A PROBABILISTIC APPROACH FOR VOLTAGE REGULATORS AND CAPACITOR PLACEMENT IN THREE-PHASE UNBALANCED DISTRIBUTION SYSTEMS

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ABSTRACT

Capacitors and voltage series regulators are widely used in distribution systems to improve the system behaviour. A mixed integer, non-linear, constrained optimization problem can be formulated for an optimal allocation of these devices. This problem is frequently solved in deterministic scenarios with Genetic Algorithms. However, the distribution systems are probabilistic in nature, leading to inaccurate and in some cases conservative deterministic solutions. This paper proposes a new probabilistic method to solve the allocation problem. The method is based on the use of a Micro-Genetic Algorithm. Two different techniques based on the linearized form of the constraints of the probabilistic optimization model and on the Point Estimate Method were tested and compared to reduce the computational efforts. The proposed approaches were tested on the IEEE 34-node unbalanced distribution system.

INTRODUCTION

Shunt capacitors and voltage series regulators are used in electrical distribution systems for several reasons; the main are to reduce the power losses and to improve the voltage profile along the feeders. The shunt capacitors operate in discrete steps while voltage regulators are transformers with variable taps; the voltage change is obtained varying the position of the tap by a control circuit.

This paper analyses the problem of contemporaneously choosing optimal locations and sizes for both shunt capacitors and series voltage regulators in three-phase unbalanced distribution systems.

In the most general case, the optimal allocation of capacitors and series voltage regulators in unbalanced distribution systems can be formulated as a mixed integer, non-linear, constrained optimization problem. This problem is usually and successfully solved in deterministic scenarios by applying Genetic Algorithms (GAs) [1].

However, the distribution systems are probabilistic in nature mainly due to the time-varying nature of load demands; then, inaccurate, and in some cases excessively conservative deterministic solutions can arise. As a result, a probabilistic optimization model seems to be most appropriate to take into account the unavoidable uncertainties affecting the problem solution.

The classical Monte Carlo simulation procedure could be applied; however, the use of such a tool in the frame of a GA can require unacceptable computational efforts. To reduce computational efforts in evaluating the state and dependent random variables features, fast techniques have to be applied. In this paper, the linearization of the model constraints and the Point Estimate Method are used and compared. Moreover, a Micro-Genetic Algorithm is also developed and tested to speed-up the convergence.

Similar probabilistic approaches were applied in [2] to solve the optimal location and size of only capacitors in unbalanced distribution systems. In this paper, the approach in [2] is extended to include series voltage regulators.

The paper is organized as follows. The mathematical formulation of the probabilistic optimization problem of the sizing and siting of capacitors and voltage regulators is firstly analysed. Then, the techniques applied to reduce the computational efforts are described. Finally, some tests on the IEEE 34-node test feeder are presented and discussed.

PROBLEM FORMULATION

The optimal siting and sizing of capacitors and of series voltage regulators can be formulated as a mixed integer, non-linear, constrained optimization problem in which an objective function (e.g., the total cost including the losses cost) has to be minimized while meeting a number of equality constraints (e.g., power flow equations) and inequality constraints (e.g., admissible ranges of the bus voltages and limits on line currents). It can be formulated as:

$$\min f_{obj}(\boldsymbol{X},\boldsymbol{C}) \tag{1}$$

$$\psi(\boldsymbol{X},\boldsymbol{C})=0 \tag{2}$$

$$\eta(\boldsymbol{X},\boldsymbol{C}) \leq 0, \tag{3}$$

where: **X** is the system state vector (magnitudes and arguments of the voltages) and **C** is the control vector, related to fixed/switched capacitors and series voltage regulators to be placed at each bus.

The capacitor banks are assumed to be fixed or switched types and are integer multiples of a capacitor unit. The series voltage regulators are assumed to come in discrete devices of pre-specified sizing. In addition, as far as capacitors are concerned, both placement and size are unknown; as far as the voltage regulators are concerned, only the placement is unknown since at each possible voltage regulator position a pre-assigned size is associated; this size is related to the maximum power demands of the loads downstream the voltage regulator position.

Because of the time load variations, the best approach for

the solution of the capacitor and regulators allocations problem should consider the input variables (mainly phase-load demands) to be random variables. Then, a probabilistic formulation of the problem (1) – (3) should be adopted to take into account the random nature of the power required by the loads.
In particular, in this paper, the loads are not assumed to be constant; instead, their random variation is characterized by a normal distribution. In the next sub-sections the objective function and the list of constraints of the probabilistic optimization model are defined in more detail.

The probabilistic objective functions

In this paper, the objective function considered is the total costs (capacitors, voltage regulator and losses). In particular, the expected value (expected value of the losses cost plus the capacitors and regulators cost) is accounted for:

$$f_{obj} = Cost_C + Cost_{VR} + \mu\left[Cost_L\right], \tag{4}$$

where: $Cost_C$ is the cost of the capacitors, $Cost_{VR}$ is the cost of voltage regulators and $\mu\,[Cost_L]$ is the expected value of the cost of the losses, depending on the expected value of the power losses.

The probabilistic equality constraints

Each minimization problem solution should satisfy the equality constraints. That is, the three-phase probabilistic load-flow equations are expressed as:

$$f(\boldsymbol{X}) = \boldsymbol{U}, \tag{5}$$

where: $\boldsymbol{U}$ is the input random vector (active and reactive load phase powers and active three-phase generation powers) and $\boldsymbol{X}$ is the state random vector (magnitude and argument of the unknown phase-voltages).
Moreover, the equations linking the dependent variables to the state variables have to be included:

$$\boldsymbol{D} = h(\boldsymbol{X}), \tag{6}$$

where: $\boldsymbol{D}$ is the dependent variables random vector. In this paper, the dependent variables are the power losses, the line currents and the unbalance factors.

The probabilistic inequality constraints

Each optimization problem solution should satisfy the limits under consideration in terms of admissible ranges of the bus voltages, line currents and unbalance factors [2]. In particular, the line current maximum values should not exceed the line ratings, and the unbalance factors 95th percentiles should not exceed the allowable value provided by the power quality standards. The Standards, or operation rules, also provide the admissible ranges of the bus voltages. In this paper, we assume the Standard EN 50160 as the reference; it suggests that under normal operating conditions, during each period of one week, 95% of the mean rms values of the supply voltage shall be within the range of ±10% of the declared voltage.
The inequality constraints considered in this paper include[1]:

$$\mu\left(I_l\right) + 3\ \sigma\left(I_l\right) \le I_{l,\max} \qquad l \in \Omega_l \tag{7}$$

$$\int_0^{k_{d,\max}} f_{k_{d,i}}\, dk_{d,i} \ge 0.95 \qquad i \in \Omega_{3p} \tag{8}$$

$$\int_{V_{\min}}^{V_{\max}} f_{V_{i,p}}\, dV_{i,p} \ge 0.95 \qquad i = 1,\ldots,N_{bus},\ p = 1,\ldots,N_{ph,i} \tag{9}$$

where: $\mu(I_l)$ and $\sigma(I_l)$ are the expected values and the standard deviation of the current at line l, respectively; $I_{l,max}$ is the current rating for line l; Ω_l is the set of system lines; $k_{d,i}$ is the unbalance factor at bus i; $f_{k_{d,i}}$ is the probability density function of $k_{d,i}$; $k_{d,max}$ is the maximum unbalance factor; Ω_{3p} is the set of three-phase busbars; $f_{V_{i,p}}$ is the probability density function of $V_{i,p}$, and V_{min} and V_{max} indicate the admissible range of voltages.
Eventually, the probabilistic optimization problem to be solved consists in the minimization of the objective function (4) subject to the equality constraints (5), (6) and the inequality constraints (7) – (9). This problem can be easily solved by means of the solution procedure illustrated in next section.

PROBLEM SOLUTION

The probabilistic optimization problem formulated in the previous section, to obtain the optimal siting and sizing of capacitors and series voltage regulators, could be solved with GAs. However, when dealing with large-scale systems, such as in the case of unbalanced distribution systems, GAs can require tremendous computational efforts. This is particularly the case of the optimization problem (4-9) where a probabilistic characterization of input/output random variables is needed and, therefore, probabilistic techniques are required to solve the problem.
First of all, to reduce the processing time while maintaining reasonable accuracy, a micro-Genetic Algorithm (μGA) can be used. This algorithm evolves with populations of only five individuals.
The μGA creates an initial population whose individuals are characterized by the following variables: allocation nodes of

[1] We assume the maximum value of the current flowing in a generic line is the sum of the mean value plus 3 times the standard deviation. When the current probability density functions are of the Gaussian type, it corresponds to assuming the value with not more than 99.86% probability.

capacitors and voltage regulators, and number of elements of a pre-assigned size of capacitors.
Once generated the initial population, the objective function (4) has to be calculated subject to the constraints (5) - (9). To do this, state and dependent random variables features have to be calculated.
In the most general case, the Monte Carlo simulation procedure could be applied for evaluating the state and dependent random variables features; however, the use of such a tool - also in the frame of a μGA - can require tremendous and unacceptable computational efforts. Then, to reduce computational efforts, fast techniques have to be applied. In this paper, the linearization of the constraints and the Point Estimate Method [3] are used.
Eventually (Fig.1), the results of either the Linearization Method or the Point Estimate Method are the inputs of the next genetic algorithm step that consists in the generation of the next population until the stopping criterion is reached.
In the following subsections some details about the Linearization Method and the Point Estimate Method are recalled [2,3].

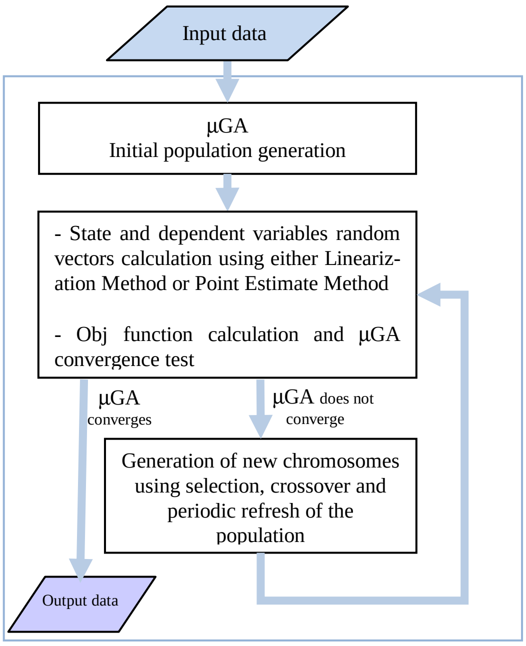

Fig. 1 – Step of the proposed approach

Linearization Method

In reference to the equality constraints (5), the three-phase load flow equations are linearized around an expected value region. In this way, each random element of the state vector is a linear combination of the random elements of the input vector. It follows that the magnitude and argument of the phase-voltages can be approximated by jointly normal correlated variables whose statistical characterization can be effected in terms of the mean values and covariance matrices (note that the input load powers are normal random variables), which can be easily calculated with simple and well known closed form relationships.
Similar considerations arise for the equality constraints (6). The analytical expressions defining such dependence can also be linearized, allowing these dependent variables to be expressed as a linear combination of the random elements of the state vector. It follows that the losses, current magnitudes, and unbalance factors can be approximated by jointly normal correlated variables whose statistical characterizations can only be effected in terms of the mean values and covariance matrices, once again obtainable with very simple and well known closed form relationships. Eventually, in the case of the linearization procedure, only the mean values and covariance matrices of the state and dependent random variables have to be known, because all of the involved random variables are jointly normally correlated variables.

Point Estimate method

The Point Estimate method has been applied in the field of the three-phase probabilistic power flow in [3], with reduced computational efforts, compared to the classical Monte Carlo simulation procedure. This method is similar to the Monte Carlo simulation procedure, since it uses deterministic routines for solving probabilistic problems; however, it allows us to obtain the first moments of the output random variables of interest through solving only a *few deterministic* three-phase power flows, compared to the enormous number of trials required by the classical Monte Carlo simulation procedure. Once the first statistical moments are known, it is possible to approximate the probability density functions of the variables of interest using analytical expressions, such as those based on the Gram-Charlier distributions.
Different schemes can be applied in the frame of the Point Estimate Method; each scheme is characterized by a different number of deterministic three-phase load flows to be solved. In this paper, the $2m+1$ scheme is used, since it provides the best solution in terms of accuracy and computational efforts. For more details about the method see [3].

NUMERICAL APPLICATIONS

The problem of the sizing and siting of capacitor banks and series voltage regulators has been solved for the unbalanced IEEE 34-bus test system, illustrated in Fig. 2 [4], where the original capacitor banks and voltage regulators have been removed. The IEEE 34-bus test system has 80 system nodes and a voltage level of 24.9 kV. The only substation on the network is located above node 800 with transformers from 69 kV to 24.9 kV. This system contains a mixture of single- and three-phase lines and loads. The complete set of network data and parameters can be found in [4].
In all the considered cases, the load demands are Gaussian-

distributed random variables. The mean values of the load powers are assumed to be the peak level reported in [4]. The standard deviation was assumed to be 10 %.

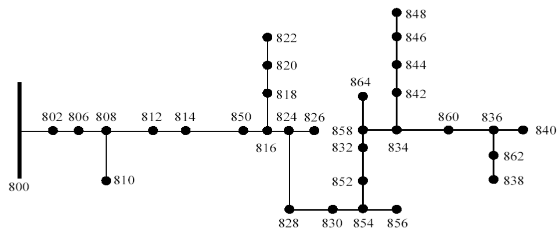

Fig. 2 - IEEE 34-bus distribution test system [4].

In reference to the constraints, the maximum line currents were fixed at the ratings reported in [4], and the maximum value of the 95th percentile of the unbalance factor was assumed to be 3%. In reference to the voltage at each busbar, the 95th percentile of the voltage was assumed to be between 90% and 110% of the nominal value. The unit capacitors available at any bus were assumed to come in discrete sizes of 50 kVar.

Table I reveals the solutions obtained by applying the Linearization Method and Point Estimation Method. We note that the objective function is normalized in reference to the value that it assumes when no capacitors and regulators are installed.

TABLE I: OPTIMAL CAPACITOR AND REGULATOR LOCATION AND SIZE

Method	Location and rating		Objective function [pu]
Linearization	Capacitor banks	450 kVar at bus #842 450 kVar at bus #844 150 kVar at bus #830 100 kVar at bus# 822	0.6895
	Regulators	Bus # 828	
Point Estimate	Capacitor banks	450 kVar at bus #836 300 kVar at bus #842 150 kVar at bus #832 150 kVar at bus# 816 50 kVar at bus# 822	0.6884
	Regulators	Bus # 814	

From an analysis of Table I, it is clear that the objective functions in both methods are very close and significantly lower that those in absence of compensation devices. Obviously, the computational efforts of the Linearization Method resulted significantly lower that that of the Estimate Point method (approximately the 1,25%).

Finally, Fig. 3 shows that the expected values of the phase a voltages obtained with the two methods depend on the position of the voltage regulator, but are anyway close (similar considerations arise for the standard deviations).

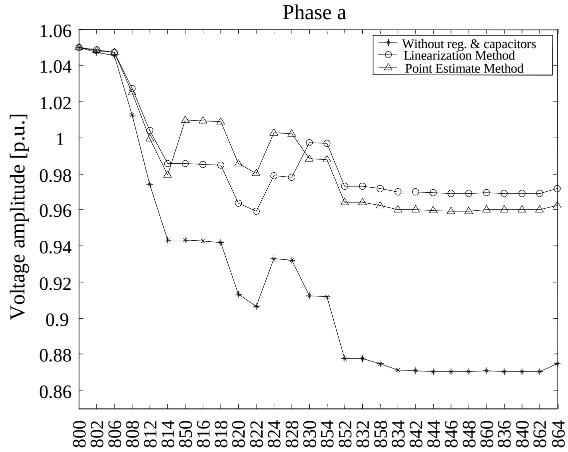

Fig. 3 – Mean values of phase a voltages

CONCLUSION

Capacitors and series voltage regulators are widely applied in the electrical distributions systems to improve the system behaviour.

In this paper, a new probabilistic method for the optimal sizes and location for both shunt capacitors and series voltage regulators in three-phase unbalanced distribution networks has been proposed. The method takes into account the time-varying nature of the distribution system load demands.

To reduce the computational efforts, a µGA was applied and two different techniques based on a linearized form of the constraints of the optimization model and on the Point Estimate Method were tested and compared. The procedure was applied to the IEEE 34-node test system.

This paper demonstrates that both methods provide good solutions in the examined cases. However, the Linearization Method requires computational efforts significantly lower than those of the Point Estimate Method.

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