EVALUATION OF NEGATIVE SEQUENCE CURRENT INJECTING INTO
THE PUBLIC GRID FROM DIFFERENT TRACTION SUBSTATION IN ELECTRICAL
RAILWAYS

Haiqun Wang
SMEPC-China
wanghq@smepec.com

Yingjie Tian
ECEPTRI-China
dsy_tianyj@ec.sp.com.cn

Qin-chang Gui
ECEPTRI-China
dsy_tianyj@ec.sp.com.cn

ABSTRACT
This paper formulated the mathematical model of traction transformers of single phase traction substation. Three phase- two phase traction substations (V/V traction transformer, Scott transformer, impedance-matching balance transformer) which are widely used in China electrified railways. The negative-sequence current injected into the public grid and three-phase voltage unbalance at PCC is quantitatively analyzed. At the same time, the influence of feeder load proportion and feeder load phase error on the negative sequence current is also discussed. This proposed method provides significant assistance for the selection of traction substation types and external power supply schemes of other electrified railways.

Keyword: Power quality, Electrical railways, Traction substation, Negative phase-currents

1 INTRODUCTION
The power supply of electrified railway is provided via single-phase or three-phase traction substations, in which the corresponding single-phase or three-phase, V/V, Scott, impedance-matching balancing transformers are used. Due to the single-phase power supply and the load characteristics, electro-train may affect the power quality. The effect mainly involves power strike, negative-sequence current and harmonics, among which negative-sequence current contributes the most. This paper makes use of power balancing equations and analyzes the phasor relationship between the single-phase current at the electro-train side and the three-phase current at the grid side. According to the symmetric component analysis, the general calculation method for sequent current is then derived. It provides a convenient way to analyze the influence on negative-sequence current due to the unbalancing load at the two arms of the electro-train. The paper evaluates the negative-sequence current injected into the grid from various traction transformers. The evaluation result may provide significant guidance for traction transformer selection in the future.

2. MATHEMATIC MODEL

2.1 General model
The two sides of the traction transformer may be considered as a multi-terminal network\(^{(1)}\) as shown in Fig. 1.

![Fig.1 Traction Transformer port model](image)

2.2 Single phase traction substation
The single phase traction transformer configuration is shown in Fig. 2.

![Fig.2 Single phase traction substation](image)

From (4), the current at the primary side of the traction transformer is:

\[
I_a = \frac{U_a e^{i\psi_a}}{K_a e^{i\phi_a}}
\]
\[ I_a + a I_a + a^2 I_c = K I_a e^{-j\phi} \] (8)

Since there is no zero-sequence current at the primary side, and \( I_b = 0 \), we may conclude that:
\[ I_a = -I_c = K I_a \] (9)

the sequence current at the primary side may be acquired:
\[ I_1 = \frac{1}{3} I_a (1-a^2) = \frac{\sqrt{3}}{3} K I_a e^{j30^\circ} \]
\[ I_2 = \frac{1}{3} I_a (1-a^2) = \frac{\sqrt{3}}{3} K I_a e^{-j30^\circ} \]
\[ I_3 = I_a = \frac{\sqrt{3}}{3} K I_a \] (10)

Negative-sequence current ratio is:
\[ \varepsilon = I_2/I_1 = 1.0 \] (11)

2.3.3 The total current at the primary side
Supposed \( I_\mu = \eta I_a \) (\( \eta \) is the load ratio at the two sides) and through the employment of superposition method, the total current at the primary side is obtained.
\[ \tilde{I}_a = K I_a e^{-j(\epsilon_\mu + 30^\circ)} \]
\[ \tilde{I}_b = K \eta I_a e^{-j(\epsilon_\mu - 90^\circ)} \]
\[ \tilde{I}_c = K I_a e^{-j(\epsilon_\mu + 150^\circ)} + K \eta I_a e^{-j(\epsilon_\mu - 90^\circ)} \]

The sequence current at the primary side is:
\[ I_1 = \frac{\sqrt{3}}{3} I_a K e^{j(\eta \phi + 30^\circ)} \]
\[ I_2 = \frac{\sqrt{3}}{3} I_a K e^{j(\eta \phi - 120^\circ)} \]
\[ I_3 = I_a = \frac{\sqrt{3}}{3} I_a K e^{j(\eta \phi - 90^\circ)} \] (14)

2.4 Scott traction substation

The configuration of SCOTT traction transformer is shown in Fig.4.

[Diagram of SCOTT traction substation]

Scott traction transformer consists of two single phase transformer M and T. The magnitude of the voltage at the secondary side is equal. The voltage angle of T transformer is 90° in lead of M.

2.4.1 Power supply through T
the voltage at the terminal is:
\[ U_a = \frac{3}{\sqrt{3}} K = \tilde{U}_a \]

where, turn ratio \( K = \frac{W_T}{W_s} \)

the current at the primary side may be calculated.
\[ I_{ba} + a^2 I_{ca} = \frac{3}{\sqrt{3}} K I_a \]

Employing the following relationship,
\[ I_{ba} = I_{ca} : I_{ba} = I_{ca} = -I_{ca} \]

Acquire the current at the primary side.
\[ I_{ba} = \frac{2}{\sqrt{3}} K I_a \] ; \[ I_{ba} = \frac{1}{\sqrt{3}} K I_a \]
\[ I_{ca} = -\frac{1}{\sqrt{3}} K I_a \]

2.4.2 Power supply through M
the voltage at the terminal is:
\[ U_\beta = \sqrt{3} K U_a e^{-j90^\circ} \]

the current at the primary side may be calculated.
\[ I_{ab} + a I_{ab} + a^2 I_{cb} = \sqrt{3} K e^{-j90^\circ} I_\beta \]
Employing the following relationship,
\[ I_{Bp} = 0; I_{cp} = I_{Bp} \]
Acquire the current at the primary side.
\[ I_{Bp} = KI_{Bp}; I_{cp} = -K\hat{I}_{Bp} \]

### 2.4.3 The current at the primary side

The total current at the primary side of the traction transformer is acquired by using superposition method.
\[ I_a = \frac{2}{\sqrt{3}} KI_a; I_b = \frac{1}{\sqrt{3}} KI_a + K\hat{I}_{Bp} \]
\[ I_c = \frac{1}{\sqrt{3}} KI_a - K\hat{I}_{Bp} \]

Supposed \( I_B = \eta I_a \), the sequence current at the primary side is:
\[ I_1 = \frac{K I_a}{\sqrt{3}} \sqrt{1 + \eta^2 + 2\eta \cos(\phi_a - \phi_b)} \]  
\[ I_2 = \frac{K I_a}{\sqrt{3}} \sqrt{1 + \eta^2 - 2\eta \cos(\phi_a - \phi_b)} \]

The negative-sequence current ratio:
\[ e = \frac{\sqrt{1 + \eta^2 - 2\eta \cos(\phi_a - \phi_b)}}{\sqrt{1 + \eta^2 + 2\eta \cos(\phi_a - \phi_b)}} \]

### 2.5 Impedance balancing transformer

The configuration of Impedance balancing traction transformer is shown in Fig.5.

**Fig.5 Impedance matching traction transformer**

In the figure,
- \( \Delta W \): The middle-phase (Phase B) external winding turns at the secondary side
- \( K_w = \frac{\Delta W}{W_2} = \frac{\sqrt{3} - 1}{2} \): The external and inner winding turns ratio of the middle-phase(phase B) at the secondary side,
- \( Z_c \): The impedance of the inner winding at the secondary side
- \( K_z = \sqrt{3} + 1 \): The middle-phase inner winding turns at the secondary side

#### 2.5.1 Power supply through phase A

The terminal voltage is: \( U_a = K(U_a - K_w U_b) \)

Due to the external winding of the middle phase, the current of phase B at the primary side may be described as:
\[ I_B = I_{Bb} + I_{Bo} \]

Where:
- \( I_{Bb} \): the current at the primary side produced by the inner winding of phase B at the secondary
- \( I_{Bo} \): the current at the primary side produced by the external winding of phase B at the primary side.

The power balance equation in (1) can be described as:
\[ U_a I_a + U_b (I_{Bb} + I_{Bo}) + U_c I_c = KUU_a I_a - KK_w U_a I_a \]

Acquire the current for the primary side:
\[ I_{Bb} + a I_{Bo} + a^2 I_c = KI_a \]

Making use of the following relationship:
\[ I_{Bb} = I_{Bo} = -\frac{I_{Ia}}{K_z + 1} \]

Get the current at the primary side:
\[ I_{Bb} = I_{Bo} = -\frac{K I_a}{K_z + 1} \]

Similarly, get the current balance equation for the primary side.
\[ I_{Bb} + a I_{Bo} + a^2 I_c = -K a^2 I_B \]

Using the relationship of the current at the primary side,
\[ I_{Bb} = I_{Bo} = -\frac{K I_a}{K_z + 1} \]

Obtain the current at the primary side:
\[ I_{Bb} = I_{Bo} = -\frac{K I_a}{K_z + 1} \]

\[ I_{Bb} = I_{Bo} = -\frac{K I_a}{K_z + 1} \]

#### 2.5.2 Power supply through phase C

The terminal voltage is: \( U_c = K(-U_c + K_w U_b) \)

Similarly, get the current balance equation for the primary side.
\[ I_{Bb} + a I_{Bo} + a^2 I_c = -K a^2 I_B \]

Using the relationship of the current at the primary side,
\[ I_{Bb} = I_{Bo} = -\frac{K I_a}{K_z + 1} \]

Obtain the current at the primary side:
\[ I_{Bb} = I_{Bo} = -\frac{K I_a}{K_z + 1} \]

\[ I_{Bb} = I_{Bo} = -\frac{K I_a}{K_z + 1} \]

#### 2.5.3 The total current at the primary side

The total current at the primary side can be acquired.
\[ I_a = I_{ka} + I_{Bp} = K(\frac{K_z + 1}{K_z + 2} I_a + \frac{1}{K_z + 2} I_{Bp}) \]
\[ I_b = I_{ka} + I_{Bp} = K(\frac{1}{K_z + 2} I_a + \frac{1}{K_z + 2} I_{Bp}) \]
\[ I_c = I_{ka} + I_{Bp} = K(\frac{K_z + 1}{K_z + 2} I_a + \frac{1}{K_z + 2} I_{Bp}) \]

Supposed \( I_B = \eta I_a \), \( K_w = \frac{1}{2} \)

The sequence current at the primary side is:
\[ I_1 = \frac{\sqrt{6} K I_a}{6} \sqrt{\eta^2 + 1 + 2\eta \cos(\phi_a - \phi_b)} \]
\[ I_2 = \frac{\sqrt{6} K I_a}{6} \sqrt{\eta^2 + 1 - 2\eta \cos(\phi_a - \phi_b)} \]

The negative-sequence current ratio is:
\[ \varepsilon = \frac{\eta^2 + 1 - 2\eta \cos(\varphi_\alpha - \varphi_\beta)}{\sqrt{\eta^2 + 1 + 2\eta \cos(\varphi_\alpha - \varphi_\beta)}} \]  

\[ \text{(20)} \]

3. EXAMPLE

In the following example, the calculation of \( I_2 \) for various traction transformers will be described.

The parameters used for the computation include:
- the primary voltage is 110kV. The secondary voltage is 25kV. The maxim load of the train is 80MVA.
- \( k = 0.22727 \) for Single-phase, SCOTT and V/V transformer; \( k = 0.39366 \) for The Impedance balancing transformer.

The maxim current \( I_a = 3200A \) for the single-phase transformer; The maxim current \( I_a = 1600A \) for V/V, SCOTT or impedance balancing transformer.

Assume that the power supplying arm \( \alpha \) is working at the maxim current and the current of arm \( B \) varies, the sequence current at the primary side is then calculated and as shown on table 1.

Table 1 The sequence current at the primary side for various traction transformers

<table>
<thead>
<tr>
<th>( \eta )</th>
<th>Single-T</th>
<th>V/V-T</th>
<th>Scott/Impedance</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_2 )</td>
<td>( I_1 )</td>
<td>( \varepsilon )</td>
<td>( I_2 )</td>
</tr>
<tr>
<td>0</td>
<td>210</td>
<td>210</td>
<td>1.0</td>
</tr>
<tr>
<td>0.1</td>
<td>231</td>
<td>231</td>
<td>200</td>
</tr>
<tr>
<td>0.2</td>
<td>252</td>
<td>252</td>
<td>192</td>
</tr>
<tr>
<td>0.3</td>
<td>273</td>
<td>273</td>
<td>187</td>
</tr>
<tr>
<td>0.4</td>
<td>294</td>
<td>294</td>
<td>183</td>
</tr>
<tr>
<td>0.5</td>
<td>315</td>
<td>315</td>
<td>182</td>
</tr>
<tr>
<td>0.6</td>
<td>336</td>
<td>336</td>
<td>183</td>
</tr>
<tr>
<td>0.7</td>
<td>357</td>
<td>357</td>
<td>187</td>
</tr>
<tr>
<td>0.8</td>
<td>378</td>
<td>378</td>
<td>192</td>
</tr>
<tr>
<td>0.9</td>
<td>399</td>
<td>399</td>
<td>200</td>
</tr>
<tr>
<td>1</td>
<td>420</td>
<td>420</td>
<td>210</td>
</tr>
</tbody>
</table>

It is concluded that:
- a) For single-phase transformer, \( I_1 = I_2 \). When the train operates under full power condition, \( I_1 = 420A \). When the train operates under half power condition, \( I_2 = 210A \).
- b) \( \eta = 0 \), which means \( I_\beta = 0 \), it have \( I_1 = I_2 = 210A \) for V/V, SCOTT and impedance-balancing transformers.
- c) \( \eta = 1 \), which means \( I_\beta = I_\alpha \), it have \( I_2 = 210A \) for V/V transformer, and \( \varepsilon = 0.5 \).
- d) \( \eta = 1 \), it have \( I_\beta = 420A \) and \( I_\alpha = 0 \) for SCOTT and impedance balancing transformer, and \( \varepsilon = 0 \).
- e) In order to restrict negative-sequence current within the allowable values, SCOTT or impedance balancing transformer is recommended with higher priority. However, prior to fulfillment allowable value \( V/V \) transformer may also be used when the load at the two arms of the train is normally in balance. When the normal load at the two arms of the train is seriously unbalanced, there will be little difference in the negative-sequence current produced by various kinds of traction transformers. Under this condition, single-phase traction transformer will be the ideal choice due to its simple structure, low production cost as well as its high operation efficiency. However, negative-sequence current control has to be implemented in accordance with the application of single-phase traction transformer.

4. CONCLUSION

The practical calculation formula for three-phase current at the primary side of the traction transformer is derived based on the power balance equation. Furthermore, the sequence current at the primary side is calculated for various types of traction transformers including single-phase, V/V, SCOTT and impedance balance transformers.

The relationship between the negative-sequence current ratio \( \varepsilon \) and the degree of load unbalance \( \eta \) at the two arms of the train is built up. In addition, the function of negative-sequence current variation with respect to the power factor difference of the load at the two arms of the train \( \varphi_\alpha - \varphi_\beta \) is determined. The negative-sequence current characteristics of various types of traction transformers can be conveniently compared then.

It provides useful guidance for the selection of reasonable traction transformer to meet the requirement of the power quality in the public grid.

REFERENCES


Biographies

Wang Haiqun (1968-) Male, Senior Engineer. He got B.S. from Shanghai Jiaotong University in 1990. He is currently working in Shanghai Municipal Electric Power Company. His major research interest covers power grid planning, special client connection into the grid and power quality.

Tian Yingjie (1969- ), male, Senior engineer, he received the M.S. degrees in power system from Hunan university, Changsha ,china, in 1993. His research interests include power system computation and analysis.

Gui Qin-chang (1944-), male, he received the M.S. degrees in power system from Jiaotong university, Shanghai ,China, in 1981. His research interests include power system plan and operation, power quality.