# A STUDY ON THE PERFORMANCE OF TCIM FULL NEWTON POWER FLOW FOR LARGE DISTRIBUTION SYSTEMS

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## *ABSTRACT*

*This work summarizes performance comparisons between the well-known forward-backward sweep power flow method and a current-injection three-phase power flow algorithm when applied to large scale three-phase distribution systems. The Three-Phase Current Injection Method – TCIM, applies the full Newton-Rapshon method to solve the nonlinear current injection equations which are derived using phase coordinates, and the complex variables are written in rectangular form. This paper intends to discuss simulation results obtained using some practical large scale distribution systems.*

## INTRODUCTION

Even though several methodologies have been proposed to solve the power flow problem in distribution systems, the Forward-Backward Sweep (FBS) [1]-[2] method has been preferred by many authors due to its robustness and simplicity of implementation. However, FBS presents some limitations when control devices are present in the system and also to solve for non-radial (meshed) distribution systems. In recent years, the Three-Phase Current Injection Method – TCIM has been proposed [3]. In this approach, the power flow equations are written in terms of the current injections in rectangular form, and the resulting set of nonlinear equations is solved using the method of Newton-Rapshon. New very efficient routines have been developed to perform matrix ordering and factorization and thus TCIM has become competitive with FBS even for purely radial systems [9]. This paper is an extension of the Panel Discussion [9] in which performance comparisons between TCIM and FBS were conducted using a number of tests. Both methodologies have been implemented using C++ and object-oriented programming techniques. In the present work, in addition to the tests reported in [9], further discussions about characteristic of the ZIP load model, ordering and layer identification performance, simultaneous factorization and ordering and a new algorithm for TCIM is presented.

## OVERVIEW OF TCIM AND FBS ALGORITHMS

### TCIM

In TCIM [3] the three-phase current injection equations are written using phase coordinates and the complex variables are considered in rectangular form, resulting in a set of 6n equations with 6n state variables (where n is the number of system buses). To solve this set of nonlinear current injection equations the full Newton method is applied. The Jacobian matrix is sparse and arranged in 6x6 dimension blocks with the same structure as the nodal admittance matrix.

### FBS

The FBS algorithm is based on following sweeps in the system until the solution is found [1]-[2]. It can be implemented in four steps.
The first step consists in identifying the different layers in the radial, or weakly meshed, system.
In the second step, the nodal current injections are calculated in each node of the system (assuming a flat voltage profile in the first iteration).
The so-called Backward sweep is performed in the third step, and consists in calculating the summation of the branch currents, beginning from the last (lower) layer and working its way up towards the upper layers.
In the fourth step, Forward sweep, the nodal voltages are corrected, beginning from the first layer towards to the bottom layers.
The second, third and fourth steps are repeated until the changes in the currents in all branches calculated are all bellow a given precision.

## METHODOLOGIES COMPARISONS

In this section important features of the two methodologies will be compared. Most of the remarks are suitable for radial or weakly meshed systems only, and without any controls. This restriction is introduced because the FBS can present some difficulties when solving highly meshed systems as well as for systems with some kind of control actions. In such cases additional routines may be required in the solution process but these will not be treated here.

### Basic Algorithms and Performance Evaluation

In order to compare the two methods, flowcharts of the implemented algorithms are shown in Figures 1, 2 and 3. In Figure 1 the basic FBS algorithm is presented, in Figure 2 the basic TCIM algorithm is presented and Figure 3 shows a TCIM algorithm where the ordering and factorization steps are made simultaneously to avoid possible problems when working with ill-conditioned power systems. The processing time of each step in the two methods will be compared; they were taken in average using the simulated systems.

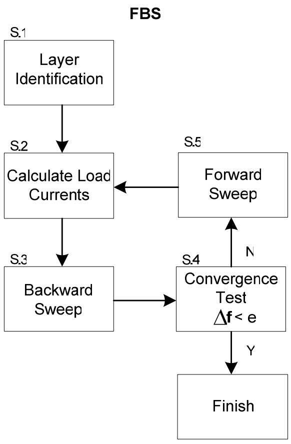


Figure 1 –FBS Algorithm

The steps A.1 and S.1 are not in the main loops. But it is important to emphasize that they are considerable, and will be treated in the results. The computational effort to evaluate step S.1 grows exponentially with the system buses number. The time needed to evaluate step A.1 is approximately the same as to realize the step A.5.
It is considered that step M.2 requires one unit of computational effort.
Thus the time taken to process steps S.2 and S.5 is almost the same as to process step A.2 (B.2). Steps A.3 (B.3) and A.6 (B.6) are also quite similar in computational effort to steps S.4 and S.3, respectively.
Since the Jacobian matrix is very similar to the admittance matrix, the time required to process step A.4 (B.4) is close to that taken by step A.2. And the computational effort needed to process step B.5 is approximately 4-5 units in the radial test systems that were selected for this work. The computational time to process step B.5 (which is composed by factorization and ordering simultaneous) is slightly higher than to process step A.5 (that is composed just of the factorization step, because in this algorithm the ordering step is done previously as shown in Figure 2), but the approach taken in TCIM-B algorithm makes the convergence process much more robust [7].

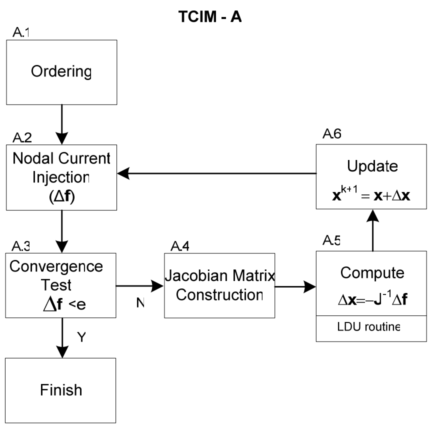


Figure 2 –TCIM-A Algorithm

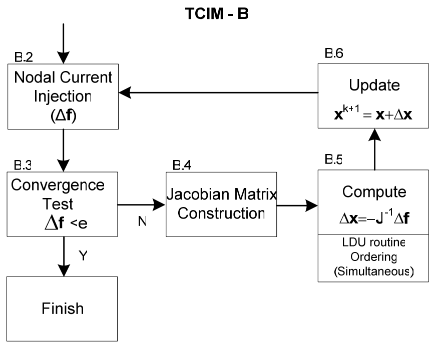


Figure 3 –TCIM-B Algorithm

It is known that in such systems, the factorization time increases almost linearly with the number of buses. When these aspects are considered, it is to be expected that each FBS iteration will be about 5-6 times faster than each TCIM iteration. It should be stressed that this evaluation is valid for radial systems only.

## Other Features

This section presents the summary of comparisons between the main characteristics of the two approaches. It is important to emphasize that the comments and results presented in this work reflect the author's views and experiences, and were taken in simulations with their software.

**Methodology and Computational Implementation**

FBS is considered a simple method and has easy computational implementation. FBS presents an excellent computational performance in radial and light loaded systems, but may present some difficulties when applied to solve systems with at least one of the following characteristics: heavy loaded; control actions (e.g. PV buses, voltage regulators, etc); very meshed; unusual equipment connections, as for example special transformers. Several papers in literature have proposed modifications in the original algorithm to allow the application of FBS to this kind of systems [4]-[5] (just for much meshed systems FBS is really not advisable). But this modifications normally raise the computational effort, where the number of iterations may increase considerably, being related many iterations to achieve convergence in this cases. And since FBS do not have quadratic convergence, higher precision of solution can be obtained only with an increase in the number of iterations.
TCIM is based on the solution of the nonlinear current injection equations using the Newton-Raphson method and presents substantial difficulties for computational implementation. In addition its performance is very much dependent on the use of optimized routines for ordering and factorization of sparse systems [6]-[7]. However once an efficient implementation is achieved, TCIM is extremely robust, and requires few iterations, because it has quadratic convergence characteristic. It works perfectly well in meshed systems and it is simple to include controls.

**Extension to 4/5 wires systems**

FBS can be easily extended for four or five-wire systems. In TCIM this extension is not so easy.
It is important to comment that a more complete algorithm to solve four-wire distribution systems based in the same

methodology of TCIM with some improvements is already developed in [8].

**Robustness**

FBS can present some difficult of convergence in some cases as shown in Methodology and Computational Implementation section of this work.

On the other side, TCIM is extremely robust, achieving the solutions in a few iterations, because it has quadratic convergence characteristic. It works perfectly in meshed systems and after the basis is implemented is simple to include controls.

**Implementation of controls**

The control implementation in FBS is very complex [1], [4] and [5]. FBS needs commonly a great number of iterations to achieve convergence when solving power flows including controls. In TCIM the control implementation is simpler than in FBS. In TCIM the convergence is not very affected by the existence of controls in the system when solving the power flow and the number of iterations does not increase considerably.

**Radial systems**

Both FBS and TCIM can handle easily radial systems.

**Very meshed Systems**

FBS can not solve this kind of system, it can be used just for weakly meshed systems. On the other side TCIM can handle any kind of system; for TCIM there is no difference in the solution process if the system is radial or meshed.

**Processing total time**

If well implemented TCIM performance is competitive with FBS in the solution of all kind of systems and better in very meshed systems and/or with many controls.

**Iteration time**

The FBS iteration time is lower than TCIM iteration time.

## RESULTS

Both methods have been implemented in C++ using Object Oriented Programming techniques. All cases were simulated on a Pentium IV, 3.0 GHz , 512 Mb RAM computer.

Three practical feeders operated by a distribution utility in Brazil have been selected to conduct the comparisons. The feeders are three-phase, radial and unbalanced, and some of them have voltage control devices. Three load levels have been adopted in every case: heavy, medium and light loading. The heavy load level is important to test the robustness of the methods.

The current deviation for convergence was set at $10^{-4}$ p.u.

As mentioned in the previous section, the timings required for steps A.1 and S.1 were included in the comparisons. And it should be mentioned that the ordering routine, a highly optimized sparse linear system solution algorithm, was developed [7].

A summary of the test cases is presented in Table I.

TABLE I
TEST CASES

| Case | Number of Buses | Load levels | Controls |
|---|---|---|---|
| 1 | 503 | 3 | No |
| 2 | 10103 | 1 (Medium) | No |
| 3 | 232 | 1 (Medium) | Yes |

In TABLES II and III some selected results of the 503 buses distribution system are presented. The total solution time is almost the same in both methodologies. In the heavy load case the FBS did not converge. The total and iteration time for TCIM-B was the same of TCIM-A.

TABLE II
503 BUSES SYSTEM, TIME IN SECONDS

| | FBS | | TCIM-A | |
|---|---|---|---|---|
| Load | Number of iterations | Total time | Number of iterations | Total time |
| Light | 8 | 0.032 | 2 | 0.041 |
| Medium | 17 | 0.062 | 3 | 0.061 |
| Heavy | ----- | ----- | 6 | 0.122 |
| | Average iteration time | | | |
| | 0.004 | | 0.0205 | |
| | Layer identification time | | Ordering time for TCIM-A | |
| | < 0.005 | | < 0.005 | |

TABLE III
TIME RATIOS TCIM /FBS, 503 BUSES SYSTEM

| Load | Total time ratios | Average iteration time ratios |
|---|---|---|
| Light | 1.281 | 5.12 |
| Medium | 0.984 | 5.12 |
| Heavy | ----- | ----- |

Two graphics are presented to illustrate the influence of the load model on the number of iterations required by each methodology to find a solution. They summarize simulation results in the 503 buses system for different load models. First the constant power load model was used in the entire system, and the results of the numbers of iterations required are shown in Figure 4. Next the load model was changed to constant impedance in the entire system and the results are shown in Figure 5. In the two cases the total load was a increase factor α.

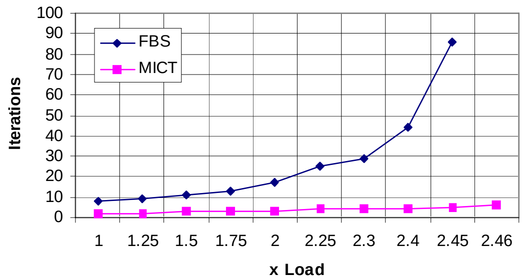


Figure 4 – Results using constant power load model

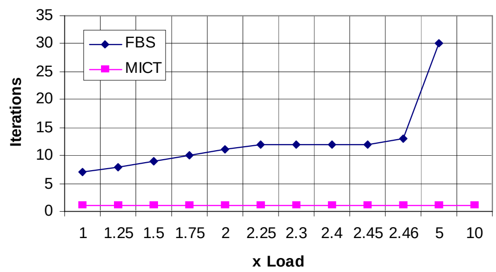


Figure 5 – Results using constant impedance load model

Comparing Figures 4 and 5, it can be seen that the forward-backward sweep method has the same convergence characteristics for both types of load models tested, whereas the TCIM takes just one iteration to find a solution when the loads are modelled as constant impedances. And it is clear that for heavier load values the TCIM requires fewer iteration to achieve convergence than FBS.

In TABLES IV and V results found for the 10103 buses test systems are presented. This system was created by connecting some smaller systems with the objective of testing the methodologies in a large scale system. In this test the TCIM has presented great advantages that can be seen comparing the total solution times and especially when the ordering step of TCIM-A is compared with the layer identification step of FBS. The total time for TCIM-B was 0.473 seconds.

TABLE IV
10103 BUSES SYSTEM, TIME IN SECONDS

| | FBS | | TCIM-A | |
|---|---|---|---|---|
| Load | Number of iterations | Total time | Number of iterations | Total time |
| Medium | 41 | 2.96 | 4 | 1.77 |
| | Average iteration time | | | |
| | 0.0727 | | 0.4401 | |
| | Layer identification time | | Ordering time for TCIM-A | |
| | 9.71 | | 0.351 | |

TABLE V
TIME RATIOS TCIM /FBS, 10103 BUSES SYSTEM

| Load | Only iteration time ratios | Total time ratios | Average iteration time ratios |
|---|---|---|---|
| Medium | 0.598 | 0.141 | 6.11 |

The results obtained for a 232 buses distribution system with two voltage control devices are shown in TABLE VI. The objective of this case is to test the performance of TCIM in the presence of controls. . It can be seen that the number of iterations required by TCIM is much smaller than FBS. Control actions tend to demand many more iterations using FBS.

TABLE VI
232 BUSES SYSTEM, TIME IN SECONDS

| | FBS | | TCIM-A | |
|---|---|---|---|---|
| Load | Number of iterations | Total time | Number of iterations | Total time |
| Medium | 42 | 0.074 | 4 | 0.029 |
| | Average iteration time | | | |
| | 0.00176 | | 0.00725 | |
| | Layer identification time | | Ordering time for TCIM-A | |
| | < 0.005 | | < 0.005 | |

TABLE VII
TIME RATIOS TCIM /FBS, 232 BUSES SYSTEM

| Load | Total time ratios | Average iteration time ratios |
|---|---|---|
| Medium | 0.39 | 4.11 |

## CONCLUSIONS

This paper has presented a performance comparison between a current-injection three-phase load flow algorithm – TCIM and a Forward-Backward Sweep – FBS method.

The authors consider as great advantages of FBS being straightforward to implement and to understand, and being very fast for radial or weakly meshed distribution systems. The overall speed in these systems arises from the low computational burden to perform each iteration. However, such advantages quickly disappear - compared to TCIM - when large scale medium and heavy loaded systems are being considered, also when voltage control devices are present in the system and specially when highly meshed systems are analyzed. The number of iterations required by TCIM does not increase considerably for such systems (in the systems tested by the authors the number of iterations never exceeded 7), whereas FBS demands a much larger number of iterations. Other TCIM advantage is that it allows the inclusion of controls in the Jacobian matrix without changing the original structure of the algorithm and the presence of many controls, as voltage regulators, in the same branch does not cause any problem to TCIM.

Although very meshed systems were not treated in this work, it is important to emphasize that TCIM algorithm and its robustness does not change when solving them, on the contrary of FBS that needs modifications in the algorithm and sometimes is not capable to achieve convergence of these systems.

**REFERENCES**

[1] C. S. Cheng and D. Shirmohammadi, 1995, “A Three-Phase Power Flow Method for Real-Time Distribution System Analysis,” *IEEE Transactions on Power Systems*, vol. 10, no. 2, pp. 671–679.

[2] D. Shirmohammadi, H.W. Hong, A. Semlyen, and G. X. Luo, 1988, “A Compensation-Based Power Flow Method for Weakly Meshed Distribution and Transmission Networks,” *IEEE Transactions on Power Systems*, vol. 3, pp. 753–762.

[3] P. A. N. Garcia; J. L. R. Pereira; S. Carneiro Jr; V. M. da Costa and N. Martins, 2000, “Three-Phase Power Flow Calculations Using the Currents Injection Methods”, *IEEE Transactions on Power Systems*, vol. 15, no. 2, pp. 508–514.

[4] R. Stoicescu; K. Miu; C. O. Nwankpa; D. Niebur; X. Yang; 2002, “Three-Phase Converter Models for Unbalanced Radial Power-Flow Studies”, *IEEE Transactions on Power Systems*, vol. 17, no. 4, pp. 1016–1021.

[5] P. Xiao; D. C. Yu; W. Wa; 2006, “A Unified Three-Phase Transformer Model for Distribution Load Flow Calculations”, *IEEE Transactions on Power Systems*, vol. 21, no. 1, pp. 153–159.

[6] W. Tinney and J. Walker, 1967, "Direct Solution of Sparse Network Equation by Optimally Ordered Triangular Factorization", *proceedings of IEEE*, Vol. 55, pp. 1801–1809.

[7] L. R. Araujo, 2000, “Applications of Oriented Object Modelling Techniques in Sparse Linear Systems”, (In Portuguese) *M.Sc. Dissertation, Federal University of Juiz de Fora*, Juiz de Fora, MG, Brazil.

[8] D. R. R. Penido, L. R. Araujo, S. Carneiro Jr., J. L. R. Pereira, 2006, "Unbalanced Three-Phase Distribution System Load-Flow Studies Including Induction Machines*" Panel on Induction Machine Modeling for Distribution System Analysis, Proceedings of PES General Meeting 2006*, Montreal, Canada.

[9] L. R. Araujo, D. R. R. Penido, S. Carneiro Jr., J. L. R. Pereira, P. A. N. Garcia, 2006, "A Comparative Study on the Performance of TCIM Full-Newton versus Forward-Backward Power Flow Methods for Large Distribution Systems*", Proceeding of 2006 Power Systems Conference & Exposition,* Atlanta, USA.