MODELLING OF NON-LINEAR ELECTRONIC LOADS FOR POWER SYSTEM STUDIES: A QUALITATIVE APPROACH

Charles CRESSWELL Institute of Energy Systems The University of Edinburgh - UK c.e.cresswell@sms.ed.ac.uk Saša DJOKIĆ Institute of Energy Systems The University of Edinburgh - UK sasa.djokic@ed.ac.uk

Ewen MACPHERSON Institute of Energy Systems The University of Edinburgh – UK ewen.macpherson@ed.ac.uk Kenneth OCHIJE National Grid Plc-UK The University of Edinburgh – UK kenneth.ochije@uk.ngrid.com

ABSTRACT

This paper analyses modelling of two important categories of non-linear electronic loads for which has been almost no previous work reported in existing literature: rectifier and dc-dc converter loads. Their consumption of active and reactive power for a range of steady state voltage supply conditions and other influential factors (e.g., bulk capacitance and total input inductance) is discussed in detail. The presented results show that existing general load models could be used for the representation of active power demand of these loads, but fail to accurately represent their reactive power demand.

INTRODUCTION

Although the importance of the correct load modelling for both steady state and transient analysis of power system performance is commonly recognised (e.g., IEEE Load Task Force, [1]), the majority of power system studies apply only general load models, in which static loads are represented by constant power/impedance/current load models, and dynamic loads with the models of small and large three-phase induction motors. Among the load types generally neglected in power system studies (e.g., highintensity/high efficiency light sources), non-linear electronic loads are expected to have the greatest rise in the ownership in residential sector, contributing to more than 30% of the total domestic electricity consumption by the year 2010 [2]. This paper considers modelling of non-linear dc (direct current) power supplies. Examples of this load type include: consumer electronics, information technology equipment, phone/fax/mobile and network communication equipment, etc. The vast majority of these devices consist of an unregulated diode rectifier with or without a switching dcdc converter (so called switch-mode power supply - SMPS), which is used when the dc voltage output should be regulated or kept constant. In this paper, these two categories of a general non-linear dc supply load are referred to as a "rectifier load" and "SMPS load", Fig. 1. The simplest and most widely used model of static, voltagedependent system loads is "Exponential model", [1]:

$$P_{A} = P_{\alpha} (V/V_{\alpha})^{\alpha} , \quad Q_{A} = Q_{\alpha} (V/V_{\alpha})^{\beta}$$
(1)

where: P_A , Q_A are the actual active and reactive power demands, P_o , Q_o are the nominal/rated active and reactive

power demands, V_o is the nominal/rated system voltage, V is the actual load voltage, α , β are the active and reactive power voltage exponents. For $\alpha = \beta = 0$ constant power load type is obtained; $\alpha = \beta = 1$ and $\alpha = \beta = 2$ correspond to constant current and constant impedance load types, respectively (e.g., [3]). In the general case, exponents α and β could be set to any other (i.e., non-integer) value, in order to provide more accurate models of different types of loads. An extension to this model is "Zip model", which introduced additional frequency-dependent terms [1]:

$$\begin{split} \mathbf{P}_{A} &= \mathbf{P}_{o} \Bigg[\mathbf{K}_{pz} \cdot \left(\frac{\mathbf{V}}{\mathbf{V}_{0}} \right)^{2} + \mathbf{K}_{pi} \cdot \left(\frac{\mathbf{V}}{\mathbf{V}_{0}} \right) + \mathbf{K}_{pc} + \\ &+ \left(\mathbf{l} + \mathbf{n}_{pf1} \Delta \mathbf{f} \right) \cdot \mathbf{K}_{p1} \cdot \left(\frac{\mathbf{V}}{\mathbf{V}_{0}} \right)^{n_{pv1}} + \left(\mathbf{l} + \mathbf{n}_{pf2} \Delta \mathbf{f} \right) \cdot \mathbf{K}_{p2} \cdot \left(\frac{\mathbf{V}}{\mathbf{V}_{0}} \right)^{n_{pv2}} \Bigg] \\ \mathbf{Q}_{A} &= \mathbf{Q}_{o} \Bigg[\mathbf{K}_{qz} \cdot \left(\frac{\mathbf{V}}{\mathbf{V}_{0}} \right)^{2} + \mathbf{K}_{qi} \cdot \left(\frac{\mathbf{V}}{\mathbf{V}_{0}} \right) + \mathbf{K}_{qc} + \\ &+ \left(\mathbf{l} + \mathbf{n}_{qf1} \Delta \mathbf{f} \right) \cdot \mathbf{K}_{q1} \cdot \left(\frac{\mathbf{V}}{\mathbf{V}_{0}} \right)^{n_{qv1}} + \left(\mathbf{l} + \mathbf{n}_{qf2} \Delta \mathbf{f} \right) \cdot \mathbf{K}_{q2} \cdot \left(\frac{\mathbf{V}}{\mathbf{V}_{0}} \right)^{n_{qv2}} \Bigg] \end{split}$$

where in (2a): K_{pz} , K_{pi} , and K_{pc} , are contributions of the constant impedance, constant power and constant current load types, respectively; K_{p1} and K_{p2} are per-unit voltage dependent terms; n_{pv1} and n_{pv2} are voltage sensitivity exponents; n_{pf1} and n_{pf2} frequency sensitivity coefficients; similar notation is applied in (2b) for reactive power.



Fig. 1 : General model of a non-linear dc supply load

GENERAL MODEL OF A NON-LINEAR DC POWER SUPPLY LOAD

Non-linear dc power supply loads are mostly one of two following configurations: a) bridge rectifier on its own, and b) bridge rectifier connected to a dc-dc converter, i.e., the SMPS, Fig. 1. They are discussed in the following sections.

Paper 0458

EMI Filter Circuit

Electromagnetic Interference (EMI) is a term used to describe unwanted high-frequency electrical noise, usually generated by rapidly changing currents, e.g., due to the switching mode of operation of some electronic circuits [4]. As the SMPS are designed to operate in exactly that mode, corresponding switching currents may cause EMI-related problems, and EMI suppression filters must be used. In most applications, bifilar wound inductors and small capacitors are the major components of the EMI filters. Their values depend on the switching frequency and amount of attenuation required, and are typically small (50μ H to 800μ H for the inductor, and in nF range for the capacitor). In the results presented in this paper, a small influence of the EMI filter to changes in active/reactive power demand is considered through the total input impedance.

Power Factor Correction Circuit

A general non-linear dc supply load will draw nonsinusoidal harmonics currents from the ac (alternate current) power supply. In 2001, regulatory standards that limit permissible harmonic emissions of these devices are introduced in Europe [4], but not in other parts of the world, e.g., USA. There is a distinction in [4] between the SMPS/rectifier devices with rated power below 75W (less stringent "Category A"), and above 75W (more tightly regulated "Category D"). Two methods are commonly used to meet regulatory requirements: passive power factor correction (pfc) and active pfc. In passive pfc circuit, an inductor is placed in series with rectifier, reducing the emission of harmonic currents and improving power factor (≈ 0.6 for passive pfc). Active pfc involves an extra dc-dc converter stage, which regulates the current drawn from the supply to a basically sinusoidal shape, resulting in power factor close to 1. The type of pfc employed will be application dependent: Category D devices with passive pfc will require physically bulky, heavy and expensive inductors to meet the regulation, and therefore they employ active pfc [5]. There is principally no difference between the SMPSs/rectifiers with passive pfc and those with no pfc (pre-2001 made and possibly currently produced in USA), as they differ only in pfc inductance. SMPSs with active pfc will also behave as a constant active power load, but their reactive power demand will be essentially reduced to zero.

Bridge Rectifier

The bridge rectifier is the industry standard circuit for the conversion of ac voltage to dc voltage. Fig. 2 shows a generic bridge rectifier, where inductance from the passive pfc circuit $L_{passive}$ is connected in series. The diodes in the rectifier convert the input sinusoidal ac voltage waveform to a rectified voltage waveform. which is then 'smoothed' by the capacitor C_{bulk} , to produce dc voltage with a lower ripple. The aggregate effects of system (source) inductance and circuit inductance are represented by the corresponding total input inductance, L_{in} :

$$L_{in} = L_{source} + L_{passive}$$
(3)



Fig. 2 : Typical bridge rectifier configuration

Rectifier draws input current only when capacitor C_{bulk} is charging, which results in non-linear operation and nonsinusoidal input current, Fig. 6a. The ripple in the rectifier's output voltage is dependent on the charge/discharge rate of the capacitor, i.e., on capacitor size, input voltage and power demanded by load. Acceptable dc voltage ripple, (4a), is application specific and is typically lower than 20%. The hold-up time is defined as a time for which rectifier's output voltage is within the specified limit after a full interruption in input voltage occurs, (4b). It also varies from application to application, but is typically 10ms-50ms [6].

$$C_{\text{bulk}} = \frac{P_{\text{load}}}{n \cdot (V_{\text{num}}^2 - V_{\text{num}}^2)} \times \frac{1}{f}$$
(4a)

$$C_{\text{bulk}} = \frac{2 \cdot P_{\text{load}} \cdot t_{\text{holdup}}}{\eta \cdot (V_{\text{nom}}^2 - V_{\text{dropout}}^2)}$$
(4b)

where: P_{load} is power delivered to the load; V_{max} , V_{min} , V_{nom} and $V_{dropout}$ are maximum, minimum, nominal and lowest acceptable dc bus voltage, respectively; $\eta = P_{load}/P_A$ is the efficiency of the rectifier.

Fig. 3 graphically illustrates several requirements for dc voltage ripple and hold-up time, and compares them with practical values of C_{bulk} found in [7]-[10], which provide 2%-5% voltage ripple and 10ms-20ms hold-up time.



DC-DC Converter

Standard dc-dc converter configuration employs a transistor switch to produce a pulse-width modulated (PWM) voltage from the input dc voltage obtained after the rectifier stage. An IC chip is usually used to control the transistor and, hence, the output voltage of the converter by implementing feedback control. It is also common to find a transformer in the converter circuit, to step up/down generated PWM voltage. The two most common of the numerous dc-dc converter topologies are Flyback and Forward converter.

Paper 0458

Flyback Converter

The Flyback converter is by far the most common SMPS topology in low power applications (\leq 75W). Fig. 4 shows a simplified diagram of a Flyback converter.



Fig. 4 : The SMPS with a Flyback converter topology

Forward converter

The Forward converter is generally found in higher power applications (from 75W up to 500W), Fig. 5.



Fig. 5 : The SMPS with a Forward converter topology

SIMULATION RESULTS

The presented models of the SMPS and rectifier loads are simulated in PSPICE software [11], for a range of rated powers, ac supply voltages, and values of C_{bulk} and L_{in}.



Fig. 6 : Input ac currents: (a) rectifier, (b) the SMPS, for different supply voltage, C_{bulk} and L_{in}

Values of input inductances in Fig. 6 represent typical source/network inductance in Europe (0.65mH-1.1mH, [12]). The results illustrate passive pfc. Active pfc will shape the input current to an essentially sinusoidal waveform, in phase with ac supply voltage. The changes of inductance and capacitance will have similar effects on the current waveforms drawn by both rectifier and SMPS load. The rectifier will draw lower current from the supply on reduced voltages, while the SMPS draws higher current due to its feedback circuitry.

Calculation of Active and Reactive Powers

Instantaneous and root-mean-square (rms) values of input current and voltage are used for the calculation of active, apparent and reactive power, [13], which were integrated using a trapezoidal numerical method, [14]. Figures 7 and 8 show results for rectifier and SMPS load with passive pfc.

$$P = \frac{1}{T} \int_{0}^{T} v(t) i(t) dt, \quad S = V_{RMS} \cdot I_{RMS}, \quad Q = \sqrt{S^{2} - P^{2}} \quad (5)$$



Fig. 8 : Active and reactive power of a 75W SMPS

Fig. 7 shows that active and reactive powers of a rectifier load are strongly influenced by changes in input ac voltage. While value of C_{bulk} and L_{in} have only a small effect on the active power, their influence is substantial on the reactive power. Larger values of L_{in} reduce the reactive power demand, which could be correlated with Fig. 6 and "stretching" effect on instantaneous current waveform.

Fig. 8 shows that active power of an SMPS is essentially constant for all simulated values of all parameters. The reactive power exhibits only small variations with changes of ac supply voltages, and is determined by C_{bulk} and L_{in} .

ANALYTICAL MODELS

The standard Exponential and Zip load models, (1)-(2), and general Polynomial load model, [15], are used for the analytical representation of active and reactive power demands of considered rectifier and SMPS load types. In the Polynomial model, selection of variables is based on their influence on the active and reactive power demands:

$$y_p = \sum_{i=0}^{M} a_i x_i$$
, $y_p = \sum_{i=0}^{M} \sum_{j=1}^{M} a_{ij} \cdot x_i x_j$ (6)

where: x_i and x_j are the selected variables, a_i and a_{ij} are the matching coefficients; M is the total number of variables. The results are compared for following ranges of variables: 10W-75W rated powers, 30μ F-275 μ F C_{bulk}, 2mH-10mH L_{in}, and 0.7pu-1.2 pu input ac voltages.

Rectifier Load

Three considered load models and above mentioned ranges of variables resulted in following analytical representation of rectifier's active, (7), and reactive, (8), power demands:

$$P_{A} = 4.6902 \cdot V^{2} - 6.7404 \cdot V + 2.3222 - 0.85852 \cdot P_{o} + 1.8969 \cdot P_{o} \cdot V \quad (7a)$$

$$P_{A} = P_{o}[0.336 \cdot V^{2} + 0.21 \cdot V - 0.053 + 0.262 \cdot V^{2.2} + 0.262 \cdot V^{2.2}]$$
 (7b)

$$\mathbf{P}_{\mathrm{A}} = \mathbf{P}_{\mathrm{o}} \cdot \mathbf{V}^{2.04} \tag{7c}$$

$$Q_{A} = 47.6 - 0.667 \cdot Q_{o} - 114.3 \cdot V - 2141 \cdot C - 2204 \cdot L +$$

$$+ 1.68 \cdot O_{o} \cdot V - 391 \cdot O_{o} \cdot C - 31.07 \cdot O_{o} \cdot L + 67.6 \cdot V^{2} -$$
(8a)

 $-3952 \cdot V \cdot C + 2494 \cdot V \cdot L + 4274 \cdot C^{2} + 3225 \cdot C \cdot L + 10565 \cdot L^{2}$

$$Q_{A} = Q_{0} [0.266 \cdot V^{2} + 0.1641 \cdot V - 0.042 + 0.243 \cdot V^{2.17} + 0.243 \cdot V^{2.17}]$$
 (8b)

$$Q_{A} = Q_{a} \cdot V^{1.7922}$$
 (8c)

where: V in pu, C= C_{bulk} in μ F, L= L_{in} in mH, Q_o in VAr, $P_o = P_{load}/\eta$ in W.

The SMPS Load

Active power of SMPS load is constant, (9), while changes in reactive power (for passive pfc) are modelled by (10): $P_{-} = P_{-}$ (9)

$$\mathbf{I}_{\mathrm{A}} = \mathbf{I}_{\mathrm{0}}$$

 $Q_{A} = -6.1297 + 18.6 \cdot V + 0.93 \cdot Q_{o} - 643.9 \cdot C - 1615 \cdot L$ (10a)

 $Q_{A} = Q_{o} [0.029 \cdot V^{2} + 0188 \cdot V + 0.272 + 0.236 \cdot V^{-0.033} + 0.236 \cdot V^{-0.033}]$ (10b) $Q_{A} = Q_{o} \cdot V^{0.24}$ (10c)

The Accuracies of Three Analytical Load Models

Table 1 shows average errors of three analytical models.

Model	Rectifier		SMPS	
	Р	Q	Р	Q
Polynomial	2.97%	4.96%	-	9.01%
Zip	2.01%	12.06%	-	13.18%
Exponential	2.91%	21.08%	-	14.20%

CONCLUSIONS

The presented results show that the existing Exponential and Zip load models can both be used to model changes in active power demand of rectifier loads (constant impedance load type) and SMPS loads (constant power load type). These two models, however, produce considerable error in modelling variations in reactive power demand, when these devices use passive pfc. The error is due to strong influence of source inductance (which is network specific), and internal circuit inductance and capacitance. This influence is better modelled with the general Polynomial load model, which will be further simplified in the future research. When active pfc is used, Exponential and Zip load models are sufficient.

ACKNOWLEDGMENTS

The authors acknowledge help from Mr Christopher Basso in PSPICE modelling of the SMPS devices.

REFERENCES

- IEEE Task Force, 1995, "Standard load models for power flow and dynamic performance simulation", *IEEE Trans. on Power Systems*, Vol. 10, No. 3, pp. 1302-1313.
- [2] Energy Saving Trust, 2006, The rise of the machines: A review of energy using products in the home from the 1970s to today, Energy Saving Trust and Market Transformation Programme, London, UK.
- [3] U. Eminoglu and M. H. Hocaoglu, 2005, "A new power flow method for radial distribution systems including voltage dependent load models", *Electric Power Systems Research*, Vol. 76, pp. 106.
- [4] IEC 61000 Series, 2001, *Electromagnetic compatibility (EMC)*, Part 6-3: Emission standard for residential, commercial and light-industrial environments, and Part 3-2: Limits for Harmonic Current Emissions.
- [5] ON Semiconductor, 2006, Power factor correction handbook, Phoenix, AZ, USA.
- [6] Rantec Power Systems Inc, 2006, *Power Supply Hold-Up Considerations*, Los Osos, CA, USA.
- [7] OMRON, 2006, Switch Mode Power Supplies, Technical Specification, Sidney, Australia.
- [8] AULT, 2006, Power Supplies, Technical Specification, Minneapolis, MN, USA.
- [9] PowerStream, 2006, *Power Supplies*, Technical Specification, Orem, UT, USA.
- [10] Power Integrations, 2006, Design Examples, San Jose, CA, USA.
- [11] Orcad, 2006, PSPICE, Software Package, San Jose, USA.
- [12]IEC, 1981, Considerations on reference impedances for use in determining the disturbance characteristics of household appliances and similar electrical equipment, Technical Report TR3 60725.
- [13] IEEE 1515, 2000, IEEE recommended practice for electronic power subsystems: Parameter definitions, test conditions, and test method, IEEE Standard.
- [14] H.J. Mathews, 1999, *Numerical methods using MATLAB*, Prentice Hall, London, UK.
- [15] M.N. Hamlaoui, M.A. Mueller, J.R. Bumby and E. Spooner, 2004, "Polynomial modelling of electromechanical devices: An efficient alternative to look-up tables", *IEE Proceedings, Electrical Power Applications*, Vol. 151, No 6, pp. 758-768.