OPTIMAL ALLOCATION OF CAPACITORS IN UNBALANCED MULTICONVERTER DISTRIBUTION SYSTEMS USING GENETIC ALGORITHMS

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ABSTRACT

Genetic algorithms (GAs) are considered very useful tools for the optimal allocation of capacitor banks in distribution systems. Nonetheless a high computational effort can be requested mainly in large scale systems. In the paper three GA-based methods for the optimal sizing and siting of capacitors in unbalanced multiconverter distribution networks are applied and compared to obtain a good solution with tolerable computational efforts. The first technique explores optimal solutions by means of a microgenetic algorithm. The other two techniques search the solution into a set of feasible candidate nodes obtained by using respectively Inherent Structure Theory of Networks and Sensitivity Analysis based methods. The three techniques are compared with a SGA (Simple Genetic Algorithm) using the IEEE 34-node unbalanced test system.

INTRODUCTION

Shunt capacitors are commonly used in distribution systems for several reasons, in particular to reduce the power losses, to improve the voltage profile along the feeders and to increase the maximum flow through cables and transformers. In order to decide on the sizes and locations for these capacitors, a rather complex mixed non-linear integer program has to be solved. The problem solution should take into account various operation and equipment limits; moreover, the presence of line and load unbalances and of nonlinear devices such as static converters has to be taken into account.

The problem of sizing and location of shunt capacitors in unbalanced distribution systems with nonlinear loads can be successfully solved by means of genetic algorithms (GA) that have been proven to find good solutions in spite of their high computational efforts [1]. The computational efforts increase for large scale systems and when load patterns are taken into account.

In order to obtain a good solution with tolerable computational efforts, several techniques can be applied; among them, in this paper the following ones are considered:

- micro genetic algorithms,
- reduction of the feasible region techniques.

In the micro genetic (μGA) algorithms very small population are employed [2].

As far as the reduction of the feasible region is concerned, in this paper two methods are applied for the identification of a proper reduced number of busbars, referred to as candidate busbars for capacitor allocation. The first technique is based on the Inherent Structure Theory of Networks (ISTN) [3, 4]; this theory that founds on the spectral representation of the admittance matrix, helps individuating the candidate busbars, as the ones in which the connection of a capacitor will give the maximum improvement of the voltage profile. The second one is based on the sensitivity of the power losses to the nodal injection of reactive power; the candidate busbars are the ones exhibiting the maximum sensitivity factors [5].

Once individuated the set of candidate busbars, the problem is solved by means of GAs and generally a good solution can be obtained with less computational efforts.

The paper is organized as follows. The mathematical formulation of the optimization problem of the sizing and siting of capacitors is firstly recalled with reference to the unbalanced distribution systems with linear and nonlinear loads. Then, the techniques applied to obtain good solutions with tolerable computational efforts are described. Many tests on the IEEE 34-node test feeder will be presented and discussed.

OPTIMIZATION PROBLEM OF SIZING AND LOCATION OF SHUNT CAPACITORS

The problem of the allocation and sizing of capacitors in unbalanced distribution systems with linear and non linear loads can be formulated as:

$$\min \ F\left(X,U\right) \tag{1}$$

subject to:

$$g\left(X,U\right)=0 \tag{2}$$

$$h\left(X,U\right)\leq 0 \tag{3}$$

where X is the system state vector (magnitudes and arguments of the phase-voltages), U is the capacitor units vector, placed at each bus. The equality constraints are the three-phase power balance equations at the fundamental frequency and at harmonic frequencies. Regarding the inequality constraints, the bus voltages at fundamental frequency have to fall in the admissible range; moreover, the individual voltage harmonics, the total harmonic voltage distortion at each bus and the line currents at fundamental frequency can not exceed the admissible values.

In this paper, two objective functions are considered; the first one, referred to as F_1, accounts for the total costs as:

$$F_1 = F_C + F_L + F_H \tag{4}$$

where F_C is the cost of capacitors, F_L is the cost of the losses evaluated at the fundamental frequency and F_H is the cost of harmonic distortions. More details on this topic can be found in [1].

The other objective function, referred to as F_2, accounts for the profile of voltage at all busbars as [6]:

$$F_2 = \frac{\sum_{i=1}^{N_{bus}} \sum_{p=1}^{N_{ph,i}} \left(V_i^p - V_{nom} \right)^2}{\sum_{i=1}^{N_{bus}} N_{ph,i}} \tag{5}$$

where V_i^p is the amplitude of voltage at busbar i with phase p, $N_{ph,i}$ is the number of phases at busbar i and N_{bus} is the number of busbars.

SOLVING METHODS

GAs have been proven to find good solutions in spite of their high computational efforts that increase for large scale distribution systems; furthermore, if load patterns and different operating conditions are taken into account, the computational burden of the solution by GAs grows. Even if the optimal design of capacitors is a off line problem, the computational time and the requirements of computer can become excessive.

The solution of large scale optimisation problem can be faced by means of sensitivity approaches that make a preselection of a set of candidate solutions and, thereby, allows a considerable reduction of the computational efforts.

In order to reduce the computational efforts and contemporaneously to maintain certain levels of accuracy, in this paper the following techniques are considered:

- micro genetic algorithms;
- reduction of the search space techniques.

Micro genetic algorithms

In this paper, the μGA already used in [7] has been applied. This μGA evolves with five individuals population and use the selection and the crossover; in particular, the selection is based on the roulette wheel method. In this algorithm, the mutation is not implemented since the diversity is guaranteed by periodical refreshes of population, that take place when the genetic operators are applied. In such cases, in fact, if the new population does not result enough diversified, their worse 4 individuals are replaced by other ones randomly generated.

Reduction of the search space techniques

In the case of siting and sizing of capacitors, the reduction of the feasible region can be obtained by individuating a proper reduced number of busbars, referred to as candidate busbars.

The following procedure is applied:

Step 1) preselection of candidate busbars;

Step 2) application of GAs to solve the optimisation problem (1)-(3); the feasible region for siting is given by the set of candidate busbars and the feasible region for sizing is given by the available standard ratings for capacitors.

Among the available techniques, in this paper the ISTN and the sensitivity technique are proposed to individuate a set of candidate solutions.

Application of ISTN

In the frame of the ISTN, it has been demonstrated that the spectral representation of the impedance matrices - direct or inverse sequence impedance matrix at fundamental frequency or harmonic impedance matrices - defines the inherent system sensitivity structure which helps in describing the sensitivity to circuit variables. When dealing with capacitor placement in unbalanced systems, the sensitivity structure is derived with reference to the three-phase impedance matrix at fundamental frequency [8].

Let us consider an $N_{ph,tot}$ unbalanced power system and its three-phase admittance matrix $\underline{\underline{\dot{Y}}}$ that is a $N_{ph,tot}$ matrix. In [3] it has been proved that the 2-norm of the voltage vector $\mathbf{V}$ (that is a $N_{ph,tot}$ vector) can be evaluated by the following approximate expression:

$$\|\mathbf{V}\| \approx \left(\dot{\lambda}_k^s\right)^{-1} \left| \left(\mathbf{\Gamma}_k^s\right)^H \mathbf{I} \right| \tag{6}$$

being $\dot{\lambda}_k^s$ the eigenvalue of minimum modulus of $\underline{\underline{\dot{Y}}}$ matrix (corresponding to the bus k with phase s), $\mathbf{\Gamma}_k^s$ the corresponding reciprocal eigenvector, $\mathbf{I}$ the current vector.

Then, having in mind that the placement of the capacitors causes a variation of the 2-norm of the vector $\mathbf{V}$ corresponding to the current injected by them, it can be expected that the solution of the optimisation problem (1) may be searched in the set of busbars at which the capacitor placement causes the highest value of the sum at the right-hand side of eq. (6). Since each term of the reciprocal eingenvector is multiplied by the corresponding term of the current vector, then, the searched set of busbars, referred to as Ω, can be quickly selected as the set including the busbars associated to the terms of the higher modulus of the considered reciprocal eigenvector.

Application of sensitivity techniques

With reference to symmetrical systems, it has been demonstrated that the sensitivity of losses to injection of reactive power can be derived on the basis of some elements of the Jacobian of the Newton Raphson single-phase load flow. When dealing with capacitor placement in unbalanced systems, the sensitivity structure is obtained by considering the Jacobian matrix used to solve the three-phase load flow.

The sensitivity of the losses P_L to active and reactive power at load busbars is given by:

$$\begin{bmatrix} \dfrac{\partial \mathbf{P_L}}{\partial \mathbf{P}} \\ \dfrac{\partial \mathbf{P_L}}{\partial \mathbf{Q}} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial \mathbf{V}}{\partial \mathbf{P}} & \dfrac{\partial \boldsymbol{\vartheta}}{\partial \mathbf{P}} \\ \dfrac{\partial \mathbf{V}}{\partial \mathbf{Q}} & \dfrac{\partial \boldsymbol{\vartheta}}{\partial \mathbf{Q}} \end{bmatrix} \begin{bmatrix} \dfrac{\partial \mathbf{P_L}}{\partial \mathbf{V}} \\ \dfrac{\partial \mathbf{P_L}}{\partial \boldsymbol{\vartheta}} \end{bmatrix} \tag{7}$$

where **P** and **Q** are the vectors of active and reactive power at load busbars, respectively, **V** and ϑ are the vectors of voltage amplitude and phases, respectively.
The sensitivity of power losses can be easily determined since, at right side of (7), the matrix can be derived from the Jacobian whereas the derivates of losses with respect to the amplitude and phase of busbar voltages can be analytically obtained.
Then, having in mind that the placement of the capacitors causes a variation of the vector of the reactive power, it can be expected that the solution of the optimisation problem (1) may be searched in the set of busbars at which the capacitor placement causes the highest variation of losses. The searched set of busbars, referred to as Ω^* can be quickly selected as the set including the busbars associated to the terms of the higher modulus of the vector $\partial \mathrm{P_L} / \partial \mathbf{Q}$.

Convergence criteria

As well known, more than one ending conditions can be chosen as GA convergence criterion. One possibility is to stop the algorithm at some finite number of generated individuals and designate the result as the best fit from the population. Another possibility is based on the evaluation for each run of the minimum and median values of the fitness function; if the difference between these two values doesn't exceed a pre-assigned minimum value, the algorithm ends. In this paper both possibilities will be explored.

APPLICATIONS

The problem of sizing and siting of capacitors has been solved for the unbalanced IEEE 34-bus test system shown in Fig. 1 [9] where, obviously, original capacitor banks and voltage regulators have been removed; the IEEE 34-bus test system is an actual distribution system located in Arizona. The total number of system nodes is 80. The voltage level of the test network is 24.9 kV. This system contains a mixture of single- and three-phase lines and loads, making it also quite suitable for testing the proposed algorithm. The complete network data and parameters can be found in [9]. The powers of linear and non linear loads are assumed at 75% of the peak level. Unit capacitors available at any bus are assumed to come in discrete sizes of 50 kVAR.
With reference to the constraints, limits on the minimum and maximum fundamental voltage, on the maximum single voltage harmonic and total harmonic distortion at all busbars have been included. In particular, the fundamental voltage at each busbar is constrained between the 90% and 110% of the nominal value; the maximum single voltage harmonic has been fixed equal to 5% and, finally, the maximum value of the total harmonic distortion is considered equal to 8%.

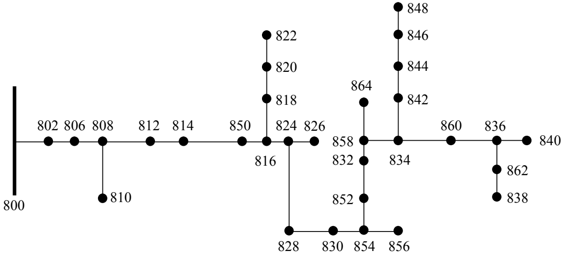

Fig. 1 - IEEE 34-bus distribution test system [9]

The GA procedure proposed in solving methods has been applied to individuate the best-compromise sizing and siting of capacitors in the network.
In the normal GA (SGA), for the application of ISTN and of sensitivity techniques, a population of 40 individuals was used, the crossover probability rate has been assumed equal to 0.7 while the mutation probability rate has been assumed equal to 0.01. In the µGA the crossover probability rate has been assumed equal to 1.
Tab. I shows the capacitor optimal placements and sizes obtained in all procedures proposed in case of F_1 objective function and assuming as convergence criterion a maximum number of generated individuals (N_{in}) equal to 500.

Table I: Optimal capacitor placement and size in case of total cost objective function and with $N_{in} = 500$

Method	Rating and size of capacitors	Total cost OF
SGA	Three-phase capacitor banks at bus #844: 600 kVar; bus #860: 300 kVar	0.7414
µGA	Three-phase capacitor banks at bus #830: 300 kVar; bus #842: 600 kVar	0.7386
ISTN	Three-phase capacitor banks at bus #844: 450 kVar; bus #862: 300 kVar	0.7356
Sensitivity	Three-phase capacitor banks at bus #834: 300 kVar; bus #862: 450 kVar	0.7357

Fig. 2 shows the minimum values of the total cost objective function, for all procedures proposed, versus the number of individuals; the OF is expressed in p.u. of the OF value without the presence of capacitors banks.

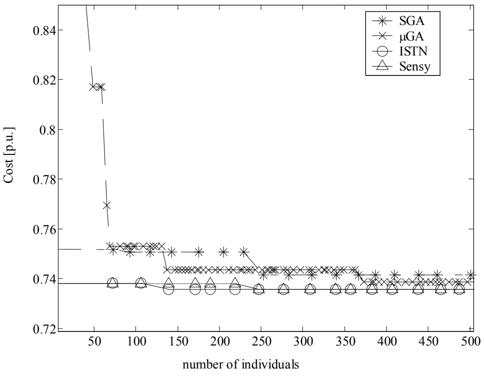

Fig. 2: Minimum values of the total cost objective function, for all GA procedure proposed, versus the number of individuals.

Tab. II reports the values of F_2 objective function corresponding to the optimal capacitor placements and sizes obtained in all procedures proposed and assuming once again as convergence criterion a maximum number of individuals N_{in} equal to 500.

Fig. 3 shows the minimum values of the voltage objective function, for all procedures proposed, versus the number of individuals; the OF is expressed in p.u. of the OF value without the presence of capacitors banks.

Table II: Optimal values of voltage objective function (N_{in} = 500)

Method	Voltage OF
SGA	0.0721
μGA	0.0700
ISTN	0.0700
Sensitivity	0.0710

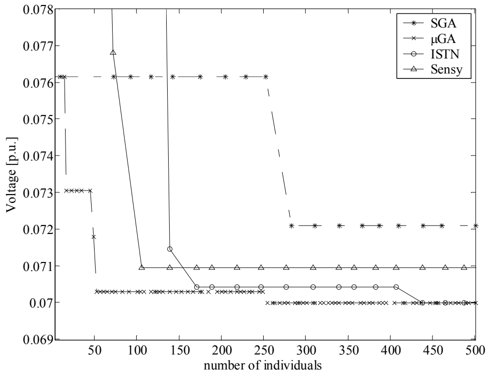

Fig. 3: Minimum values of the voltage objective function, for all GA procedure proposed, versus the number of individuals

Table III shows the objective function values in case of the convergence criterion based on the difference between the minimum and median values of the fitness function.

Table III: Objective function value in case of the second convergence criterion

Method	Voltage OF	Total cost OF
SGA	0.0721	0.7391
μGA	0.0670	0.7317
ISTN	0.0700	0.7355
Sensitivity	0.0710	0.7350

From the analysis of Tables I, II and III and Figs. 1 and 2 the following considerations arise. The results of all the methods applied are very close each other; however, with reference to the first convergence criterion, from Tab. I, it arises that the application of ISTN gives the best results whereas, from Tab. II, it arises that the application of ISTN and μGA give the same objective function. With reference to the second convergence criterion, for both objective functions, the application of μGA gives the best results but with computational time higher than feasible region techniques.

It is important to point out that even if the application of the ISTN should be more efficient if referred to the voltage profile and then it is related to the objective function F_2, it has given good results also with F_1 (see Tabs I and III).

CONCLUSIONS

In this paper, three GA-based methods for the optimal sizing and siting of capacitors in unbalanced multiconverter distribution networks have been applied and compared to obtain a good solution with tolerable computational efforts. The first technique is the application of microgenetic algorithms. The other two techniques are based on the determination of a set of feasible candidate nodes by using respectively the Inherent Structure Theory of Networks and the Sensitivity Analysis. The three techniques have been compared with a SGA (Simple Genetic Algorithm) using the IEEE 34-node unbalanced test system.

The results obtained have shown that, even if the optimal solutions are close each other, globally the ISTN based method give almost always the better results.

Further studies are in progress to test the ISTN based method for different test networks and for multiple operating conditions.

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