

## The Distribution Electric Price with Interruptible Load in the Electricity Market

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### ABSTRACT

*In the smart distribution system environment, Interruptible Load (IL) could play an active role on the safe and reliable operation of power system. This paper discusses the principles of IL contracts, and presents a multi-period distribution price model integrating IL contracts and optimal power flow from the win-win of power companies and consumers, then gets the optimal interruptible dispatching load. Numerical results of an IEEE14-bus example have showed the efficiency of the proposed model.*

### INTRODUCTION

Interruptible Load is one of important demand response measures that consumers response to the interrupt request signal from IL implementing organization on the basis of prior contract engagement between power companies and consumers during peak times<sup>[1]</sup>, especially for large industrial and commercial consumers, which can decrease the peak load of system, ease pressure of dispatching, and reduce the risk of the power company furthermore in the electricity market with opening generation side and demand side<sup>[2]</sup>.

In the electricity market with opening demand side, power companies need to cope with three issues if IL as one distributed generator involve in dispatching: firstly, power companies can not fully grasp consumers' information because of anisomerous information between them and consumers. If consumers' declaration in pursuit of high interest are full trusted, it may result in losses on account of hiding information, so it need to think over how to encourage reporting consumers' true cost by setting of interruptible contracts<sup>[3-5]</sup>. Secondly, how to dispatch IL optimizing the distribution of power resources<sup>[6-10]</sup>, most literature is to study the optimal power flow framework on the basis of IL optimal allocation. Thirdly, the distribution price should be how to adapt to the electricity market with the introduction of IL, namely the interaction between the distribution price and IL<sup>[11-15]</sup>. Although there are many research on the three issues, there is little literature taking account of the above issues, how to set IL contracts, how to dispatch loads, and calculate the influence on the distribution price in the electricity market environment. This article will proceed in that aspect, do research on how to dispatch IL combined with IL contracts and optimal

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power flow, while simplifying the optimization process of IL.

From the incentive compatibility principle, the optimal interruptible contracts is got on the basis of principal-agent model and mechanism design theory of game theory to guide consumers reveal their true cost of power shortage in reporting to reach the win-win. Then, in real-time power market environment, based on the classic real-time pricing model of F C Scheppe, we put the cost and corresponding benefits of IL into the objective function minimizing power companies' cost and maximizing economic benefit, while satisfying network constraints and interruptible constraints. Lastly, the optimal dispatching of IL and its impact on the power system was derived by primal-dual interior point method through Kuhn-Tuck conditions.

### MATHEMATICAL MODEL

In the electricity market with opening demand side, when capacity shortage occurred in high peak period, ISO would dispatch IL as DG together with generators on the basis of prior signed IL contracts between power companies and consumers to ensure economic and security operation of power system.

#### Objective Function

We think about multi-period IL dispatching problem, putting the compensation fee of IL into the cost of power companies. The objective function aims to minimize the cost of power companies and maximize economic benefits mathematically expressed as follows, while operating costs include the cost of purchasing electricity, network losses and compensation fee. The cost of purchasing electricity can be expressed as a quadratic function of active power in a fixed moment, the network loss is translated into corresponding cost through a coefficient, and compensation fee is based on the optimal interruption amount and interruption compensation of IL contracts.

$$\min f = \sum_{t=1}^T \left[ \sum_{i=1}^n C(P_{Gi,t}) + \sum_{i=1}^n u_{i,t} S_i^*(\theta_j) + \alpha \cdot \left( \sum_{i=1}^n P_{Gi,t} - \sum_{i=1}^n P_{Di,t} + \sum_{i=1}^n u_{i,t} x_i^*(\theta_j) \right) \right] \quad (1)$$

Where  $C_i(P_{Gi,t}) = aP_{Gi,t}^2 + bP_{Gi,t} + c$  is cost function of purchasing electricity in the period  $t$ ,  $t$  is the period,

$t = 1, 2, \dots, T$ ,  $\alpha$  is conversion coefficient of network loss,  $P_{Gi,t}$  is the active input of node  $i$  in the period  $t$ ,  $P_{Di,t}$  is the active load of node  $i$  in the period  $t$ ,  $x_i^*(\theta_j)$ ,  $S_i^*(\theta_j)$  is the optimal interruption amount and interruption compensation based on reference [5].

$$S_i^*(\theta_j) = K_1 (x_i^*(\theta_j))^2 + K_2 x_i^*(\theta_j) (1 - \theta_j) \quad (2)$$

$$+ K_2 \sum_{m=1}^{j-1} [(\theta_{m+1} - \theta_m) x_i^*(\theta_j)]$$

$$x_i^*(\theta_j) = \frac{\lambda_i p_i(\theta_j) - K_2 p_i(\theta_j) (1 - \theta_j)}{2 p_i(\theta_j) K_1} \quad (3)$$

$$= \frac{K_2 (\theta_{j+1} - \theta_j) \sum_{l=j+1}^J p_i(\theta_l)}{2 p_i(\theta_j) K_1}$$

Where  $\theta_j \in [0, 1]$  means consumers' interruptible type in accordance with the wishes of interruption from small to large order,  $j = 1, 2, \dots, J$ , that is, the smaller  $\theta_j$ , the higher reliability requirement consumers have, and the more unwilling to be interrupted,  $p_i(\theta_j)$  is probability of type  $\theta_j$  of node  $i$ , characterized as  $[p_i(\theta_1), p_i(\theta_2), \dots, p_i(\theta_J)]$ .

### Constraints

1. Shortage capacity constraints of the power grid, that is, sum of interruptible amount at one moment must be greater than or equal to the total system shortage load.

$$\sum_{i=1}^n u_{i,t} x_i^*(\theta_j) \geq P_{que,t} \quad (4)$$

Where  $u_{i,t}$  is a decision variable (0-1 variable), where  $u_{i,t} = 1$  means IL node  $i$  participates in dispatching in the period  $t$ , and  $u_{i,t} = 0$  means IL node  $i$  isn't involved in dispatching in the period  $t$ ,  $P_{que,t}$  is total shortage of power system in the period  $t$ .

2. Maximum interruption number constraint, preventing consumers' requirement aren't met in the actual interruption to guarantee consumers willing to participate in IL.

$$\sum_{t=1}^T u_{i,t} \leq \max(i) \quad (5)$$

Where  $\max(i)$  is the maximum interrupt frequency of node  $i$ .

3. Active power constraints, ignoring reactive power constraints, where equality constraints is the active balance equation, and inequality constraints includes constraints of electricity purchasing, node voltage and branch active power.

$$P_{Gi,t} - P_{Di,t} - u_{i,t} x_i^*(\theta_j) - U_i \sum_{j \in i} U_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0$$

$$P_{Gi,t}^{\min} \leq P_{Gi,t} \leq P_{Gi,t}^{\max} \quad (6)$$

$$U_{i,t}^{\min} \leq U_{i,t} \leq U_{i,t}^{\max}$$

$$P_{ij,t}^{\min} \leq P_{ij,t} \leq P_{ij,t}^{\max}$$

Where  $j \in i$  is node  $j$  connects with node  $i$ ,  $G_{ij}, B_{ij}$  is the element of node admittance matrix,  $\theta_{ij}$  is the phase difference between node  $i$  and node  $j$ ,  $U_i, U_j$  is the voltage amplitude of node  $i, j$ ,  $P_{ij}$  is the active power of branch  $ij$ , from  $i$  to  $j$ .

### Solving Process

The price model is a nonlinear mixed-integer optimization problem, which is difficult using conventional optimization method, so we use the primal-dual interior point method to solve it. According to short-term marginal cost pricing theory, the distribution price is equal to increment ratio of system cost to each node's active load. The multi-period model can made up by multiple single-period model, but there is the maximum interruption number constraint between adjacent time, such as when one consumer reached the maximum interruption number, then in the next period, optimal operation doesn't excluding this consumer.

We took period  $T_1$  for example, the main steps were as follows:

- 1) Input the data, including the original parameters of the distribution network, consumers' information participating in IL management (interruptible probability distribution, electricity costs), etc.
- 2) Get the optimal interruptible amount and optimal compensation according to reference [5].
- 3) Determine the combination type of IL as "interruptible amount-compensation" meeting the shortage of the power system, which can be solved by enumeration. Assuming there has  $h$  consumers existing  $C_h^1 + C_h^2 + \dots + C_h^h$  combinations, remove combinations not satisfying inequality (4), and remain are required combinations.
- 4) Put the  $kth$  composition type into the optimal

dispatching model(1)-(6), and solve the above optimization problem by primal-dual interior point method, then get the corresponding short-term marginal cost of power  $\lambda^k$ , that is  $\frac{\partial f}{\partial P}$ , where  $f$  is the new lagrangian function structured.

5) Put the  $k+1$ th composition type into the optimal dispatching mode, and compare  $\lambda^{k+1}$  with  $\lambda^k$ , to choose a smaller value meaning that composition has more effect on the distribution price.

6) Repeat 5) step until all the combinations finished obtaining the minimum short-term marginal cost of power system (equivalent to bubble method), that is optimal interruptible load programs and best price in this state.

**NUMERICAL EXAMPLES**

This example was on the basis of IEEE14-bus example<sup>[13]</sup>, consumers' type interval was [0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9]. Assume nodes 1,4,7,8 and 13 like to participate in interruptible management, and maximum interrupt frequencies a month were 6,4,3,8 and 4 times respectively. According to the reference[5], interruptible probability distribution and the optimal interruptible amount was shown in Tab.1, that the first line was load probability, and the second line was the optimal interruptible amount under the probability. For space restrictions, there only listed the load of each node at  $T_1$ , as shown in Table 2. At this moment, system shortage load was 19MW, the quadratic and primary term coefficient of consumers' interruption cost were 238.8\$/MW<sup>2</sup> and 31.25 \$/MW.

Tab.1 Interruptible probability distribution and the optimal interruptible amount (unit:MW)

Node\ Type	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1							0.1	0.7	0.2
							0.1	6.5	7.3
4	0.3	0.5	0.2						
	1.1	3.0	3.9						
7					0.2	0.4	0.4		
					4.4	7.0	8.3		
8		0.3	0.6	0.1					
		1.2	3.2	4.0					
13							0.9	0.1	
							6.0	6.7	

After the optimal solution of IL dispatching, we chose the nodes 4,7,8 and 13 in IL management, the optimal break quantity/optimal compensation for interruption were 3.0MW/105.8\$,7.0MW/226.39\$,3.2MW/103.88\$, 6.0MW/160.97\$, total compensation cost was 597.11\$,

and node loads and the distribution price before and after were in Table 3 and Figure 1.

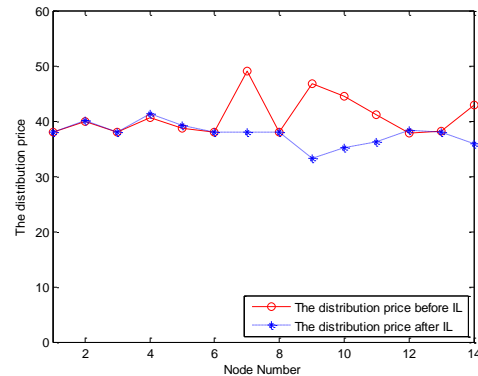


Fig.1 The distribution price with IL in period  $T_1$

Tab.2 Node loads with IL in period  $T_1$  (unit: MW)

Node	Load before IL	Load after IL
1	10	10
2	21.7	21.7
3	93.2	93.2
4	47.8	44.8
5	7.6	7.6
6	21.2	21.2
7	17	10
8	15	11.8
9	29.5	29.5
10	9	9
11	13.5	13.5
12	6.1	6.1
13	33.5	27.5
14	14.9	14.9

Tab.3 Active power input with IL(unit: MW)

Node	Active input before IL	Active input after IL
1	177.35	178.75
2	50	50
3	64.89	60.10
6	39.05	18.46
8	24.44	30.55
Total	355.73	337.76
Network loss	15.73	16.96

As can be seen from Tab.3, the total load of power system after the implementation of ILM decreased from 340MW to 320.8MW decreased by 5.6%, the average distribution price is also from 40.81 \$/MW decreased to 37.7 \$/MW decreased by 7.6%, indicating that inhibit price fluctuations caused by high loads. The changes active power input of power system were as shown in Table 3. From Tab.3 and 4, the active input from 355.73MW to 337.76MW decreased by 5.0%, network loss increased from original 15.73MW to 16.96MW, power purchase fell about 8% in power costs, and network loss fee increased by 7.8%, and all expenses decreased by 1.4%. Taking into

account all cost, electricity price fluctuations had significantly improved, which is conducive to the stability and healthy development of power market. The above data show that the participation of IL is benefit for power companies, consumers and the electricity market.

Tab.4 The electric cost with IL (unit: \$)

	The electric cost before IL	The electric cost after IL
Cost of purchasing	9812	9021.53
Cost of loss	597.74	644.48
Cost of IL		597.11
Total cost	10409.74	10263.12

## CONCLUSIONS

In this paper, we introduce the principle of incentive compatibility to real-time pricing model of power companies to study the impact on the power system especially the distribution price, and do a brief benefit analysis. The example shows that IL is an effective way to participate in the electricity market, inhibiting the excessive price fluctuations of nodes, easing the pressure of peak power grid dispatching, reducing the electric cost, and bringing greater economic benefits.

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