PROBABILISTIC ASSESSMENT OF WIND FARM ENERGY YIELD CONSIDERING
WAKE TURBULENCE AND VARIABLE TURBINE AVAILABILITIES

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ABSTRACT

Existing wake models used for power output and energy yield calculation usually provide deterministic wind speed values at wind turbines. However, wind inside the wind farm can be affected by several factors which can alter the wind speed at each turbine. A probabilistic wake model is proposed in this paper considering dynamics of wind inside the wind farm. The model can be used during prefeasibility studies for energy yield calculations as well as for power output estimation when wind speed and direction forecast is available. Effect of variable turbine availabilities on energy yield is also investigated in this paper.

INTRODUCTION

Due to stochastic nature of wind it can vary as it passes through turbines (WTs) inside the wind farm (WF). Physical constraints and flow conditions can affect its speed and behaviour making it difficult to predict. For this reason, complex simulation methods such as Finite Element Method, Navier Stokes equations and Computational Fluid Dynamics (CFD) are often used for simulating wind inside a wind farm. These models can lead to reliable results but on the other hand they are complex and cumbersome to implement. Most of them can significantly increase simulation time depending on the computing power available.

Wind farms are becoming an increasingly popular choice in many countries as a future source of electricity. Normally, wake effect models used by practitioners in wind power industry calculate mean wind speed (WS) incident at turbines under wake. In reality however, this may not be entirely realistic as dynamics of wind inside the WF is very complex and can influence ‘effective’ wind speed at each turbine. Effective wind speed is defined as the mean wind speed that affects the power output of a turbine. Wind characteristics are dependant on the way wind interacts with the group of turbines under certain atmospheric conditions. Therefore, a better way would be to estimate a range of possible effective wind speeds that a turbine can face.

A novel way to approximate effective wind speed variation at each turbine is presented in this paper. The proposed approach models stochastic effects inside a wind farm by using deterministic wake model and a turbulence model. This combined model is aimed to reduce computation time while remaining simple in implementation. The methodology is tested on a large wind farm and results validated through measurement data.

PROBABILISTIC WAKE MODEL

Wake effects

Kinetic energy of wind is extracted by the wind turbine leaving wind downstream that is both reduced in speed and turbulent, known as ‘wake’ of a turbine. Thrust coefficient of a turbine is vital to analyse reduction in velocity as well as turbulence generated behind a turbine, since it determines the amount of momentum that is extracted from wind. Those turbines that operate in wake of another turbine face horizontal wind shear and reduced wind speeds. Most often wake models employed in power industry and commercial software provide deterministic results. One such a model is shown in the following section. For instance, the software will calculate mean wind speeds and hence mean power output from each turbine. In reality however, as recorded by site data there are many wind speeds are variable leading to a range of power outputs from each turbine. Deviation of power output from mean value can occur because of several reasons such as wind shear, turbulence, surface roughness, thermal effects causing vertical motion of wind, density of air, wake vortices, shear-generated turbulence, and overlap of wakes. To be able to simulate such deviations two models for wakes have been combined together.

A deterministic wake model is used to calculate the effective mean wind speeds and a turbulence model to estimate deviations in effective mean wind speeds. Normally, turbulence refers to variation of wind speed on a relatively fast time-scale and it is levelled out mostly by rotor inertia. In this study, however, range of effective wind speed is estimated through a turbulence model, which can be caused by any of the processes described above. The turbulence model is used to represent stochastic effects (to model wind behaviour inside WF) that cause these variations.

Deterministic wake model

A commonly used model for power calculations is Jensen’s wake model [4, 5] and this is used in this paper for single wake calculation. The mean wind speed at turbine under single wake is calculated using Jensen’s model as:

\[ v_i = u \left[ 1 - \left( \frac{c}{(r_i + c_x)} \right) \left( 1 - \sqrt{\frac{c}{c_x}} \right) \right] \]

where \( c \) is the entrainment constant which represents the effects of atmospheric stability, \( r_i \) is the radius of the WT, \( u \) is the speed of free-stream wind received by the turbine.
with the thrust coefficient $C_t$, and $v_1$ is the WS at a distance $x_o$ from a WT.

Partial shadowing is a phenomenon that occurs when one or more upwind turbine casts a 'single' shadow on the downwind turbine. If the WT is under partial wake condition the WS at the rotor disc of interest is determined by calculating the ratio (weighting factor, $\beta$) between the rotor area in wake and the total rotor area. WS into the WT is then given by [6]:

$$v_n = u \left(1 - \frac{1}{\sqrt{\sum_k \rho_{k,n} \left(1 - v_{n,k}/u\right)}}\right)$$ (2)

where $j$ is the WT under wake, $k$ is the upwind turbine, $u$ is the initial WS entering into the WT $k$, $v_{n,k}$ is the shadow of WT $k$ falling on WT $j$.

In a larger WF several WTs may be arranged one behind the other. Turbines downwind in the same row receive less wind due to two or more upwind turbines. This effect is referred as multiple wakes and WS approaching the third turbine and onwards in a row is calculated using [7]:

$$v_n = u \left[\left(1 + s + L_u + C_{n,w}\right)\right] \left[\left\{v_n - \sqrt{C_{n,w} - \left(\frac{u}{v_1}\right)^2}\right\} - 1\right] + 1$$ (3)

where $v_n$ is the turbine of which wind speed is being calculated and $v_{n,k}$ and $C_{n,w}$ is the wind speed of the turbine immediately at front shading turbine $n$.

The value of wake decay constant (entrainment constant) $c$ depends on site location, it is usually set to 0.075 for onshore and 0.04 for offshore sites [8]. The model ignores effects of wind shear on blades and other stochastic phenomenon and gives a mean value of wind speed at a turbine. These phenomenons are assumed to be represented by a wake turbulence model.

**Turbulence model**

Generally, turbulence intensity is defined as a measure of overall level of turbulence and is expressed by:

$$I = \frac{\sigma}{\bar{U}}$$ (4)

where $\sigma$ is the standard deviation of wind speed over a period of 10 min and $\bar{U}$ is the mean wind speed.

The model employed for wake turbulence calculation in this paper can be used with single, multiple and partial wakes. It is described [9] as:

$$I = I_o \left(1 + \alpha \exp \left(-\left[\frac{s}{\beta}\right]^2\right)\right)$$ (5)

$$\beta \equiv 0.5 \left\{180/\pi \cdot \tan^{-1} \left(\frac{1}{s} \right) + 10^6\right\} = 25/s[\text{deg}]$$ (6)

where $\beta$ is the characteristic width of the wake, $s$ is the distance between the turbines in separate rows, $x$ is the angle between line connecting two turbines and the wind direction (WD), $\alpha$ is a constant expressed by $I_o$ (ambient turbulence) and $I_w$ (wake added turbulence):

$$\alpha = \sqrt{\left(I_w / I_o\right)^2 + 1} - 1$$ (7)

The prediction of wake added turbulence is usually evaluated based on the wake model being used. $I_w$ is the maximum wake added turbulence intensity at hub height in centre of the wake, which in this case can be expressed as:

$$I_w = \frac{1}{\sqrt{1.5 + 0.35 \sqrt{U}}}$$ (8)

or if thrust coefficient, $C_t$ of turbine is known for every wind speed then the following formula can be used:

$$I_w = \frac{1}{\sqrt{1.5 + 0.1 \sqrt{C_t}}}$$ (9)

where $s$ is the distance between two wakes which wake one or the other, $u$ is the mean wind speed.

**Results from the probabilistic wake model**

The effective mean wind speed at a turbine is calculated using (1)-(3) while the range of speed variation is calculated using (4)-(9) where $\sigma$ defines the width of this range. The distribution is assumed to be Gaussian at each turbine as shown for WT 21 in Fig. 1 (ii). Wind distribution for turbines arranged in a row is plotted using this approach in Fig. 1.

**CASE STUDY**

The proposed method is applied to a wind farm consisting 49 turbines as shown in Fig. 2 (i). Each turbine has a rated power of 2 MW with hub height of about 80 m and rotor radius of 40 m. Rated power of the wind farm is 98 MW. The wind farm is located at sea with surface roughness of 0.0002. Distance between two turbines in the same row is 400 m.

Figure 2 (ii) illustrates probabilistic wind speed received by WT13 inside the WF from all directions. This shows that using deterministic model fixed results are obtained whereas if probabilistic method is used a spread is observed.
Energy Yield Analysis

Traditionally energy yield (EY) is calculated by using Weibull distribution and then power curves of WTs or if wind measurements at the site are available power of each turbine is calculated for every WS and then multiplied by total number of turbines. Both ways overestimate energy yield because wake losses are ignored. In this paper, wake effects are modelled by using both deterministic and probabilistic models, the results are tabulated in Table I. Wind speed and direction measurements for a site in North Sweden recorded with 10-min intervals were available for year 2000. Using probabilistic model, some powers in the year were higher while some were lower than the mean power (calculated using (1)-(3)), equalling out the rise and fall in energy yield. However, the difference observed after several simulations is shown below.

<table>
<thead>
<tr>
<th>EY ignoring wake effects</th>
<th>EY with deter. wake model</th>
<th>EY with prob. wake model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>-15.41%</td>
<td>-15.41% ± 0.2%</td>
</tr>
</tbody>
</table>

It can be seen that considering deterministic wakes results in energy yield losses of about 15.41% (compared to the case when the wake is completely ignored) while inclusion of probabilistic nature of wind “converts” these loses into a range of (15.41± 0.2)%.

EFFECT OF WT AVAILABILITIES ON EY

Turbines operating under wakes of other turbines experience increased loads as compared to turbines in free flow [9, 10] that can result in reduction in lifetime of turbine components. As a consequence turbine may have to undergo regular maintenance during the year preventing it from producing any power for the time it is switched off. Unavailability of turbines is assumed to be dependant on how frequently it stays under wakes during the year. Method presented in [11] is used to calculate frequency of high and low WS every turbine faces, this is to see how much each WT receives free-stream wind and how much it remains under wake. WTs are then grouped into five clusters (high to low WS) based on level and frequency of WS they face in a year. This is shown in Fig. 5 where turbines that face highest WS are placed in Cluster 1 while those that are mostly under wake and face lowest WS are in Cluster 5. Unavailability of WTs is dependant on which cluster they belong to. Two steps of grading, 5% and 10% are used. For instance, in Case 1, turbines under least wake (receiving most WS during the year) are considered to be 100% available, this will decrease to 95% for those WTs which receive slightly less WS, then to 90% for those that receive even lower WS and so on. Similarly, Case 2 is performed with steps of 10%. Case 0 is the reference case when all WTs are available. Wind speed measurements available for
year 2000 from a site in North Sweden are used in all cases. Deterministic wake model was employed for this section.

Conventional way to estimate EY is as described in the previous section which is by ignoring all effects including wake and turbine availabilities. If say turbine availabilities are considered, the same availability factor is assumed for all turbines which may not be realistic as different WTs face different wake loads.

![Figure 5. WTs are placed into clusters based on the level and frequency of WS they receive](image)

TABLE II. ENERGY YIELD DIFFERENCE FOR TWO CASES

<table>
<thead>
<tr>
<th>Cluster Number</th>
<th>Case 0</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-8.65</td>
<td>-17.3</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Results in Table II show reduction in EY in both cases where in Case 2 it is double that of Case 1. It is shown that if unavailability of WTs is estimated based on WF layout, position of WTs and wind site location the effect is significant and cannot be ignored during EY calculations.

CONCLUSION

A probabilistic wake model to account for WF power output variation due to stochastic nature of wind (inside the wind farm) is presented. Using this method, the effect of power output and energy yield can be estimated. It is an attempt to make wake effect models more probabilistic which are rather deterministic at present. The deterministic models do not take into account the dynamic characteristics of the wind inside the wind farm. It is computationally efficient in comparison with complex models available. Main advantage of this approach includes estimating a range of possible WF power output for an available forecast of WS and WD of a few minutes ahead. This method is beneficial since many large wind farms are installed in the network and a range of power output from each WF is needed by system operator to allocate spinning reserve and generator dispatch. The results of the model are dependant on location, layout and type of WTs installed inside a wind farm more precisely on distance between WTs, thrust coefficient, speed and direction of wind entering the wind farm. Impact of different WT availabilities on EY is also presented. Such analysis is useful during pre-feasibility studies to estimate energy loss due to WT unavailability. It considers unavailability of each turbine based on its position in the WF, layout of WF and site measurements. Straightforward inclusion of wake effects reduced EY by about 15% while additional modelling of stochastic nature of wind inside the WF (turbulence) contributed to ±1.3% variation in this value or ±0.2% variation in energy yield. The probabilistic modelling of wake therefore introduces about 0.4% uncertainty in energy yield.

When variable unavailability of wind turbines is considered EY loss varied between between 9% and 17%. Since both wakes and turbine availabilities are actual factors that affect EY they cannot be ignored as they influence on energy yield can be quite significant.

REFERENCES