

## COST ALLOCATION BY COOPERATION AMONG DISTRIBUTED GENERATORS INSIDE A MICRO GRID, USING THE COOPERATIVE GAME THEORY

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### ABSTRACT

*This paper presents the benefits obtained by cooperation with the distributed generators inside a micro grid, with Cooperative Game Theory (CGT), in special, applying the Shapley value to allocate the shared costs. The paper presents a case study where 3 diesel distributed generators providing energy to 3 buildings. The paper shows the load curve, the specific consumption curve of each generator, the economic dispatch of them in each period of load, and share to them the profits from the cost reduction due the cooperation of the distributed generators. The paper makes a comparison between the independent generation (without cooperation) with the cooperative generation, showing the costs and benefits for the customers in both situations. The paper discuss some technical elements that are necessary to form the cooperation, like: connection and use of the utility grid, necessity of coordination to dispatch the power, operation, automation, security and the necessary laws to the cooperation among the distributed generators agents to obtain the real benefits from the cooperation. Another approach in the paper is related with the possibility of the utility to dispatch the capacity from the distributed generators to provide energy to others loads inside the micro grid.*

### INTRODUCTION

Nowadays, the units of distributed generation are turning more and more appropriate to be used in places where the utility presents expansion restrictions or difficulties in the attendance of new loads, be for technical, environmental or regulatory reasons. In a micro grid, where the units of distributed generation (DG) are responsible to provide energy inside the utility area, exist the opportunity of distributed generators to operate, with reduced costs in the case where these generators operate through a cooperative model.

In the micro grid, where the clients are near themselves and acquire energy from theirs distributed generators the connection with them or trough the utility grid is possible, doing bilateral contracts to sell their surplus. In this paper the DG are considering only generators that are small, between 10 to 1.000 kW like in [1].

Suppose that each distributed generator supply energy to one client, the costs will be paid only for this client.

Considering that the client load curves may be different along the day and there are instants where the GD are idle, perceive that the individual generation turns “inefficient” because the generator needs to be project to support the maximum of the load curve or the peak load and in the remain of the time it stay with idle capacity.

Assume now that the clients perceive the possibility to reduce costs cooperating their generators. In this moment a coalition is formed. In this case, it’s necessary to allocate correctly the costs from the coalition.

One way to minimize the costs in the cooperation is to realize an economic dispatch among the clients [2], because they are in the minimum marginal cost. In this case, the generators need to be linked and the resultant profit can be allocated for the clients using the cooperative game theory. In special, with the Shapley function, it’s possible to allocate the profits or the shared costs among the clients [3].

This method always considers the maximum capacity from the generators to determine the economic dispatch. With this, presuppose that the generators always have the maximum capacity available, or that the clients cooperate with 100% of the capacities from their generators. In this case, the profits from the cooperation not always get a maximum value, but only a value maximized for that condition with the total participation when each generator is available.

In this moment, it's interesting to analyze the possibility to get a maximum benefit in the cooperation where this value is greater than the profit obtained with the economic dispatch only.

To get the maximum value or maximum benefits in the cooperation we consider that each client can participate in the game with a partial contribution of their generators, that is, each generator contribute with a partial capacity from their maximum capacity. In this case the economic dispatch is realized and we can find a surplus capacity that can be to sell to the utility or participate in another game.

This surplus capacity is get because the main concept of cooperative game is used to choice a better coalition and not only realize the allocation the shared costs from the economic dispatch.

### THE DISTRIBUTED GENERATORS COSTS

We can consider that the individual costs of the distributed generator are related with the fuel, maintenance and operation (O&M) costs and the cost are related with the power generated along the time. Each diesel generator can be expressed as follows:

$$G_i : \quad G_{i_{\min}} \leq G_i \leq G_{i_{\max}}; \quad (1)$$

$$F_{\text{cost } G_i} = (A_i + B_i \cdot P_i + C_i \cdot P_i^2) \cdot C_{ci} \quad (2)$$

$$\text{Price to sell energy} = P \cdot G_i \quad (3)$$

Where:

$F_{\text{cost } G_i}(P_i)$  = specific cost of distributed generator  $G_i$  (R\$/h)  
 $A_i, B_i, C_i$  = coefficients of distributed generator  $G_i$   
 $C_{ci}$  = cost of fuel (R\$/litre) = R\$1,00/litre (adopted)  
 $P_i$  = Active power of the generator "i" (kW)

In this paper we are considering 3 generators with the characteristics showed in Table 1.

### ECONOMIC DISPATCH

One way to minimize the total costs is realizing the economic dispatch [2]. For this, its necessary realize cooperation with the clients, linking their generators directly or by the utility grid.

For the economic dispatch the traditional method is the Lagrange Function. For this method we need to determine the minimum marginal cost that the generators are supplying the load in each time.

The problem is to minimize the total cost function  $F_{T_{\text{cost}}}(\mathbf{G}(\mathbf{P}_i))$  subject to the constraint that the sum of the powers generated must equal the received load. That is,

$$F_{T_{\text{cost}}}(\mathbf{G}(\mathbf{P}_i)) = F_{\text{cost}}G_1(P_1) + F_{\text{cost}}G_2(P_2) + F_{\text{cost}}G_3(P_3) \quad (4)$$

Subject to

$$dF_{T_{\text{cost}}}(\mathbf{G}(\mathbf{P}_i))/dP_i = \lambda \text{ (Lagrange coefficient)} \quad (5)$$

$$P_{i_{\min}} \leq P_i \leq P_{i_{\max}} \quad (6)$$

$$\sum P_i = P_{\text{load}} \quad (7)$$

The clients can cooperate with themselves to reduce the total costs. One bilateral contract of O&M can be reduced in 10% if there are two generators or more than 20% if the number of generators increases. This percentage can be negotiated with the responsible company for the O&M.

Table 1 – Characteristics of Distributed Generators

	A	B	C	Max. (kW)	Min. (kW)
G1	6	0,1868	0.0005	98,5	6
G2	20	0,1224	0.0003	320	20
G3	53,7	-0,0306	0.0004	509	53,7

### COOPERATIVE GAME THEORY AND THE SHAPLEY VALUE

Game Theory includes not only strategic conflicts but also a possibility of cooperation and coalition forming. In a coalition is assumed to win some total profit that is distributed among its members, and the players tend to make this total pay-off as high as possible and to negotiate their individual participation in it as large as possible [4].

When the clients decide cooperate and if the generators are linked, the first problem is how to allocate correctly the total profit or shared costs with themselves.

The most usual way to allocate shared costs is the Shapley Function [Shapley, 1953]. This function from Cooperative Game Theory is used to divide with the "fair" way the total shared costs or benefits from the cooperation among the players. In our case, the clients decide cooperate their generators and get reduced costs with the economic dispatch and with the O&M negotiation.

The equation that represents the Shapley function is:

$$\phi_i(v) = \sum_{i \in S: S \in 2^N} \frac{(N - |S|)! (|S| - 1)!}{N!} \times [v(S) - v(S - \{i\})] \quad (8)$$

Where:

"i" is the player

$\phi_i(v)$  is the cost allocated to player "i" (Shapley value)

"S" is the coalition with the player "i"

"N" is the number of total players in the game

$v(S)$  is the value of the allocated cost or profit in "S"

$S - \{i\}$  is the coalition "S" without the player "i"

$v(S) - v(S - \{i\})$  is the incremental cost that the player "i" contribute for the coalition "S", normally called of marginal cost of the coalition "S"

The Shapley value represents the profit (or cost) allocated to each generator in the coalition.

### CASE STUDY WITH THE DISTRIBUTED GENERATORS AND DISTRIBUTION UTILITY

Suppose that the game is formed with 3 generators G1, G2 and G3 and the distributed utility D, showed in the Figure 1.

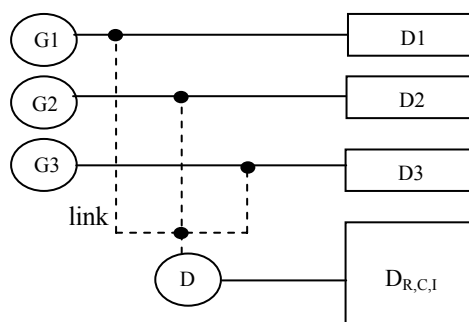


Figure 1: Three distributed generators in coalition.

**The Load Curve of the Clients D1, D2 and D3**

Now we are doing analyze some situations: First when the clients are no linked, second when they are linked without the utility and the last when all the generators are linked with them and with the distribution utility. The load curves for the clients D1, D2, D3 and respectively G1, G2 and G3 are showed in Figure 2.

**First situation: without cooperation (not linked)**

Suppose that the clients are not in cooperation, where the generators G1, G2 and G3 are supplying energy to D1, D2 and D3 respectively. In this case the individual costs are:  
 Cost of D1 =  $F_{cost}G_1(P_{1 \text{ in } t_1}) * t_1 + F_{cost}G_1(P_{1 \text{ in } t_2}) * t_2 + F_{cost}G_1(P_{1 \text{ in } t_3}) * t_3 = 14,27*10 + 19,01*11 + 29,25*3$ ; then Cost of D1 = R\$ 439,56. For D2 and D3 we have:  
 Cost of D2 = R\$ 1.510,61; Cost of D3 = R\$ 2.221,14  
 The total cost (without coalition) is: R\$ 4.171,31

**Second situation: only the generators are linked (not linked with the utility). Optimization case.**

Suppose that the clients are now in a coalition, where the generators G1, G2 and G3 are supplying the total power for the clients. In this case, the total cost is obtained with the economic dispatch using the Lagrange method.

For t1 we have the total load  $DT_1 = 340kW$ , with  $G1_{optim} = 6kW$ ;  $G2_{optim} = 81,57kW$ ;  $G3_{optim} = 252,43kW$   
 For t2 we have the total load  $DT_2 = 780kW$ , with  $G1_{optim} = 98,5kW$ ;  $G2_{optim} = 280,14kW$ ;  $G3_{optim} = 401,36kW$   
 For t3 we have the total load  $DT_3 = 807,5kW$ , with  $G1_{optim} = 98,5kW$ ;  $G2_{optim} = 295,86kW$ ;  $G3_{optim} = 413,14kW$

With the coalition, the shared (total) cost in each time is:  
 For t1: R\$1.105,83; for t2: R\$ 2.342,32 for t3: R\$ 663,17  
 Now, to allocate the shared costs for the clients we use the Shapley function, getting for one day:  
 Cost of D1 = R\$ 421,54; Cost of D2 = R\$ 1.490,82  
 Cost of D3 = R\$ 2.198,96  
 The total cost (with coalition) is: R\$ 4.111,32  
 The difference from the first and second situations is the profit of the coalition. The total benefits is R\$59,99 for each day of operation with coalition.

**Third situation: the generators are linked with the utility.**

In this case, it's possible to determine a best coalition among the generators where make possible to sell the power surplus obtained. For this, its necessary determine the minimum capacity for each generator in the coalition to preserve the benefits from the economic dispatch and to allow selling the surplus. The upper part of Figure 2 shows this situation.

To determine the minimum capacity for each generator it's necessary to consider the worst situation in the coalition, where the demand is maximum in the day. In our case we have the worst situation when the generators are at t3 time. At t3 period, the total demand of load is  $DT = 807,5kW$ . At this time the minimum capacity for the generators are:

$G1_{optim} = 98,5kW$ ;  $G2_{optim} = 295,86kW$ ;  $G3_{optim} = 413,14kW$ .  
 Then, the surplus capacity for the coalition is  $Ds = 120kW$ .

Obviously that these surplus exist only when all the generators are in coalition. Then the benefits obtained from the sale of surplus to the utility needs to be shared to them.

For each generator we have the surplus:  
 For G1:  $Ds_1 = 98,5 - 98,5 = 0$   
 For G2:  $Ds_2 = 320kW - 295,86kW = 24,14kW$   
 For G3:  $Ds_3 = 509kW - 413,14kW = 95,86kW$

For this case we are considering that each client contributes partially for the coalition and sell the difference from their maximum capacity to the distribution utility [5]. To sell the surplus we adopted that the value of the energy is 30% more than their individual costs.

Obvious that the cost for the distribution can be expensive considering a traditional condition (buying from great generators through line transmission), but in restriction cases, where the distribution cannot attend satisfactory the clients, this procedure becomes interesting for the utility, making possible to buy the surplus from the cooperated generators.

Considering that the power surplus will be sell to the distribution utility, them to divide the benefits from the efficient coalition we will use the Shapely function again.

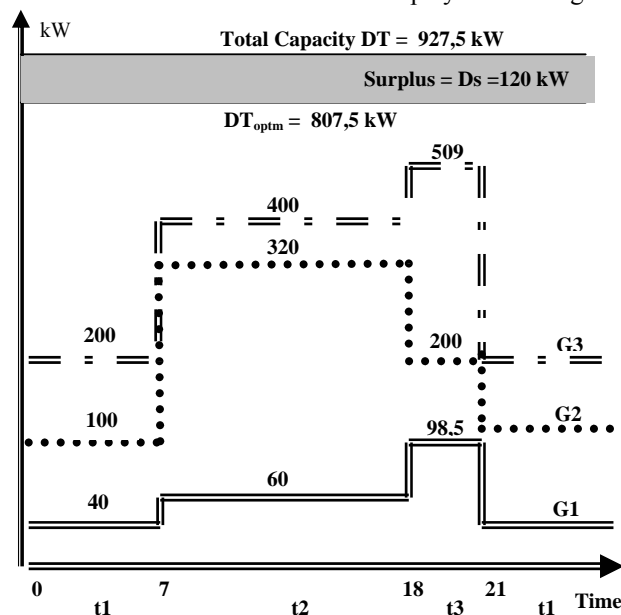


Figure 2: Load curve of clients D1, D2, D3 in one day.

For  $t_1$  we have the new total load  $DT_1 = 460$  kW, where  $G1_{optim} = 20,66$  kW;  $G2_{optim} = 141,77$  kW;  $G3_{optim} = 297,57$  kW  
 For  $t_2$  we have the new total load  $DT_2 = 900$  kW, where  $G1_{optim} = 98,5$  kW;  $G2_{optim} = 320$  kW;  $G3_{optim} = 481,50$  kW  
 For  $t_3$  we have the new total load  $DT_3 = 927,5$  kW, where  $G1_{optim} = 98,5$  kW;  $G2_{optim} = 320$  kW;  $G3_{optim} = 509$  kW

The new total costs and the new benefits (or profits) for each generator (or client) in the game can be calculated with the Shapley function. The Table 2 shows the final results for the efficient coalition with the sale of power surplus to the utility and their respective costs.

### TECHNICAL PROCEDURES FOR COALITION WITH THE DISTRIBUTION UTILITY

To make possible the cooperation with the clients in the game it's necessary to accomplish some technical procedures. On reference standard to distributed generation is IEEE 1547. In Table 3 we can see some typical interconnection equipments used in DG applications. Based on the installations today, the principal elements of a typical generator interconnection to the primary feeder for larger DG, includes: CT and VT to measure the current and voltage conditions; Relaying protection package; Low-voltage circuit breaker; Disconnect switch; Step-up transformer; High-side protection device (fuse, recloser or circuit breaker).

In the case where the generators are small (less than 20kW) we can have more simplified characteristics. But in all cases, it's necessary to have one load and generator meter, to measure the total contribution for the generators in the coalition.

Its very important knows that there are different procedures depending on the scale of the distributed generators. The Table 3 shows some comparison.

### CONCLUSION

The efficient coalition allow the clients (players) to get more benefits because they cooperate to obtain the minimum capacity for each generator and can sell the power surplus to the utility distribution. It's important to note that others coalitions could be formed, like G2 with G3 only, but in this case the total benefit for each one would be smaller than the grand coalition. The game theory helps us to allocate the shared cost through the Shapley function.

### REFERENCES

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- [5] Gama, P.H.R.P., 2007, "Cost Allocation among distributed generators through fuzzy cooperative game theory", Thesis, University of São Paulo, Brazil.
- [6] EPRI, 2001, "Integrating Distributed Resources into Electric Utility Distribution System", Technical Report, USA.

Table 2 – Costs and benefits for an efficient coalition.

Coalition	Cost per day (R\$)			
	G1	G2	G3	Total
1 <sup>a</sup> Situation	439,56	1.510,61	2.221,14	4.171,31
2 <sup>a</sup> Situation	421,54	1.490,82	2.198,96	4.111,32
3 <sup>a</sup> Situation	452,62	1.653,80	2.770,20	4.836,62
Revenue from distribution utility (R\$)				
3 <sup>a</sup> Situation	31,08	162,98	571,24	765,30
Benefits per day (R\$)				
1 <sup>a</sup> Situation	0	0	0	0
2 <sup>a</sup> Situation	18,02	19,79	22,18	59,99
3 <sup>a</sup> Situation	26,80	51,70	182,36	260,85

Table 3 – Comparison of Typical Interconnection Equipment Used in DG Application [6].

Description	Small Scale Residential DG	Small Scale: Commercial and Industrial DG	Intermediate Scale: Commercial and Industrial DG
Type of DG size, and operating voltage	Usually single phase, less than 20 kW, 120/240 V -typically a single unit	Usually 25-1 00 kW, three phase, 208Y/1 20 or 480Y/277 V -typically a single unit	Usually 100-1 000 kW, three phase, 208Y/120 or 480Y/277, volts - often multiple units in parallel
Primary protection device and disconnect means	Use existing primary fuse	Depends on layout of facility	3 pole switch and/or breaker may need to be added
Low voltage disconnect switch for generator	Yes (must be visible break, load break, lockable disconnect switch)	Yes (must be visible break, load break, lockable disconnect switch)	Yes (must be visible break, load break, lockable disconnect switch)
Low voltage protection device	Fuse or breaker suitable	Breaker – typical molded case circuit breaker - unit may also use contactor in addition	Likely motor-operated circuit breaker
Generator and utility system protection relays	Basic protection as per IEEE 929/UL1741 for inverters and basic anti-island protection for rotating equipment	Fairly intensive protection but may be lacking some functions of the larger generators	Fairly intensive protection but may be lacking some functions of the larger generators