LEAST SQUARE METHOD AND BILEVEL PROGRAMMING APPLIED TO WORKS SELECTION IN THE DISTRIBUTION SYSTEM

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ABSTRACT

In this paper we propose a work selection methodology to the medium-voltage distribution system aiming to adequate the reliability continuity index (DEC) to the limits defined by the regulatory agency. The Portuguese abbreviation DEC corresponds to the English abbreviation SAIDI (System Average Interruption duration Index). That step comes after the technical and financial classification phase of the works. Based on minimum square method concept and using bilevel programming, it presents a mathematical treatment and applies this methodology in part of COPEL´s annual works planning program. COPEL is a Brazilian utility company of generation, distribution and commercialization of electricity.

INTRODUCTION

Traditionally, the utility company makes an annual works planning program for the medium-voltage distribution system, where it uses previously elaborated criteria to prioritize the planned works. Generally this decision is made targeting the optimization of the society's general benefit through analysis of voltage drop, losses and not-supplied energy.

However, with the increase in the rigour of the continuity index standards and the introduction of monetary penalties to the utilities, not always the selection of works by old criteria reaches the desired aims. It now needs a careful planning in order to control the demand of directed investment to increase the reliability. It is fundamental to the utility company to develop methodologies which measure the continuity indexes and their correspondent variations to each intervention in the network. Consequently, it is necessary a method which helps the identification of the type and sequence of investments that will be applied in the electric network in order to reach the demanded limits by the regulatory agency.

In order to help the utility company in the decision-making process, we propose a mathematical methodology based on the least square method and the bilevel optimization. From a range of planned works, already approved by a technical and economic point of view and with the data of investments and the expected earning to each one, this methodology selects the sets of planned works which brings the greatest avoided DEC benefit to the company and/or includes the greatest number of sets of works into the target. Different scenarios were analyzed, resulting in the best list of works, limited by the board of directors' approved budget.

The simulations were run on a LINGO application using mathematical language. It was tested in parts of a real program of works in an electricity company in Brazil (COPEL) and compared to the planned works list obtained by the method of cost-benefit (R$/DEC). It was also made an analysis in relation to the penalties applied by the regulatory agency.

To estimate the benefits of each work, it is used the CHI (abbreviation for consumer-hour of interruption), given by the following equation:

$$CHI = \sum_{i=1}^{n} Ca(i).t(i) \quad (1)$$

$Ca(i)$ – Number of consumption units interrupted in one event (i), in the period of verification

$t(i)$ – Duration of each event (i) in hours, in the period of verification;

i – Index of events occurred in the system that provoke interruption in one or more consumption units.

n – Maximum number of events in the considered period.

The benefit calculation of each work is obtained a priori using the Payoff Method [1] which results in a value estimate to be saved in CHI. The cost is estimated with the average value calculation for each work.

It is considered that each work presented had been already studied before and considered viable from a technical and economic point of view.

LEAST SQUARES METHOD APPLIED TO WORKS SELECTION

The central idea of the least square method is to find a function which adjusts best to the set of given spots, minimizing the square difference among them.

Its use applied to work selection consists in making the opposite, which means, from the target of the set, that is known, select the best spots (works), in a way that the distance between the target and these spots will be minimized.

The application can be better explained in chart 01 below.

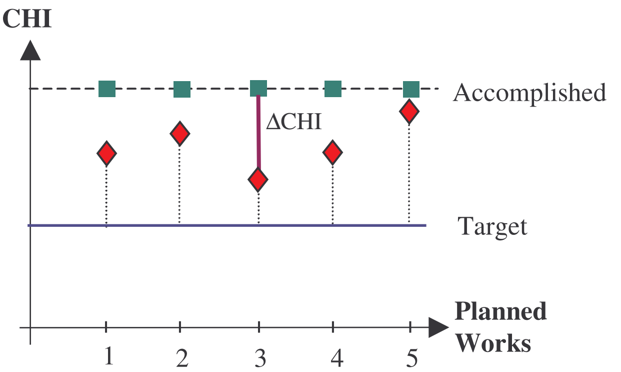


Chart 01- Least Square Method applied to work decision-making.

The green squares show CHI before work. The red diamonds show CHI after work has been done. Being ΔCHI (reached benefit) - which is the CHI saved with the work execution – the distance between them. Least square method applied to this case consists of minimizing the sum of the squared differences between ΔCHI and the target, in the various works, differentiating whether they are being done or not and choosing the set of work which is nearer to the target.

Just to exemplify, let us suppose that the five works had the same execution value, and the budget was enough to execute just one work. The chosen work would be work 3 as visually we can realize that it is the one which has shortened most its distance to the target. And it is the one which has the largest ΔCHI, therefore the one which shows the best benefit. With one work this task is easy to be executed but the same is not true when there are hundreds of works with different costs. If this example was extrapolated to a distribution utility company, the target could be general and works would be chosen following a priority order until they had run out of budget or the target had been reached. The more are the works, the more complex are the calculations which makes necessary specific computer tools.

Another way to explain chart 01 is that the proposed works would be in the same set and there would be a set target. In the same way, this methodology would select the best works until the target was reached.

The function which represents mathematically this idea, called objective function, is described as:

$$\min\left[\sum_{i=0}^{ma}\left(\frac{CHI(X_i).\beta_i}{CHI_p}\right)-\alpha\right]^2 \qquad (2)$$

subject to restriction:

$$\sum_{i=1}^{n}\beta_i.X_i \le R \qquad (3)$$

X_i is the amount of money invested in the work.
ma is the number of proposed works.
β_i is the decision variable (1 do the work / 0 do not do)
CHI_p is the predicted CHI which means, the target. If equals to 1 means, 100% of the target
R is the approved budget to be used in a work.

MATHEMATICAL MODELLING APPLIED TO SETS

Another common situation in an electricity distribution company, is when the company is in the general continuity target, but there are various sets of works which have extrapolated their targets. As the budget generally is not enough to execute all the planned works, there is a doubt where use the budget to include the great majority of the sets in the target. This situation is represented in chart 02, where the doted line is the accomplished benefit (in CHI) and the blue line is the target.

There are three sets: A (works 1,2 and 3), B (works 4 and 5) and C (just work 6). The sets A and C show accomplished CHI beyond the target.

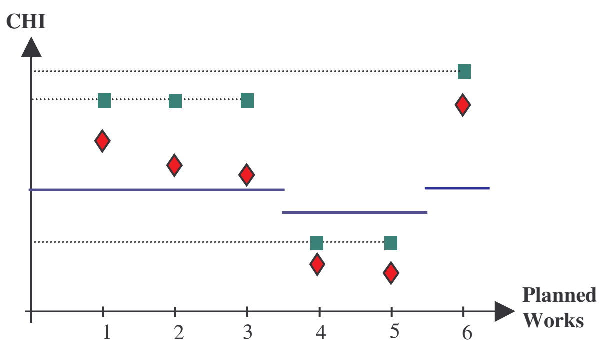


Chart 02 – Least square method applied to works selection, considering the sets of works.

In order to help the sets maximization, a new equation is proposed:

$$\left(\frac{CHI(j,k)}{CHI_p(j)}-\delta\right)\le\varepsilon \qquad (4)$$

where:

$CHI(j,k)$ is an array with benefit in works CHI in relation to the one registered the year before, being j the set and k the works.
$CHI_p(j)$ is a vector with the targets of the sets.
$CHI_a(j)$ is the CHI accomplished.
δ - If equals to 1, means 100% of the target.
ε - It is an input parameter which measures the tolerance in relation to the target. If equals to 0, set has reached 100% of the target.

Then, there is an inclusion of a new restriction to the sets from 1 to n:

$$\left[\left(\frac{CHI_a - \sum_{i=1}^{ma} \Delta CHI(X_i)}{CHI_p}\right) - 1\right] \le 0 \qquad (5)$$

BILEVEL OPTIMIZATION

The problem with bilevel programming is that it contains an optimization problem with a hierarchical structure where a subset of variables is restricted to be a solution to an optimization problem with the remaining variable as parameters [2,3]. The singular structure of the bilevel problem or else the multi-level problem, offers a greater facility in the definition of various problems which involves hierarchical decision processes.

Bilevel optimization is associated to a model which involves two decision agents. The first agent – superior or leader – decides resorting to the first set of variables (x), while the second agent – inferior or follower - controls the second set of variables (y). The superior makes a decision according to its objective. Once made this decision, the inferior reacts according to the objective which is associated to it. A high level decision-maker is capable of influencing the decisions made in a lower level without having a full control of the actions.

The superior agent decision may influence not just the inferior choice possibilities but also in its criteria of choice. The reaction of the inferior agent makes the superior agent rethink its strategy, making new decisions. The decision are top-down throughout the hierarchical chain, however low level decisions affect the high level ones.

The general formulation of a bilevel programming problem is [2]:

$$\min_{x,y} \quad F(x,y)$$
$$\text{s.t.} \quad G(x,y) \le 0$$
$$\textit{where } y \textit{ resolves}$$
$$\min \ f(x,y)$$
$$s.t. \ g(x,y) \le 0$$

where $x \in \Re^{n1}$ e $y \in \Re^{n2}$. The variables of problem are divided into two classes, namely the upper-level variables $x \in \Re^{n1}$ and the lower-level variables $y \in \Re^{n2}$. Similarly, the functions $F : \Re^{n1+n2} \to \Re$ and $f : \Re^{n1+n2} \to \Re$ are the upper-level and lower-level objective functions respectively, while the vector-valued functions $G : \Re^{n1+n2} \to \Re^{nu}$ and $g : \Re^{n1+n2} \to \Re^{nu}$ are called the upper-level and lower-level constraints respectively. Upper-level constraints involve variables from both levels.

For the case proposed in this paper, work selection modelling could be shown as:

LEADER – Main office – Controls the amount of resources to be invested.
Responsible for the general benefit.
Sets priorities according to the company's policies.

FOLLOWER – Regional planning (responsible for sets).
Proposes and selects sets of works (determination of costs and benefits of each work in the sets).

In this case, the objective function and the inferior level restriction would be eq. (2) and eq. (5) respectively, minimizing the budget according to the proposed works in each set.

Superior level objective function would be defined by company's board of directors, which in this case is the maximization of sets included in the target, subject to the company's budget restrictions (R). Consequently the objective function and superior level restriction would be:

$$\text{Max } \{ \text{ N } \} \qquad (6)$$

where N is the number of sets to be defined in the target, subject to the budget restrictions (eq. 3).

COST-BENEFIT RELATION MODEL (CBR)

To be compared to the least square method, we will use a work selection technique used in the electricity sector – the cost-benefit method (CBR). Generally expressed in values, this metric shows how much benefit results to the applied unitary cost. In other words, how much money is applied to obtain an improvement in liability.

Cost-benefit relation proposed can be defined by cost divided by saved CHI as in the formula below:

$$CBR = \frac{Work\ Cost}{Saved\ \ CHI} \tag{7}$$

Translating this mathematical relation and adapting it to the optimization:

$$CBR(CHI(i)) = \max\left(\sum_{i=1}^{ma} \frac{X_i}{\Delta CHI(i)} * \beta_i\right) \tag{8}$$

EXAMPLE

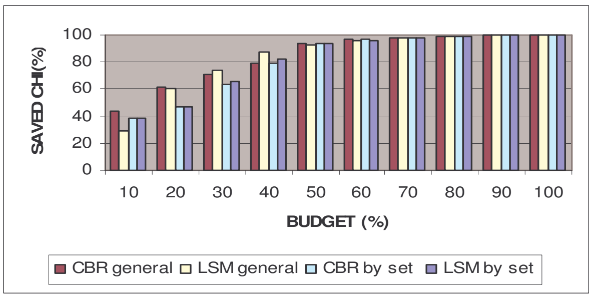
The example below is part of the work planning program from COPEL – The electricity company from Paraná state in Brazil, whose general data are summed up in table 01below:

Number of works	124
Total benefit (CHI)	2.109.137
Total number of sets which can be benefited with works	32
Sets which can be defined in the target	21
Sets which cannot be defined in the target or reached the target	11
Total investment (R$)	49.616.657,00
Total avoided fine (R$)	519230,00

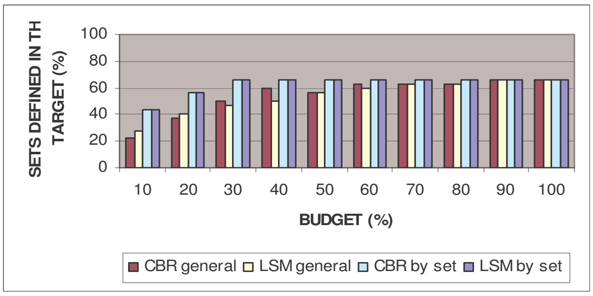
Table 01 – Example –General data.

Simulations were made using least square method and compared with cost-benefit model. They will be presented in two scenarios. The first scenario does not use the concept of sets – aiming the company general target – called general. The second uses bilevel programming and try to maximize the number of sets defined in the target.

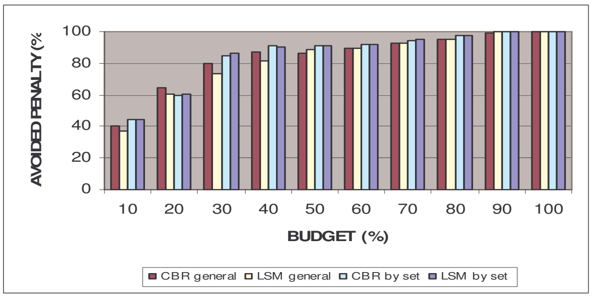
The results of the simulations are shown in the pictures 3, 4 and 5 respectively:



Picture 3 – Benefit according to the applied budget.



Picture 4 – Number of sets defined in the target according to the budget.



Picture 5 – Avoided penalties according to the applied budget.

CONCLUSION

Selection of works in a distribution system is not a trivial task, seeing that it involves a lot of variables and a great volume of works. To reach the objective of defining the greatest number of sets in the target, it is viable the development of a specific methodology to help in the decision-making.

The best chosen works in order that the sets reach the target not always bring the greatest benefits to the company. We can observe that least square method does not interfere in the budget reduction, but shows the best way to use a defined budget.

The least square method used in the works selection together with bilevel optimization, is still being developed, but seems to be a useful tool to works evaluation in the distribution system.

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