STATISTICAL ANALYSIS OF VOLTAGE SAGS IN DISTRIBUTION NETWORK
ACCORDING TO EN 50160 STANDARD – CASE STUDY

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ABSTRACT
Voltage events introduce considerable economic losses and have the high impact on consumers. From an economic point of view the frequency and duration of voltage sags are very important because they can cause huge damage in industrial processes. This paper presents results of voltage dips measurements in several transformer stations in eastern Croatia. Power network analysers (LEM MEMOBOX 800, LEM MEMOBOX 808 and LEM TOPAS 1000, supported by powerful mathematical software) were used for measurements and analysis. The paper presents voltage dip probability functions calculated from the actual measurement data. The intention of the authors is to show a statistical method which could be useful for assessing the total annual event.

INTRODUCTION
Voltage dips are the most frequent cause of power quality problems. They introduce considerable economic losses and have the high impact on industry and other consumers. The most sensitive applications are continuous production lines, lighting and safety systems and computer equipment. From an economic point of view the dip frequency, i.e. the annual number of dips, is very important [1]. When assessing the total annual dip related cost one has to find out how many dips are expected. Some rough estimation can be acquired from measurement over a shorter period. Another approach is to use stochastic mathematical methods for assessing more precise figures.

Measurements of voltage events for a number of domestic transformer stations using LEM MEMOBOX 800, LEM MEMOBOX 808 and LEM TOPAS 1000 power quality analyzers were performed, which enabled the detailed statistical analysis and derivation of probability density functions.

Furthermore, a hill climbing algorithm used for minimizing the chi squared criterion and the best probability distributions used for fitting the measured data are presented in the paper.

PROBABILITY DISTRIBUTION FUNCTIONS
Probability distribution functions are mathematical equations allowing a large amount of information, characteristics and behaviour to be described by a small number of parameters [2]. A probability distribution function has an associated density function, \( f(x) \), that represents the likelihood that a random variable \( x \) will be a particular value. In this paper, lognormal and Weibull probability functions are used for describing voltage dip distributions.

FITTING CURVES TO MEASURED DATA
When probability distribution curves are used to represent empirical data, the information associated with thousands of data points can be modeled with one or two parameters.

Chi Squared Criterion
Chi squared criterion \( (\chi^2) \) indicates how well a model matches the data that is supposed to represent [2]. It is based on density functions and data bin densities:

\[
\chi^2 = \sum_{\text{bins}} \left( \frac{\text{Observed Freq. in Bin} - \text{Expected Freq. in Bin}}{\text{Expected Freq. in Bin}} \right)^2 
\]

\[
\text{Observed Freq. in Bin} = \frac{\text{Number of Samples in Bin}}{\text{Total Number of Samples}} 
\]

\[
\text{Expected Freq. in Bin} = \int_{a}^{b} f(x) dx = F(b) - F(a) 
\]

\( \text{Bin} = a \leq x < b \)

A hill climbing algorithm is used for minimizing the chi squared error of a curve fit. This algorithm guarantee that the parameters are locally optimal.

VOLTAGE EVENTS ANALYSIS
Measurements were performed on several (13) LV transformer stations with domestic consumers. A measurement period used for each consumer was one week, according to the European standard EN 50160. According to EN 50160, a voltage dip is a sudden reduction of the voltage supply to a value between 90% and 1% of the nominal voltage, with duration between 10 ms and 1 minute [3]. Table 1 represents summed voltage events for domestic transformer station measurements. Figure 1 (3D view) which derives from Table 1, shows that most voltage dips are found in two depth categories: 10 – 15 % \( U_n \) and 15 – 30 % \( U_n \).
measurements.

<table>
<thead>
<tr>
<th>Phase L1, L2, L3</th>
<th>Surge &gt; 10 %</th>
<th>Dip &gt; 10 %</th>
<th>10…&lt; 15 %</th>
<th>15…&lt; 30 %</th>
<th>30…&lt; 60 %</th>
<th>60…&lt; 99 %</th>
<th>Interruption</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 20 ms</td>
<td>1040</td>
<td>10</td>
<td>152</td>
<td>15</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>20…&lt; 100 ms</td>
<td></td>
<td></td>
<td>30</td>
<td>9</td>
<td>5</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>100…&lt; 500 ms</td>
<td></td>
<td></td>
<td>10</td>
<td>20</td>
<td>3</td>
<td>3</td>
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<tr>
<td>500…&lt; 1 s</td>
<td></td>
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<td>14</td>
<td>4</td>
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<td>3</td>
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<td>1…&lt; 3 s</td>
<td></td>
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<td></td>
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<tr>
<td>3…&lt; 20 s</td>
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<tr>
<td>20…&lt; 60 s</td>
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<td></td>
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<tr>
<td>&gt;= 1 min</td>
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</tr>
</tbody>
</table>

Figure 2 and figure 3 represent 10 - 15 % $U_n$ and 15 – 30 % $U_n$ voltage dip distributions, and it is obvious that there were much more voltage dips in 10 – 15 % $U_n$ depth category (206), than in 15 – 30 % $U_n$ (48). The data are also retrieved from Table 1.

The next step was to compare those distributions to lognormal and Weibull probability distributions. Hill climbing algorithm is performed by Visual basic program. Each time, the algorithm is initialized with parameters that already represent the general shape of the data set.

![Voltage Events](image)

Fig. 1. Voltage events for domestic transformer stations measurements.

![Voltage dips 10-15 % U_n](image)

Fig. 2. 10 - 15 % $U_n$ voltage dip distribution.

![Voltage dips 15-30 % U_n](image)

Fig. 3. 15 - 30 % $U_n$ voltage dip distribution.
10 - 15 % \( U_n \) voltage dip distribution

Optimal parameters for lognormal function are \( \sigma = 15,8 \) and \( \mu = 12,5 \) with \( \chi^2 = 30,32373 \). Thus, lognormal probability distribution which best describes 10 – 15 % \( U_n \) voltage dip distribution is:

\[
f(x) = \frac{1}{x\sqrt{2\pi}} \exp\left[-\frac{(\ln x - 12,5)^2}{2 \cdot 15,8}\right].
\]

Optimal parameters for Weibull function are \( \alpha = 1, \beta = 0,04 \) and \( \chi^2 = 43,51809 \). So, the best Weibull probability function which describes 10 – 15 % \( U_n \) voltage dip distribution is:

\[
f(x) = 0,04x^{0.04-1}\exp[-x^{0.04}].
\]

Although both probability distributions (lognormal and Weibull) fit 10 – 15 % \( U_n \) voltage dip distribution pretty well and their curves have similar shapes (figure 4), lognormal probability distribution has lower chi square error (\( \chi^2 = 30,32373 \)). So, lognormal probability distribution with parameters \( \sigma = 15,8 \) and \( \mu = 12,5 \) is most suitable for describing 10 – 15 % \( U_n \) voltage dip distribution.

15 - 30 % \( U_n \) voltage dip distribution

Optimal parameters for lognormal function are \( \sigma = 1,398 \) and \( \mu = 0,773 \) with \( \chi^2 = 12,08993 \). Lognormal probability distribution which best describes 15 – 30 % \( U_n \) voltage dip distribution is:

\[
f(x) = \frac{1}{x\sqrt{2\pi}} \exp\left[-\frac{(\ln x - 0,773)^2}{2 \cdot 1,398}\right].
\]

Optimal parameters for Weibull function are \( \alpha = 2,099, \beta = 1,439 \) and \( \chi^2 = 13,86839 \). The best Weibull probability function which describes 10 – 15 % \( U_n \) voltage dip distribution is:

\[
f(x) = \frac{1,439x^{1,439-1}}{2,099^{1,439}}\exp\left[-\frac{x}{2,099}\right].
\]

Lognormal probability distribution has lower chi square error (\( \chi^2 = 12,08993 \)). So, lognormal probability distribution with parameters \( \sigma = 1,398 \) and \( \mu = 0,773 \) is most suitable for describing 15 – 30 % \( U_n \) voltage dip distribution.

Both probability distributions (fig. 5) don't fit 15 – 30 % \( U_n \) voltage dip distribution as well as in the previous case.
That's because there were less voltage dips in this depth category, so there are also less data. Nevertheless, both probability curves in this case have significantly different shape (first ascending than descending) than in the previous case (only descending shape).

**CONCLUSION**

In this paper analysis of voltage dips measurements in several (13) LV transformer stations with domestic consumers in eastern Croatia were presented. A measurement period used for each consumer was one week, according to the European standard EN 50160. Most measured voltage dips were in two depth categories: 10 – 15 % $U_n$ and 15 – 30 % $U_n$. The chi squared criterion is used to identify distribution function parameters. This criterion is also used to compare the fit of lognormal and Weibull functions to a voltage dip distribution. Furthermore, a hill climbing algorithm used for minimizing the chi squared criterion and the best probability distributions used for fitting the measured data were presented and analysed in the paper.

In both cases (both depth categories of voltage dips), lognormal probability distribution is most suitable for describing measured voltage dip distributions.

**REFERENCES**


