielding and so on [8, 23]. Therefore, this model is then used to suggest approachable parasitic elements to optimize the design of a medium or high voltage pulse transformer. Besides, using two auxiliary winding for improvement of technical characteristics of output pulse is explained and new prove based on characteristic roots method is done.
same mathematical model. For example, we can implement a simulation using fluxes or current as state variable. In our case, we will pick the total flux linkages of the two windings as the state variables. In terms of these two state variables, the voltage equations can be written as

\[ v_1 = i_1 r_1 + \frac{1}{\omega_b} \frac{d\psi_1}{dt}, \]  
\[ v_2' = i_2' r_2' + \frac{1}{\omega_b} \frac{d\psi_2'}{dt}, \]  

where \( \psi_1 = \omega_b \phi_1 \) and \( \psi_2' = \omega_b \phi_2' \), and \( \omega_b \) is the base frequency at which the reactances are computed. The flux linkage per second of the windings can be expressed as

\[ \psi_1 = \omega_b \phi_1 = x_{11} i_1 + \psi_m, \]  
\[ \psi_2' = \omega_b \phi_2' = x_{12} i_2' + \psi_m, \]  

and

\[ \psi_m = \omega_b L_m (i_1 + i_2') = x_m (i_1 + i_2') \]  

The current \( i_1 \) can be expressed in terms of \( \psi_1 \) and \( \psi_m \) using Equation (3) similarly, \( i_2' \) can be expressed in terms of \( \psi_2' \) and \( \psi_m \) using Equation (4).

\[ i_1 = \frac{\psi_1 - \psi_m}{x_{11}}, \]  
\[ i_2' = \frac{\psi_2' - \psi_m}{x_{12}}. \]  

Substituting the above expressions of \( i_1 \) and \( i_2' \) into Equation (5), we obtain

\[ \frac{\psi_m}{x_m} = \frac{\psi_1 - \psi_m + \psi_2' - \psi_m}{x_{11} + x_{12}}. \]  

Collecting the \( \psi_m \) terms to the right, we obtain the desired expression of \( \psi_m \) in terms of the two desired states, that is

\[ \psi_m \left( \frac{1}{x_{11}} + \frac{1}{x_{12}} \right) = \frac{\psi_1}{x_{11}} + \frac{\psi_2'}{x_{12}}, \]  

Letting

\[ \frac{1}{x_M} = \frac{1}{x_{11}} + \frac{1}{x_{12}} \]  

Equation (9) can be written more compactly as

\[ \psi_m = x_M \left( \frac{\psi_1}{x_{11}} + \frac{\psi_2'}{x_{12}} \right). \]  

Using Equations (6) and (7) to replace the currents, Equations (1) and (2) can be expressed as integral equations of the two total flux linkages, that is

\[ \psi_1 = \int \left( \omega_b \psi_1 - \omega_b r_1 \frac{\psi_1 - \psi_m}{x_{11}} \right) dt \]  
\[ \psi_2' = \int \left( \omega_b \psi_2' - \omega_b r_2' \frac{\psi_2' - \psi_m}{x_{12}} \right) dt \]  

Collectively, Equations (6), (7), (11), (12), and (13) from a basic dynamic model of a two winding transformer to which magnetic nonlinearity and iron losses may be added if necessary. In this model, the flux linkages are the internal variables, the terminal voltages are the required inputs, and the winding currents are the main outputs. In the next section, pulse transformer modeling based on this modeling has been described.

**SIMULATION OF PULSE TRANSFORMER**

In many applications of pulse transformer, we need a flat-top portion of the high voltage output pulse and fast rise time. In order to achieve a rise time that is less than 400nS we must be improved the design of a pulse transformer by trade off among the droop, the core size, and the rise time [4, 8, 9, 15, 16, and 24]. For investigation on effects of leakage parameters in output pulse, simulation is done. Figure 1 shows the SIMULINK simulation that is in accordance with the flow diagram based on flux linkage as state variable. In addition to, leakage capacitance and load model is defined in accordance with bellow equations:

\[ v_2' = \frac{1}{C_{eq}} \int i_1' dt = \omega_b B' \int (-i_2' - \frac{v_2'}{R_O}) dt. \]  

**FIGURE 1-** Simulation of pulse transformer based on flux linkage as state variable

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calculated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 )</td>
<td>0.1</td>
</tr>
<tr>
<td>( R_2' )</td>
<td>0.14</td>
</tr>
<tr>
<td>( x_{11} = x_{12} )</td>
<td>0.62</td>
</tr>
<tr>
<td>( R_0' )</td>
<td>50</td>
</tr>
<tr>
<td>( \omega_b )</td>
<td>6283</td>
</tr>
<tr>
<td>( x'c )</td>
<td>79577</td>
</tr>
<tr>
<td>( x_{m1} )</td>
<td>50</td>
</tr>
</tbody>
</table>
Initial simulation has been done in f=1 kHz (parameters of pulse generator: amplitude: 500V, periods: 0.001, pulse width 20% of period) with calculated parameters (Table 1). Result of simulation is illustrated in Figure 2. This simulation is repeated in f=10 kHz (Table 2 and Figure 3). Effects of leakage inductance and inter-winding capacitance in medium frequency (1 kHz) are shown in Figures 4 and 5. Therefore we can predict output waveform of pulse transformer based on changing in period of input pulse, leakage inductance and inter-winding capacitance.

Table 2- Transformer parameters (f=10 kHz, Transformer ratio: 1:10)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Calculated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>0.1</td>
</tr>
<tr>
<td>$R'_2$</td>
<td>0.14</td>
</tr>
<tr>
<td>$x_{11} = x_{22}$</td>
<td>6.28</td>
</tr>
<tr>
<td>$R'_o$</td>
<td>50</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>62831</td>
</tr>
<tr>
<td>$x'c$</td>
<td>3200</td>
</tr>
<tr>
<td>$x_{m1}$</td>
<td>500</td>
</tr>
</tbody>
</table>

FIGURE 2- Wave form of $v'_2$ (periods of pulse transformer: 0.001)

FIGURE 3- Wave form of $v'_2$ (periods of pulse transformer: 0.001)

FIGURE 4- Wave form of $v'_2$ (periods of pulse transformer: 0.001, $x_{11} = x_{22} = 1.24\Omega$ others according to Table 1) that is unwanted output.

FIGURE 5- Wave form of $v'_2$ (periods of pulse transformer: 0.001, $C = 2nF$ others according to Table 1) that is unwanted output.

SIMULATION RESULT COMPARING WITH MODELS BASED ON DISTRIBUTED PARAMETERS
For very high speed pulse, transformers were considered as a distributed parameter circuit [9]. In this method, for a non-inverting type transformer (n: 1), the pulse rise time response can be expressed as follow [9]:

\[ v_o(t) = \frac{E_G}{n(k+1)}[u(t)+\sum_{n=1}^{\infty}u(t-2nT_b)A_n] \]  \hspace{1cm} (15)

where \( G = \frac{R_3}{R_3+R_1} \), \( k_1 = \frac{\omega_b}{n} \),

\[ A_n = (1-\rho)(-\rho)^{-1}, \rho = \frac{1-k_1}{1+k_1} \]

Equation (15) comply with the corresponding simulations results. The discrepancies are almost 3% up to 32% in different conditions of our study. Thus our investigation is useful for analysis of pulse transformer and has good points for selecting and setting leakage parameters of pulse transformer.

**RISETIME REDUCTION AND NEW MATHEMATICAL PROVE**

One of the techniques usually adopted to decrease the leakage inductance of the transformer adds two auxiliary windings to the transformer. If properly used, these auxiliary windings reduce the leakage flux and, therefore, the leakage inductance. As a result the pulse rise time is reduced.

Firstly, analysis of transformer with subtractive connection of the auxiliary windings based on previous studies is explained. We consider the transformer supplying power to a load and the auxiliary windings connected as in Figure 6. Now \( i_2 = -i_4 \) where \( i_4 \) is the load current. The third and fourth windings are connected in subtractive mode (terminals five and six are connected to terminals seven and eight, respectively). This circuit exemplifies the normal operating condition of the transformer. Considering that \( i_4 = -i_3 = i_{aux} \), in Figure 6, applying to basic equation of four winding transformer and taking into consideration that \( N_3 = N_4 = N_{aux} \), yields, respectively

\[ v_1 = R_1i_1 + N_1\frac{d\phi}{dt} + l_1\frac{di_1}{dt} - l_2\frac{di_2}{dt} + (l_{14} - l_{13})\frac{di_{aux}}{dt} \]

\[ v_2 = -R_2i_2 + N_2\frac{d\phi}{dt} + l_3\frac{di_3}{dt} - l_2\frac{di_2}{dt} - (l_{14} - l_{13})\frac{di_{aux}}{dt} \]

\[ v_3 = -R_3i_3 + N_3\frac{d\phi}{dt} + l_4\frac{di_4}{dt} - l_3\frac{di_3}{dt} + (l_{14} - l_{13})\frac{di_{aux}}{dt} \]

\[ v_4 = R_4i_{aux} + N_4\frac{d\phi}{dt} - l_4\frac{di_4}{dt} + l_3\frac{di_3}{dt} + (l_{14} - l_{13})\frac{di_{aux}}{dt} \]  \hspace{1cm} (16)

Given that the third and fourth windings are connected as in Figure 6, then \( v_1 = v_4 \), which yields

\[ (l_{13} - l_{44})\frac{di_1}{dt} + (l_{42} - l_{32})\frac{di_2}{dt} = R_{aux}i_{aux} + l_{aux}\frac{di_{aux}}{dt} - M_{aux}\frac{di_{aux}}{dt} \]  \hspace{1cm} (17)

where \( R_{aux} = R_3 + R_4 \), \( l_{aux} = l_{33} + l_{44} \) and \( M_{aux} = l_{44} + l_{43} \).

The current across the auxiliary windings is ruled by (17). It is interesting to observe that \( i_{aux} \) is independent of the time derivative of the resultant flux, \( \phi \). If the auxiliary windings have the same number of turns, \( i_{aux} \) exists only as the consequence of the leakage coupling between the primary and secondary leakage flux linking the auxiliary windings. Moreover, the current across the auxiliary windings results from the difference between the leakage mutual inductance coefficients of the primary and the secondary with respect to the third and fourth windings, times the time derivative of the primary and secondary currents, as shown in (17).

The auxiliary current produces a flux that opposes the primary and secondary leakage flux. We can say that the auxiliary current, \( i_{aux} \), function of the leakage flux coupling between the primary and secondary with the third and fourth windings, generates a magnetic flux [23] that reduces the leakage flux of the primary and secondary windings. Consequently, the leakage inductance in the transformer is reduced.

Secondly, New Proof Based on Characteristic Roots Method is presented as below:

By considering resistance \( R_a \) between third and fourth windings and \( x_1 = i_1, x_2 = i_2, x_2 = x_3, i_{aux} = x_4 \) as state variables, state equations can be written. Computing poles of transfer function with different amount of \( R_a \) shows rise time reduction for small amount of \( R_a \) (it is proof for connecting two auxiliary winding) due to relation among distance of poles from origin point and rise time.

We calculated poles of the case study pulse transformer for values shown in table 2 that is changed by adding auxiliary windings \( (N_3 = N_4 = 25) \).

For \( R_a = 50\Omega \), \( s = 1.0 + \frac{1006* (-2.9905 + 0.0017 + 0.0920i)}{-0.0017 - 0.0920i} \).
For $R_a = 100\Omega$, $s = 1.0e+006 \times (-5.9095, -0.0017 + 0.0920i, -0.0017 - 0.0920i, -0.0)$.
For $R_a = 0\Omega$, $s = 1.0e+004 \times (-0.1388 + 9.2294i, -0.1388 - 9.2294i, -7.2036, -0.0011)$.

These values prove our approach about connection of auxiliary winding for rise time reduction in pulse transformer.

CONCLUSION

High voltage pulse transformers are often used in association with high voltage pulse generating circuits to further increase the pulse output voltage level. However because of the transformer parasitic elements involved, the transformer is the critical device in shaping the rising characteristics of the output pulse. In this paper, we simulated a two winding pulse transformer based on linkage flux as state variable method. It has been considered leakage inductance and capacitance in simulation. We have obtained conditions for these parameters causing unexpected output pulse. Finally submitted model of pulse transformer was compared by distributed parameters model. Our investigation is useful for analysis of pulse transformer and has good points for selecting and setting leakage parameters of pulse transformer. Especially, it helps us in stages of pulse transformer designing. Furthermore, using two auxiliary winding for improvement of technical characteristics of output pulse is discussed and new proof based on characteristic roots method is done.

REFERENCES


