PARK THREE-PHASE APPROACH TO ELECTRIC ARC DYNAMIC ANALYSIS

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ABSTRACT

The approach to the circuit theory based on the method of the state equations is introduced and applied to single-phase networks that include circuit breakers. After put in evidence the asymmetry of the three-phase circuits in presence of circuit breakers, the state analysis in terms of phase variables is worked out. The further use of Park transform finally emphasizes the role of the imaginary power on the energetics in the three-phase interruption.

INTRODUCTION

Traditionally, the dynamic analysis of the three-phase power systems assumes time the symmetry and the linearity of the circuit. This allows to make use of the equivalent single-phase networks and the Fortescue transform. The following introduction, usually made upon an experimental base, of the electric arc model, usually in the form elaborated by Mayr and Cassie, removes, with a significant conceptual and computational effort, such preliminary hypothesis.

The present paper shows that the three-phase power networks, usually considered as physically symmetric, become asymmetrical during the opening operation, because of the presence of non-linearity of the three electric arcs. As a matter of fact, the three different arcs are characterized by the same constitutive relation *R=F(i)*, but the non linearity of the functional *F* leads to an instantaneous asymmetrical condition. In this condition, the transient analysis by using single-phase or sequence components network is not possible. Instead, the transient analysis requires an approach based on state equations and on phase variables. This method is used in the present paper.

Furthermore the paper shows that, applying Park approach to the obtained phase quantities, it is possible to analyse all the power and energy aspects related to the three-phase interrupting processes by using the Park imaginary power. This quantity is usually adopted in the three-phase power quality analysis. The Park imaginary power affects the Joule integral and the breaking work of the switch present in the power system. Consequently, it is necessary to take it into account during the system design.

Some numerical examples, related to single- and three-phase applications, show the validity of the proposed method.

SWITCHING ARC

For what concerns the analysis of simple switching transients and for carrying out large system studies, it is often sufficient to model a circuit breaker as an ideal switch. When, studying arc-circuit interaction, the influence of the electric arc on the system elements is of importance, a thorough knowledge about the physical processes between the circuit breaker contacts is absolutely necessary.

Arc modelling has always been one of the main topics in circuit breaker research. Arc models can be classified in three categories. Physical arc models are based on the equation of fluid dynamics and obey the laws of thermodynamics in combination with Maxwell's equations. They consists of a large number of differential equations. Black box models, where the arc is described by a simple mathematical equation and provides the relation between the arc conductance and measurable parameters such as arc voltage and arc current. Usually, black box models consist of one or two differential equations and they are very useful to simulate arc-circuit interaction in network studies.

Parameter models are a variation on black box models in the sense that more complex function and tables are used for the essential parameters of the black box models.

The classical black box models are the Cassie model and Mayr model or a combination of the two models [1-3].

SINGLE-PHASE CIRCUIT

When opening a circuit breaker, the dimensions and the frequencies of the electric arc fully allows the theory of the electric networks with lamped constants. The transient analysis of circuits with electric arc – become complicated because of the non-linearity of the arc – may be carried out with the systematic, topological and numerical approaches that belong to the modern state equations theory [4]. In such case, the electric arc is formally considered as any other one port of the network, *a priori* non-linear and time-varying, of which is known in advance the input-otput relation.

Under the systematic aspect (Fig.1), the networks during the operating phases , adopt as electrical state variables the flux ψ or the current i for the inductors and the charge q or the voltage v for capacitors. In addition to these is added up a suitable state variable r that represents the component circuit breaker now seen as a dynamic element.

The Mayr Cassie black box models brings to a state equation as follows:

$$(1)\quad \begin{cases} \dfrac{d}{dt}r = F\left(r,i\right) \\ v = r \cdot i \end{cases}$$

where the algebraic relation voltage-current represents a resistance and F depends on the used arc models [1-3].

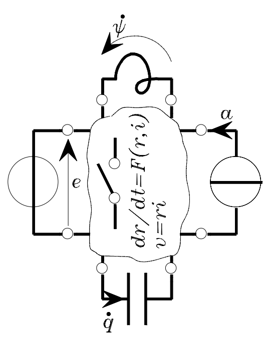

Fig.1. The systematic reading.

For what concerns the topological approach, confirming as preliminary hypothesis, in case of non-degenerating network, the placing of the inductors on the chords and the capacitors on the branches, for the dynamic one port breaker its inserting on the tree or cotree branches does not depend on any preliminary rule and it is carried out time by time according to the typlogy of the network in which is located. This comes from the algebraic characteristic of the voltage-current relation (1) that matches formally the dynamic component arc to any other resistance of the network. The numerical integration of the complete network model brings to the descriptive port quantities $\{v(t),i(t)\}$ associated with the breaker (Fig.2). In such conditions, naming t_i and t_f respectively the starting and the ending instants of the interrupting process, it is possible to deduce, in particular, the maximum arc current I_M, related to the requested interruption power, and the recovery voltage $v_r(t)$, from which depends the potential subsequent restart of a new arc:

$$(2)\quad \begin{cases} I_M = \max\{i(t)\} & t_i \le \forall t \le t_f \\ v_r = v_r(t) & \forall t \ge t_f \end{cases}$$

With reference to the electrical energetics of the interruption phenomenon, are then defined the Joule integral:

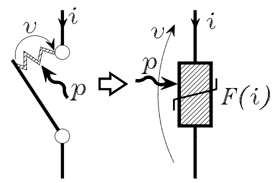

Fig.2. Single phase arc

$$\int_{t_i}^{t_f} i^2 \cdot dt = I_e^2 \cdot (t_f - t_i) = I_e^2 \cdot T$$

linked to the rms arc current I_e and, as a measure of the thermal stress of the component, the electrical breaking work elaborated by the breaker:

$$(3)\quad L = \int_{t_i}^{t_f} v(t) \cdot i(t)\, dt = P \cdot (t_f - t_i) = P \cdot T$$

In the Fig.3a, as an example, is presented the elementary case of a serie RLC circuit during the opening phase. The breaker (Fig.3b) is in this case positioned on the tree. In the Fig.3c are respectively presented, as a confirmation of the uniformity of the approach, the two different network models according to the two approaches of Mayr and Cassie. The network in Fig.3d, of the second order, is instead referred to an electrical line in the operating state. In such a case, given the presence of the capacitor on the tree, the breaker is necessarily located on the cotree (Fig.3e). The model derived, characterized by the resistance r of the arc dynamics (1), is formalized in the following way:

$$\frac{d}{dt}\begin{bmatrix} i_L \\ v_c \end{bmatrix} = \begin{bmatrix} -r/L & -1/L \\ -1/C & -1/(rC) \end{bmatrix} \cdot \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 0 \\ 1/(rC) \end{bmatrix} \cdot e$$

In the Fig.3f is depicted the whole interrupting and recovery transient of the circuit.

Concerning the method with the Laplace transform [5], depending on the hypothesis of linearity of the network, could be used, once interrupted the current, only for the recovery phase. With two essential reserves that cannot be ignored. The first one is related to the initial conditions $\{i(0), v_c(0)\}$ to be adopted in the approach with Laplace. These are concerned with the final part of the transient of a non-linear circuit, which is possible to investigate only through a numerical approach. The second reserve is related to the fact that the investigation with Laplace of the recovery phase, assuming the extinguished arc to the vacuum, is not able to study the subsequent potential restart.

THREE-PHASE CIRCUIT

Usually the three-phase analysis, thanks to the symmetry typical of these networks, leads, in terms of sequence components, to a more compact complex single-phase expression. In such conditions is sufficient to read in a complex form the equations in real form deducted from the single-phase case and substitute the circuit elements with the three-phase sequence correspondent ones. Concerning the dynamic analysis of three-phase networks, under the hypothesis of symmetric network, it is caried out in terms of instantaneous sequence components by means of the Park transform. The obtained three-phase state model can advantageously make use the same numeric algorithms and the same topologic approach already consolidated for the single-phase case. Likewise there fully applies all the energetic approaches that belong to the Park formalism. Among these ones, important for the role in presence of distorsion and asymmetry, is the imaginary power.

In the case of three-phase networks with electric arc the situation becomes completely different: it is now not possibile an approach based on the sequence components. This comes from the fact that, even although the three distinct arcs may be considered (for evident reasons of construction symmetry) with the same constitutive relation R=$F(i)$, as a matter of fact the non-linearity of the functional F introduces in the three arc resistances a condition of instantaneous asymmetry of the following type:

$$\begin{cases} R_a = F(i_a) \\ R_b = F(i_b) \Rightarrow R_a \ne R_b \ne R_c \Rightarrow \\ R_c = F(i_c) \end{cases} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} R_a & 0 & 0 \\ 0 & R_b & 0 \\ 0 & 0 & R_c \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

for which is not possibile to define the equivalent single-phase sequence circuits. The dynamical analysis, not anymore single-phase as in the case of symmetric networks, must then be referred to the phase variables $[y_{abc}(t)]$.

The Park method for the three-phase arc

In case of three-phase networks with electric arc, once integrated the model with the method of the state equations the matrix functions $\{[i_{abc}],[v_{abc}]\}$are obtained as port variables of the three-phase arc.

Hence the (2) can be read in the following three-phase form:

$$\begin{cases} I_M = \max\{[i_{a,b,c}(t)]\} & t_i \le \forall t \le t_f \\ [v_r] = [v_{r_{a,b,c}}(t)] & t_f \le \forall t \end{cases}$$

The passage to the Park variables $[dq0]$ can be achieved in a second moment after the integration process, but always in an exclusive energetic range.

The three arc currents $[i_{a,b,c}]$ can be matched the correspondent three-phase Park arc current [2]:

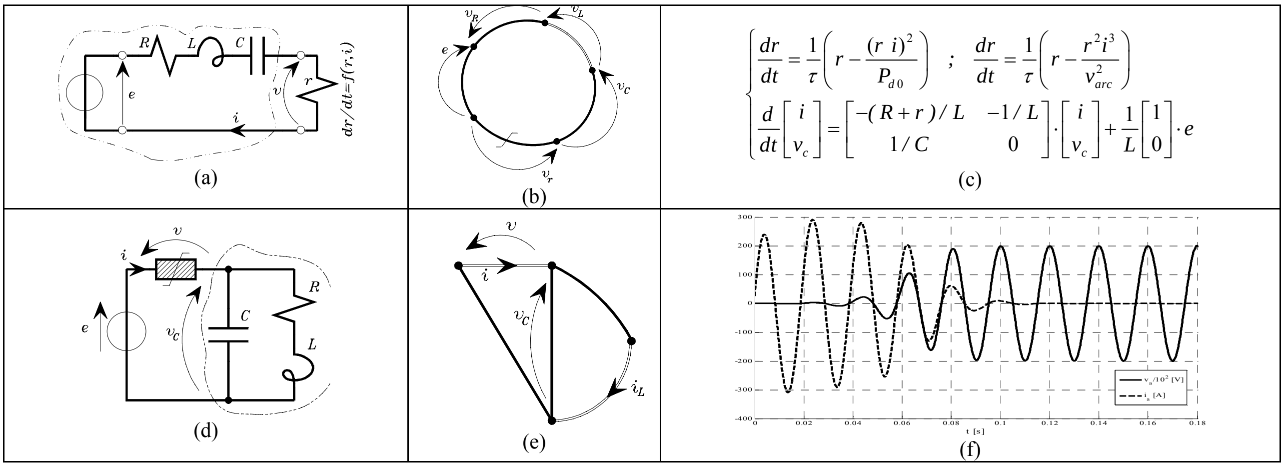

Fig.3 Dynamic analysis, according to the models of Mayr and Cassie, of opening transients in power networks.

(4) $$\begin{cases}\bar{i}(t)=\sqrt{\frac{2}{3}}\left\{ i_a(t)+\bar{\alpha}\cdot i_b(t)+\bar{\alpha}^2\cdot i_c(t)\right\}=i(t)e^{j\vartheta_I(t)} \\ \bar{\alpha}=\exp(j2\pi/3)\end{cases}$$

Also to the three arc voltages $[v_{a,b,c}]$ correspondents the three-phase Park voltage:

(5) $$\bar{v}(t)=\sqrt{\frac{2}{3}}\left\{ \mathrm{F}(i_a)\cdot i_a(t)+\bar{\alpha}\cdot\mathrm{F}(i_b)\cdot i_b(t)+\bar{\alpha}^2\cdot\mathrm{F}(i_c)\cdot i_c(t)\right\}= v(t)\cdot e^{j\vartheta_V(t)}$$

As can be observed, even although it is a resitive circuit, the Park arc voltage results to be instantaneously shifted of the angle $\vartheta_I(t)$-$\vartheta_V(t)$=$\varphi(t)$ respect to the Park arc current. Even although it is not related to any process of energy storage, it is consequence of the instantaneous arc asymmetry introduced by the non-linearity.

The Park approach confirms anyhow its conceptual and practical importance under the energetic aspect. Once associated to the three-phase arc functions $\{[v_{a,b,c}],[i_{a,b,c}]\}$ the correspondant Park quantities (4,5) it is definable, with a formalism that confirms the metrological correspondence with the single-phase case, the energetics of the “three-phase electric arc” based on the instantaneous complex power $\bar{a}(t)=\bar{v}(t)\cdot\underline{i}(t)=p(t)+jq_P(t)$.

From this, can be deducted the real component of the power:

(6) $$\begin{cases}p(t)=\left[v_{a,b,c}(t)\right]_t\cdot\left[i_{a,b,c}(t)\right]=\Re e\left\{\bar{a}(t)\right\}=v(t)\cdot i_f(t) \\ P=\Re e\frac{1}{T}\int_{t_i}^{t_f}\bar{a}(t)\cdot dt=\left\langle p(t)\right\rangle\Big|_T\end{cases}$$

and, linked to both the distortion of the waveforms and the asymmetry introduced by the non-linearity of the three-phase electric arc, the imaginary component:

(7) $$\begin{cases}q(t)=\Im m\left\{\bar{a}(t)\right\}=v(t)\cdot i_q(t) \\ Q_P=\Im m\frac{1}{T}\int_{t_i}^{t_f}\bar{a}(t)\cdot dt=\left\langle q(t)\right\rangle\Big|_T\end{cases}$$

This physical quantity affects, as index of the “three-phase unbalance” instantaneously caused by the arc, the Joule-Park integral as results from the following expression:

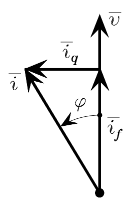

Fig.4. Instantaneous components of the Park arc current.

(8) $$\int_{t_i}^{t_f}\bar{i}\cdot\bar{i}^*\cdot dt=I_e^2\cdot T=\int_{t_i}^{t_f}\frac{p^2+q_P{}^2}{v^2}dt=\left\{I_{e_p}^2+I_{e_q}^2\right\}\cdot T>I_{e_p}^2\cdot T$$

Always due to the presence of the quadrature component $i_q(t)$, the modulus of three-phase breaking work becomes as follows:

(9) $$L=\left|\int_0^T\bar{v}(t)\cdot\underline{i}(t)\;dt\right|=\sqrt{L_p^2+L_q^2}=\sqrt{P^2+Q_P^2}\cdot T$$

It could be, together with the sizing classical three-phase power:

(10) $$S=\sqrt{\frac{1}{T}\int_0^T\bar{v}(t)\cdot\underline{v}(t)\;dt}\cdot\sqrt{\frac{1}{T}\int_0^T\bar{i}(t)\cdot\underline{i}(t)\;dt}=V_e\cdot I_e$$

a useful element for the breaker design.

SOME EXAMPLES

We consider the simple but representative case of Fig.4a. When the breaker is closed the network is linear and symmetric. The breaker opening introduces the three non-linearities due to the electric arc, and, together them, the asimmetry. It must be investigated in the phases domain with the help of method of the state equations. In such a case the three arc resistors must be placed onto the tree. Fig.5a shows the state matrix equation. This model, as consequence of the three-phase network degeneracy, does not appear in the normal form. In Figg.5b-e are reported the obtained diagrams. The energetic approach, developed with the Park formalism (Table 1), confirms the role of the complex power and work.

CONCLUSIONS

The analysis connected to the arc phenomena has been incorporated, under the systemic and topologic aspect, in the modern state approach. The obtained results show new perspectives of investigation. In particular, given the flexibility and computational power of the method, there is no limitation for the arc model and the network complexity. The three-phase case presents a very specific limit: the three-phase arc, because of its non-linearity, causes dissimetry. The Park approach, linked to instantaneous se-

Table 1. Significant computed quantities.

	Three phase	Equivalent Single-phase
Arc dissipated power [W]	$4.85 \cdot 10^6$	$3.72 \cdot 10^5$
Breaking energy [J]	$9.69 \cdot 10^6$	$7.10 \cdot 10^6$
Arc three-phase rms voltage [V]	$5.44 \cdot 10^6$	$3.84 \cdot 10^6$
Arc three-phase rms current [A]	$1.38 \cdot 10^4$	$0.95 \cdot 10^4$
Three-phase sizing power [VA]	$7.52 \cdot 10^{10}$	$3.67 \cdot 10^{10}$

quence circuits, is therefore not allowed. The state analysis has to be performed through real three-phase method. The following energetic approach with Park emphasizes the role of the three-phase arc as an imaginary power source and the subsequent need to adopt, in a new form, the complex breaking work.

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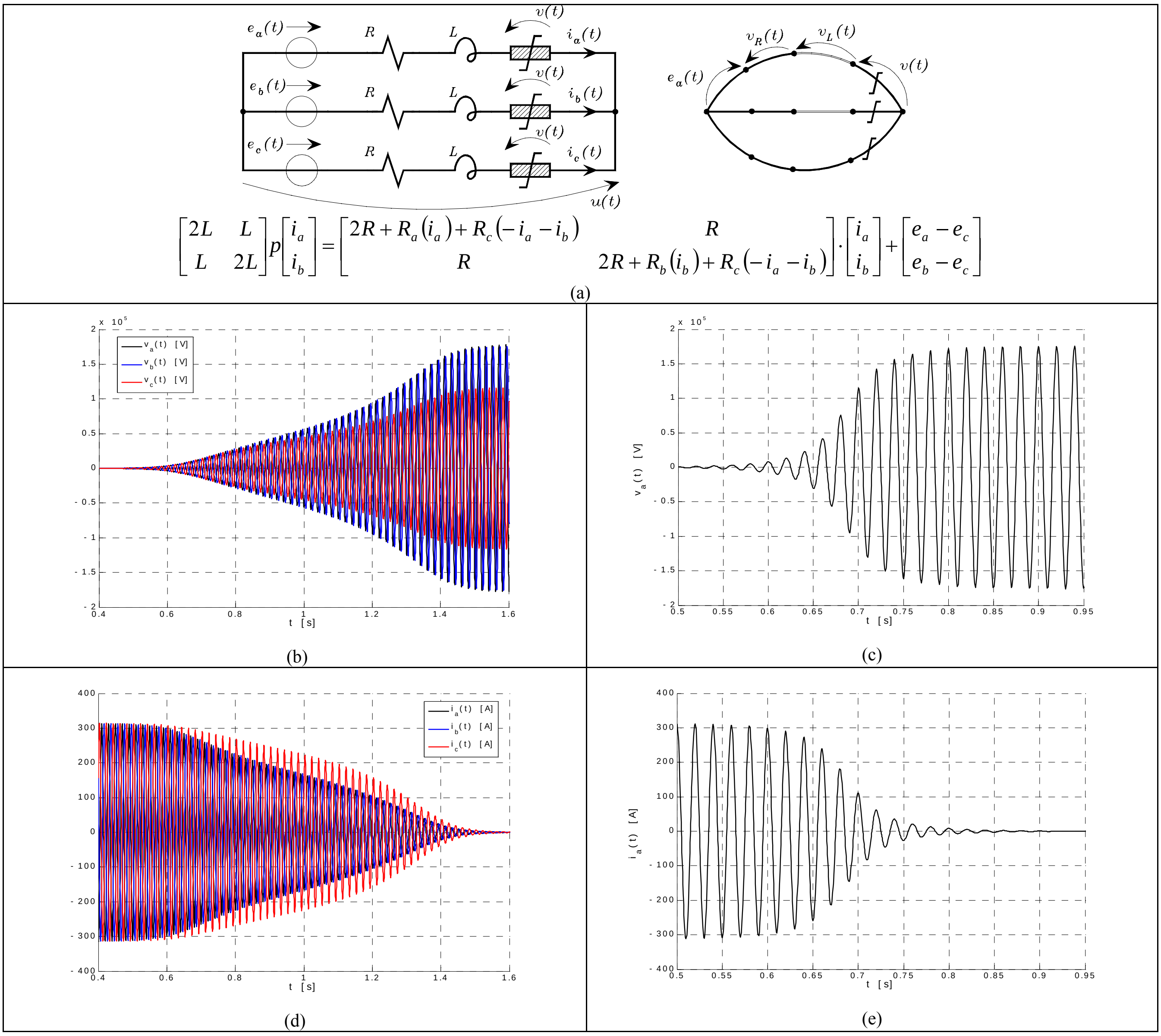

$$\begin{bmatrix} 2L & L \\ L & 2L \end{bmatrix} p \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} 2R + R_a(i_a) + R_c(-i_a - i_b) & R \\ R & 2R + R_b(i_b) + R_c(-i_a - i_b) \end{bmatrix} \cdot \begin{bmatrix} i_a \\ i_b \end{bmatrix} + \begin{bmatrix} e_a - e_c \\ e_b - e_c \end{bmatrix}$$

Fig.5. Three-phase arc circuit.