EXPANSION PLANNING OF DISTRIBUTION NETWORKS USING SIMULATED ANNEALING TECHNIQUE

Željko N. POPOVIĆ  
Elektrovojvodina - Serbia  
zeljko.popovic@su.ev.rs

Vojin Dj. KERLETA  
Technical faculty of Zrenjanin - Serbia  
kerleta@tf.zr.ac.rs

ABSTRACT
This paper presents a hybrid metheuristic procedure for expansion planning of distribution networks. The proposed procedure combines simulated annealing technique and mixed integer linear programming algorithm in order to overcome the shortcomings of each of individual approaches. The obtained numerical results show that proposed approach has the potential to improve the quality of solutions of real size planning problems.

INTRODUCTION
Distribution expansion planning is a hard combinatorial optimization problem with long history of contributions for improved solutions [1],[2]. Two main groups of methods have been proposed for solving such a complex problems: artificial intelligence (AI) based methods and mathematical programming based methods.

The proposed AI models [3]-[6] can provide good solution for real (large) size distribution networks but the quality of obtained solutions is uncertain since they have not been compared with the true global optimal solutions so far. Mathematical programming based methods mostly employ mixed integer linear programming (MILP) algorithms [7],[8]. Although the MILP models can guarantee that global optimal solution will be found, due to significant computational complexity they are limited to smaller size problems.

This paper proposes a new algorithm for expansion planning of real size distribution networks that is based on the local network concept, branch-exchange algorithm, MILP model and simulated annealing (SA) technique. The goal is to combine the advantages of AI and mathematical programming based methods in order to improve the quality of solutions of larger size planning problems.

In the first step of the algorithm the initial solution is obtained by dividing the considered network into a number of sub-networks (local networks) and then solving each of them by applying the properly designed MILP algorithm aimed to minimize investment cost. This solution is further iteratively modified using simulated annealing technique to search for the minimum total cost solution. At each iteration a random neighbors are generated. The new neighbor structures (solutions) are generated by random modification of the local networks that exist in the current solution as well as by random creation of new local networks, i.e. by division of the entire network into a number of new local networks. In the first of described ways the exploitation of the current solutions (intensification) is performed while in the second way the exploration of the search space (diversification) is enabled. Every neighbor solution is obtained by applying the MILP model at each local network modified (created) during the intensification (diversification) phase. It should be emphasized that proper implementation of the above mentioned search mechanisms, especially the second one, can significantly improve the effectiveness of SA algorithm, i.e. it improves the quality of solutions obtained by the SA algorithm [9]. Neighbor solutions that improve the cost function are always accepted while nonimproving solutions are accepted with certain probability. This process is repeated until the appropriate stopping criterion is reached and the best solution is found. The obtained numerical results show that proposed approach can produce the same quality solutions as MILP based methods and thus has a potential to improve the quality of planning process in real size distribution networks.

SOLUTION APPROACH
The simulated annealing algorithm for solving distribution expansion problems can be summarized as follows:

Input: Cooling schedule /*Initial temperature (Tmax), Final temperature (Tmin), Cooling rate, Number of iterations at a fixed temperature*/.
s = s0 ; /* Generation of the initial solution */
Tk = Tmax ; /* Starting (initial) temperature*/
k=0;
Repeat /*General iteration, k*/
   Repeat /*At a fixed temperature Tk (at iteration k)*/
      Generate a random neighborhood solution s′;
      ΔE = f (s′) − f (s) ; /*Cost difference*/
      If ΔE ≤ 0 Then s = s′ ; /*Accept s′ as the current solution*/
      If ΔE > 0 Then
          If f (s′) − f (sbest)<0 Then sbest = s′ ; /*Accept s′ as the best solution found so far (sbest)*/
          Else Accept s’ with a probability e−ΔE/Tk ;
      Until Equilibrium condition /*e.g. a given number of iterations executed at each temperature Tk */
      Tk+1 = g(Tk ) ; /* Cooling rate*/
      k=k+1;
Until Stopping criteria satisfied /*e.g. Tk < Tmin*/
Output: Best solution found.
The elements of the above algorithm are described in more details in the sequel.

**Cooling schedule**

The cooling schedule is the control strategy used from the beginning until the convergence of the simulated annealing algorithm. The parameters to consider in defining a cooling schedule are the starting temperature, the equilibrium state (number of iterations carried out at each temperature), a cooling rate (the rate of temperature reduction), and the final temperature that defines the stopping criteria. In this study the initial temperature is determined in the following way [10]:

\[
T_{\text{max}} = \frac{\mu}{- \ln \phi} f(x^0)
\]

where it is assumed that \( \phi \) [%] of the uphill moves, which are \( \mu \) [%] worse than the initial solution \( f(x^0) \), are accepted at the initial temperature level \( T_{\text{max}} \). The determination of \( T_{\text{max}} \) from (1) has the advantage of being simple and direct. Geometric cooling rate can serve as a suitable function for deriving the set of temperatures required by the schedule in the distribution expansion problems [9],[10]. This function is defined as follows:

\[
T_{k+1} = \alpha \cdot T_k
\]

where \( T_k \) and \( T_{k+1} \) are temperatures at the iteration \( k \) and \( k+1 \), respectively, and \( \alpha \) is a constant generally lying on the range \((0.5-0.99)\) [10].

The number of iterations to be carried out at each temperature depends on the problem dimension and is defined in the next sections. The SA algorithm is stopped whenever the temperature reaches a lower bound value (e.g. \( T_{\min} = 0.01 \) ) or whenever the solution is not improved for the defined number of consecutive temperatures.

**Determination of initial solution**

The initial network, which consists of the branches and nodes that already exist in the considered network and of all possibly new branches and nodes, is arbitrarily divided into the number of local networks (sub-networks/sub-problems). The possible way of generating local networks is presented in the defined number of consecutive temperatures.

**Intensification**

Intensification mechanism proposed here is based on well known branch-exchange algorithm [13], local network concept and MILP algorithm. The branch-exchange algorithm starts from the current solution at iteration \( k \) (at temperature \( T_k \)) and the neighborhood structure is formed in the following way. One NO switch (tie switch) is chosen by chance from the set of all NO switches (tie switches) that potentially connect adjacent local networks (sub-networks) that exist in the initial current solution at iteration \( k \). It should be noted that chosen NO switch connects not only the two adjacent local networks but also the two adjacent feeders belonging to those networks. Therefore, the branch-exchange will be conducted on the set of switches that belongs to those feeders. Hence, the chosen NO switch will be closed and the adjacent one, which was closed, will be opened. Then, voltage and thermal constraints in this configuration (state) are checked out (tested) using the same method that is used in the MILP model [12]. If the constraints are not violated, the next branch exchange is performed in the same way and in the same direction as the previous one. This process is repeated while constraints (voltage or thermal) are not violated or while all possible branch-exchanges are made on the considered feeders. If any of these conditions is fulfilled, then calculations are performed using the MILP algorithm for each of the considered local networks. It should be noted that the initial (starting) configuration of the two considered local networks is changed during the branch-exchange process. Therefore, the MILP algorithm is used to solve planning problem for each of modified local networks, starting from their initial states (states that they have in the initial network). In this way the new neighbor solution of the overall problem is obtained. This solution is accepted if its cost \((f(S_j))\) is less than that of the current solution \((f(S_i))\). If the cost of the neighbor solution is higher than the cost of the current solution, it still can be accepted with a certain probability. This ability to perform uphill moves allows simulated annealing to escape from local optima. The entire acceptance process at iteration \( k \) (temperature \( T_k \)) can be summarized as follows:
▪ ΔE= f(S_i)- f(S_j); if ΔE > 0, then neighbor solution becomes current solution (S_j=Si); furthermore, if the cost of the neighbor solution is smaller than that of the current best solution, then it becomes the current best solution; the new local networks are stored along with the NO switches between adjacent local networks

▪ If ΔE < 0 then the following is applied:
  ○ If e^−ΔE/T_k > random [0,1], then neighbor solution becomes current solution; the new local networks are stored along with the NO switches between adjacent local networks
  ○ If e^−ΔE/T_k < random [0,1], then neighbor solution is not accepted.

The intensification procedure described so far will be repeated for each not yet processed NO switch, which belongs to the set of NO switches at iteration k (temperature T_k). This procedure generates new solutions that are “close” to the initial current solution at temperature T_k. However, if all new solutions generated at a temperature T_k are accepted according to probabilistic criterion or no new solution is generated (accepted) then the diversification procedure (mechanism) will be applied.

**Diversification**

Diversification is an algorithmic mechanism that tries to alleviate the above mentioned problem by forcing the search into previously unexplored areas of the search space.

In order to perform the diversification in the search space the neighbor structure is formed in the following way. Starting from the initial current solution (network state) at iteration k, the neighbor structures are obtained by constructing new local networks. In the current network state a single feeder is randomly chosen and the local network is formed that consists of the chosen feeder and its adjacent feeders, i.e. feeders that have normally open switches toward the chosen feeder [11]. Then, using the MILP algorithm, planning problem is solved for the obtained local network, starting from its initial state (state that the local network has in the initial network). In this way a neighborhood solution is generated. Acceptability of this solution is tested according to the acceptance rules defined in the previous section. The described procedure is repeated at iteration k for each of the remaining (not yet processed) feeders that are not part of the already processed local networks. This procedure generates larger neighborhood structures, compared to that generated during the intensification process, and thus enables exploring of so far unexplored areas of the search space. After the diversification procedure is finished, in the next step (iteration) of the algorithm the obtained solution will be explored more thoroughly through the above described intensification mechanism.

**NUMERICAL RESULTS**

The proposed approach has been used to find solution with minimal investment cost for 20 kV test network shown in Fig. 1. The test system consists of one supply node (supply substation), 44 existing branches (solid lines) and 14 possibly new branches (dashed lines). In the supply substation three transformers 110/20 kV/kV exist (TRF1&2&3, branch 0-1) with the capacity of 31.5 MVA per transformer. It is assumed that substation capacity can be upgraded by adding maximally two additional transformers of the same size, with capital (installation) cost of 725 000 $ per transformer. Also, the four size possibilities for constructing and/or upgrading each branch are considered. Table I shows physical and assumed unit cost data for each size. Assumed upgrading unit costs are given in Table II. In Fig.1 capacity of each existing branch in the initial (present) period is shown in MVA (bold numbers) while the lengths of all branches are given in kilometers. It is assumed that there is a switch in each branch. Branches with normally opened switches (unloaded lines), which ensure radiality condition in the initial state of the network, are marked with "X" in the middle of the branch. Existing demand nodes (36) are marked with empty circles while 6 future demand nodes are shown as full circles. Forecasted load in each demand node is given in MVA in Fig. 1. Voltage in supply node (1) is assumed 20.5 kV, lower voltage limit is set to 19 kV and power factor to 0.95. Optimization package TOMLAB (CPLEX) [14] has been used for MILP calculations. Taking into account that the cost of the obtained initial solution is 1,818,300 $, φ =40% and μ = 30% the starting temperature, according to (1), becomes T_max=595. The SA algorithm is terminated if solution is not improved after 3 consecutive temperatures or if the temperature reaches 0.01 (T_min=0.01). The constant α in (2) is chosen to be 0.9.
Table 1 - Branch data

<table>
<thead>
<tr>
<th>Size [MVA]</th>
<th>Installation cost [$x10^3/km]</th>
<th>$r$ [Ω/km]</th>
<th>$x$ [Ω/km]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>60</td>
<td>0.383</td>
<td>0.23</td>
</tr>
<tr>
<td>8</td>
<td>80</td>
<td>0.265</td>
<td>0.22</td>
</tr>
<tr>
<td>10</td>
<td>100</td>
<td>0.191</td>
<td>0.2</td>
</tr>
<tr>
<td>14</td>
<td>140</td>
<td>0.123</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table 2 - Upgrade costs

<table>
<thead>
<tr>
<th>To</th>
<th>From</th>
<th>Upgrade cost [$ x 10^3/km]</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>10</td>
<td>91</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>72</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>85</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>105</td>
</tr>
</tbody>
</table>

The best solution of the problem has been found after 16 global iterations (different temperatures) of the proposed SA algorithm. In those iterations the MILP calculations are performed 116 times. The cost of the best obtained solution was 1,326,500 $. This solution was found at temperature $T=390$. The same problem has been solved by applying the MILP model [12] for the entire network from Fig.1, i.e. the problem has been solved without dividing the network from Fig.1 into a number of sub-networks. In this case the cost of the obtained solution was 1,326,500 $. Hence, the proposed approach generates the same solution as the mathematical-programming based model, i.e. it generates the global optimal solution of the considered problem. This result shows that proposed hybrid metaheuristic approach can noticeable improve the quality of distribution expansion planning process.

CONCLUSION

A new hybrid metaheuristic approach for expansion planning of distribution networks has been proposed. The proposed approach employs branch-exchange algorithm, local networks concept and MILP algorithm for generating initial solution as well as a number of alternative (neighborhood) solutions in each step of the SA algorithm. The SA algorithm is designed to ensure proper search intensification and diversification and thus to enable obtaining of high quality solutions. The presented results show that proposed approach has a potential to improve the quality of solutions of real size planning problems.

REFERENCES