However, the distribution grid and the corresponding load feed-in of e.g. wind turbines by stochastic modelling. The recent development of electricity market models focused on the consideration of uncertainties of the intermittent effects of growing distributed generation penetration. This leads to an increasing interest in optimization models which are able to estimate the economical and technical effects of distributed generation given to the power producers. Like fixed feed-in tariffs, since there are no incentives for an improved grid operation due to the altered power flow characteristic. Moreover, the changed voltage quality has also to be considered when defining aggregated points with common power feed-in and consumption. Hence, the alternating load flow between individual nodes of the grid and consequently the voltage profile at individual nodes cannot be considered. This may lead in many cases to unrealistic and inaccurate results. To improve the consideration of the grid and the load flow in stochastic electricity market models, several approaches can be applied. However, these approaches lead to different results and are differentially applicable. To investigate the effects of various methods to describe the load flow, results of several approaches are compared and evaluated.

ABSTRACT

To evaluate the integration of distributed generation, a stochastic optimization model will be applied. Existing stochastic optimization models show a rough and therewith not adequate description of the grid. For an improved consideration of the distribution grid, several methods to describe the load flow during optimization are analysed by using an exemplary case study. The obtained results are compared and evaluated.

1. INTRODUCTION

Distributed generation is expected to have an important role in future electricity markets. The increase of distributed generation is enforced by the promotion of regenerative energy sources (RES) as well as combined heat and power plants. However, the integration of distributed generation causes financial and technical impacts on the operation of the power system. These impacts are expected to increase with an increasing share of distributed and renewable generation. In general, the integration of fluctuating and not perfect predictable electricity generation by RES based power plants, like wind turbines and solar power plants, influences the unit commitment of conventional power plants leading to more frequent part-load operation and start-ups. Furthermore, the predominant unidirectional load flow may be changed to a bi-directional one due to spatial scattered generation units that are connected to existing distribution grids. Hence, new bottlenecks may occur and the amount of grid losses may significantly be influenced due to the altered power flow characteristic. Moreover, the changed voltage quality has also to be considered when evaluating the integration of distributed generation. These effects are strengthened with existing promotion schemes like fixed feed-in tariffs, since there are no incentives for an efficient grid operation given to the power producers. This leads to an increasing interest in optimization models which are able to estimate the economical and technical effects of growing distributed generation penetration. Recent development of electricity market models focused on the consideration of uncertainties of the intermittent feed-in of e.g. wind turbines by stochastic modelling. However, the distribution grid and the corresponding load flow is generally not modelled in detail. In fact, such models feature a rough description of the electricity grid by defining aggregated points with common power feed-in and consumption. Hence, the alternating load flow between individual nodes of the grid and consequently the voltage profile at individual nodes cannot be considered. This may lead in many cases to unrealistic and inaccurate results. To improve the consideration of the grid and the load flow in stochastic electricity market models, several approaches can be applied. However, these approaches lead to different results and are differentially applicable. To investigate the effects of various methods to describe the load flow, results of several approaches are compared and evaluated for an exemplary case study.

This paper is organized as follows: Section 2 provides a brief overview on the optimization model. In section 3, applied approaches to describe the load flow are presented. The results of the case study are discussed in section 4. Finally, a conclusion is given in section 5.

2. THE OPTIMIZATION MODEL

The fundamental stochastic model analyses the unit commitment of a given distribution grid based on an hourly description of generation, transmission and demand. It will be applied for the investigation of the variation of system operation costs and electricity prices caused by integration of new technologies. Furthermore, integration measures like grid extension and implementation of storage facilities can also be evaluated.

The model is defined as a stochastic programming model [1], [2]. The stochastic part is represented by scenario trees describing possible wind power forecasts and the application of rolling planning. A detailed description of the modelling approach is given with [3]. The objective function minimises the operation costs in the whole system, compare equation (1). The formulation of the objective function considers the probability of occurrence of individual wind power forecast scenarios. Thus, the minimum of the expected value is determined. The first two summands in equation (1) describe the operation and start-up costs of the power plants. Thereby fuel and additional operation and maintenance costs are considered. The third summand models the transmission costs. The totals of the value of power plant units being online at the end of the
scenario tree reduce the total operation costs (last
summand).
\[
\text{min } V_{\text{obj}} = \sum_{i \in I} \sum_{s \in S} \sum_{t \in T} \pi_i \left( c_{\text{operation}}^{\text{ONLINE}} (p_{i,k,t}, p_{k,l,t}) + s_{i}^{\text{START}, \text{UP}} (p_{i,k,t}^{\text{ONLINE}}, p_{k,l,t}^{\text{ONLINE}}) \right) + \\
\sum_{i \in I} \sum_{s \in S} \sum_{t \in T} \pi_i \left( 1 - c_{\text{TRANS}, \text{START}}^{\text{ONLINE}} (p_{i,k,t}^{\text{ONLINE}}, p_{k,l,t}^{\text{ONLINE}}) \right) - \\
\sum_{i \in I} \sum_{s \in S} \sum_{t \in T} \pi_i s_{i}^{\text{TRANSPORT}} (p_{i,k,t}^{\text{ONLINE}}, p_{k,l,t}^{\text{ONLINE}}) \right)
\]

Technical consequence of the stochastic description of wind power forecasts is the consideration of three balance equations:
- One balance equation for the scheduled delivery of real power considering scenarios of wind power forecasts, called day-ahead market. This balance equation is determined at 12 o’clock for the hours of the following day.
- One hourly balance equation for handling deviations between expected real power productions agreed upon the day-ahead market and the realized values of real power production in the actual operation hour, called intraday market. Hence, the demand is caused by the forecast errors connected to wind power production.
- One hourly balance equation for delivery of reactive power.

Furthermore, decision variables for power output and for transmitted power have to be partitioned: one part describes the different quantities of power sold or bought at the day-ahead market. They are fixed and do not vary for different scenarios. The other part describes contributions at the intra-day-market both for up and down regulation. The latter consequently depends on the scenarios.

Capacity restrictions for electricity producing units are defined for maximum and minimum electric power output. Start-up costs may considerably influence the unit commitment decisions of plant operators. To avoid the use of binary variables, an approximated formulation for modelling start-up costs is used [4]. In order to prevent that units are always kept online, efficiency at part load is considered to be lower than at full load. Further restrictions describing the maximal transmittable apparent power have to be taken into account.

3. DESCRIPTION OF THE LOAD FLOW

To determine the power flow between individual nodes of a distribution grid and the corresponding voltage quality, restrictions on the power flow have to be taken into account during the optimisation of the unit commitment. In the following, several methodologies to describe the power flow are presented. Therefore, a network with two nodes is considered, compare Figure 1.

![Two node network](image)

**Figure 1.** Two node network with \( S = \) apparent power consisting of real power \( P \) and reactive power \( Q \), \( U = \) node voltage, \( Z = \) line impedance consisting of resistance \( R \) and reactance \( X \).

**AC load flow**

The determination of the AC load flow between two nodes is based on the apparent power \( S_{12} \) at node 1 flowing to node 2, compare equation (2).

\[
S_{12} = P_{12} + jQ_{12}
\]

Considering the line current, magnitude and angle \( \delta \) of the voltage, equations (3) and (4) are derived for the active and reactive power load flow at node 1 toward node 2, compare e.g. [5], [6]:

\[
P_{12} = \left( U_{12}^2 - U_1^2 \right) \cos \delta_{12} \frac{R_{12}}{R_{12}^2 + X_{12}^2} - U_1 U_{12} \sin \delta_{12} X_{12}
\]

\[
Q_{12} = \left( U_{12}^2 - U_1^2 \right) \cos \delta_{12} \frac{X_{12}}{R_{12}^2 + X_{12}^2} + U_1 U_{12} \sin \delta_{12} R_{12}
\]

These equations can be used for the determination of the AC load flow within the optimization model, thereby considering the individual wind power forecast scenarios. Real and reactive power line losses are determined implicitly. The magnitude of the transmitted apparent power has to be determined by equation (2). To ensure that given standards of steady-state voltage quality are met, further equations that restrict the steady-state voltages and the power angles within permissible ranges have to be introduced. However, equations (2), (3) and (4) are non-linear and non-convex. Hence, applied to large stochastic optimization models, a solution of the problem may not be obtained in any case.

**Fast decoupled load flow (FDLF)**

Electric power systems operating in the steady-state condition show a strong interrelation between active power flow and voltage angles and between reactive power flow and voltage magnitudes. Whereas the other interdependencies are relatively weak. This characteristic is utilized by the decoupled Newton load flow considering the corresponding Jacobian elements. With the assumption \( U_1 = U_2 = 1 \text{ p.u.} \), the Jacobian elements become constant. The resulting fast decoupled power flow is described with equations (5) and (6). A more detailed derivation is provided e.g. by [6].
\[ P_{12} = \frac{\sqrt{3}}{2} U_{1} Z_{12} \delta_{12} \]  (5)

\[ Q_{12} = \frac{1}{Z_{12}} (U_{2} - U_{1}) \]  (6)

These equations are linear. However, non-linear equation (2) is still necessary to describe the transmitted apparent power. Hence, mathematical problems to solve the optimization model may occur in extreme load flow situations. Furthermore, line losses are not considered.

**DC load flow**

The DC load flow is derived from the AC load flow with following approximations:
- no reactive power balance equations
- all voltage magnitudes set to 1 p.u.
- no line losses

The DC load flow is described with linear equation (7) using the line susceptance \( B_{12} \):

\[ P_{12} = B_{12} \delta_{12} \]  (7)

However, voltage magnitudes cannot be determined. To consider real power line losses, equation (7) is extended as follows, compare e.g. [7]:

\[ P_{12} = B_{12} \delta_{12} + \frac{R_{12}}{R_{12}^2 + 1/B_{12}^2} (1 - \cos \delta_{12}) \]  (8)

Therewith the problem becomes non-linear as well.

**Combination with power system simulator**

The accurate description of the load flow requires a non-linear optimization model. However, the consideration of large distribution systems with stochastic description of uncertainties leads to large optimization problems intractable to be solved. On the other hand, the linearization of the load flow may unacceptably simplify the description of the grid operation. Therefore, the linear optimization model with DC load flow is combined with the power system simulator PSS\textsuperscript{TM}NETOMAC to verify obtained results and to determine the resulting voltage magnitudes [8]. The sequence of actions between the optimization model and the power system simulator works as follows:

1. Determination of the optimal unit commitment and DC load flow by the optimization model.
2. Read-out of resulting real power generation and of real and reactive power demand at individual nodes to power system simulator.
3. Determination of resulting load flows and voltages by power system simulator considering the AC load flow.

## 4. EXEMPLARY CASE STUDY

### Case description

To analyze the presented approaches to describe the load flow, a three node test network was chosen. For each node, the balance equations, capacity restrictions as well as the restrictions of the power flow have to be met. The structure of the network is presented in Figure 2. It simulates a 20 kV medium voltage distribution grid containing a connection to the high voltage grid as well as distributed generation constituted by a wind power farm with stochastic real power feed-in. The connection to the high voltage grid is located in node N1 which is simultaneously the slack node. Power generation in the high voltage grid is modelled as a gas turbine providing real and reactive power. The wind power farm is connected to node N2; an electrical consumer with real and reactive power demand is located in node N3. The parameters of the transmission lines are given in Table 1.

![Figure 2. Structure of the test network.](image)

### Table 1. Line parameters.

<table>
<thead>
<tr>
<th>From Node</th>
<th>To Node</th>
<th>( R' ) [( \Omega/\text{km} )]</th>
<th>( X' ) [( \Omega/\text{km} )]</th>
<th>L [km]</th>
<th>No. of parallel lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td>N3</td>
<td>0.2</td>
<td>0.34</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>N2</td>
<td>N3</td>
<td>0.2</td>
<td>0.34</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Typical load curves and stochastic wind power forecasts for an exemplary time period of two days have been used. Figure 3 shows the assumed real and reactive power demand at node N3. Figure 4 shows exemplary wind power forecast scenarios at node N2 that are expected when the day-ahead market for the second day is cleared. The value of the steady-state voltage at each node has to be within the range of \( \pm 10 \% \) of the nominal voltage, according to DIN EN 50160 [9].
Results

With regard to load flow, the load of the individual transmission lines, the voltage magnitude at individual nodes and line losses represent the main criteria to evaluate the integration of distributed generation. Figure 5 shows the resulting power flow at node N1 derived with the individual methodologies presented above. Thereby, demand at N3 and feed-in at N2 are considered as preset. For the considered time period, the problem can be solved with the AC load flow by different scaling of the model. The results are identical with the results of the power system simulator (PSS) and describe the transmission in reality. The transmission of apparent power derived with the fast decoupled load flow (FDLF) gives a good approximation. Whereas the application of the DC load flow without and with losses (DC and DC LOSS, respectively) lead to major deviations due to non-consideration of the reactive power flow.

The resulting relative errors of the power flow at node N1 are summarised in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>FDLF</th>
<th>DC</th>
<th>DC LOSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>1.30%</td>
<td>10.03%</td>
<td>9.95%</td>
</tr>
<tr>
<td>Max</td>
<td>2.13%</td>
<td>28.41%</td>
<td>28.35%</td>
</tr>
<tr>
<td>Average</td>
<td>1.77%</td>
<td>16.79%</td>
<td>16.72%</td>
</tr>
</tbody>
</table>

The resulting load flow at node N2 is shown in Figure 6. All methodologies lead to same results because there is no reactive power feed-in.

Figure 7 shows the resulting voltage magnitudes at each node derived with those load flow methodologies that consider voltages. The outcome of the power system simulator and the AC load flow are identical. The voltage at node N1 remains at 20 kV because it is defined as slack node of the grid. At node N3, the voltage declines further than at node N2. The application of the fast decoupled load flow results in an overestimation of the voltage level. Furthermore, the voltage has the same magnitude at node N2 and N3 because this approach does not consider reactive power losses between these nodes. The resulting relative failures errors of the voltage magnitude are summarised in Table 3.

Table 3. Resulting relative errors of voltage magnitude at node N1 derived with individual descriptions of the power flow.

<table>
<thead>
<tr>
<th></th>
<th>PSS / AC</th>
<th>FDLF</th>
<th>DC</th>
<th>DC LOSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>1.30 %</td>
<td>1.50%</td>
<td>10.02%</td>
<td>9.95%</td>
</tr>
<tr>
<td>Max</td>
<td>2.13 %</td>
<td>2.50%</td>
<td>28.41%</td>
<td>28.35%</td>
</tr>
<tr>
<td>Average</td>
<td>1.77 %</td>
<td>1.75%</td>
<td>16.79%</td>
<td>16.72%</td>
</tr>
</tbody>
</table>

Figure 7. Resulting voltage magnitudes at individual nodes.
Table 3. Resulting relative errors of voltage magnitudes at node N2 and N3 derived with fast decoupled load flow.

<table>
<thead>
<tr>
<th></th>
<th>Node 2</th>
<th>Node 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>0.06 %</td>
<td>0.22 %</td>
</tr>
<tr>
<td>Max</td>
<td>0.71 %</td>
<td>0.83 %</td>
</tr>
<tr>
<td>Average</td>
<td>0.40 %</td>
<td>0.53 %</td>
</tr>
</tbody>
</table>

Only the power system simulator, the AC load flow and the extended DC load flow approach consider line losses. Figure 8 compares the resulting line losses between the individual nodes. The DC load flow with line losses considerably underestimates line losses.

![Figure 8. Resulting line losses between the individual nodes.](image)

5. CONCLUSION

By the use of a test network, several approaches to describe the power flow in a stochastic optimization model have been analyzed. Compared to the results of the power system simulator, the AC power flow precisely reproduces the power flow, voltage magnitudes and line losses. However, a solution of the problem is hardly found due to the non-linearity of the optimization model. The use of the fast decoupled load flow gives a good approximation of real and reactive load flow and of voltage magnitudes. On the other side, line losses are neglected. Furthermore, the optimization model may have mathematical solving problems in extreme load flow situations. If only real power has to be considered, the use of linear DC load flow is adequate. However, lines of distribution grids, especially on medium voltage level, show a high portion of reactance compared to the total impedance [10]. Hence, the load of individual transmission lines is underestimated. Moreover, the voltage quality is neglected. The presented extension of the DC load flow to consider line losses is not satisfying. Comparing the properties and results of the individual approaches, it is proposed to apply the fast decoupled load flow for analyzing the integration of distributed generation into grids with sufficient line capacities. For further validation of obtained results and in the case of extreme load flow situations, a combination with the power system simulator is used.

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REFERENCES


