ABSTRACT

In this paper, a new Total Transfer Capability (TTC) based on the Static Voltage Stability Region (SVSR) method is developed and incorporates a new transmission probabilistic nodal loading model (PNLM). The uncertainty of the nodal loading model is determined based on the feeder-head load data, which is computed using a probabilistic three-phase load flow (PTPLF) considering residential load aggregation. An accurate parametric normal distribution is adapted to describe the uncertainty of feeder-head load data. For the transmission-level, a hyper-cone-like model is used to calculate the SVSR boundary based on the proposed nodal load model. Case studies using the IEEE 118 node test system demonstrate the effectiveness of this approach in comparison to the standard Monte-Carlo (M.C.) method and a conventional probabilistic nodal loading model.

INTRODUCTION

Total Transfer Capability (TTC) is the basis of Transmission Available Transfer Capability (ATC) calculation. Probabilistic TTC is more effective in providing uncertainties of system operations. A new fast calculation probabilistic TTC based on the SVSR method is proposed in [1]. The SVSR is classified as a “region” concept that does not change with power injection variations, which has the added capability of dealing with the uncertainties of system operations. A reliable PNLM in parameter space that describes the uncertainties of the load power injections is very important for uncertainty modelling of the SVSR method. Both PNLMs proposed in [1][2] are based on top-down approaches, which determine load growth and load profiles based on measurements. Such approaches yield an empirical, single-lumped model for voltage stability assessment, and cannot reflect the complicated uncertainties contributed by many different types of end-use appliances and equipment in a detailed distribution feeder simulation.

In this paper, the quantitative component-level description is used at the distribution-level to investigate the status of individual residential appliances which switch on and off according to appropriate responses to changes in the outdoor environment, thermostat setpoints, and human behaviour induced effects. To obtain the load aggregation uncertainties, a new probabilistic three-phase load flow (PTPLF) algorithm is presented to model the probability density functions (PDF) of the feeder-head load data through a Monte-Carlo simulation. At the transmission-level, in order to completely describe possible changes in the operational trajectory within the power injection space, we construct the original hyper-cone model which provides a close fit to the actual PDF model extracted from the feeder-head load data to replace the rough one presented in [1]. Through a similarity transformation, we obtain sub-hyper-cone model to determine possible changes in operational trajectories from the current operating point, and achieve a fast calculation (F.C.) method for TTC based on a smaller SVSR boundary area.

DISTRIBUTION-LEVEL

Physically-based residential load modelling

Physically-based residential loads consist of various end-use appliances which are most important for residential load aggregation. An equivalent thermal parameter (ETP) approach is used to represent the loads of a house. Noting that the thermostatically controlled load (TCL) is typically the largest portion of a residential load, the Heating, Ventilation and Air Conditioning (HVAC) system and the water heater are selected as two examples of TCL to be analyzed in this paper. (1) HVAC system

In the ETP method, the building is decomposed into different thermal masses, connected through coefficients of heat transfer. The thermal and electrical behaviours of a typical HVAC system are shown in Fig.1.

Figure 1. The status of power usage and critical parameters of a typical HVAC system in an individual household

In this case, the thermostat setpoint $\theta_{sa}$ is set as 20 ($^\circ$C), the change dead band $\delta_{sa} - \theta_s$ is 1 ($^\circ$C) and the outdoor temperature $\theta_o$ is extracted from climate data. During the winter heating-conditioning months, the rising curves represent the heating mode, when the heating system is
“ON”, and the falling curves represent the drop in indoor temperature, when the HVAC is “OFF”.

(2) Water heater
Another similar device is the water heater. The thermostat logic and the connection to the power system are similar to the HVAC case. For the following case, as shown in Fig.2, the tank setpoint \( \theta_{\text{set}} \) is 50.0 °C and the tank volume is 60 (gal).

Thus the corresponding complex power consumption can be calculated as,
\[
\bar{S}_{\text{light}} = P_{\text{light}}[1 + j(\phi_{\text{light}} - 1)^{1/2}]
\]

(4) Probabilistic three-phase load flow (PTPLF) considering residential load aggregation

For system integration, the slow dynamics of typical TCL and non-TCL models described in the last section can be traced by time-series load flow calculations. In this work, GridLAB-D [4], an open-source software platform developed by Pacific Northwest National Laboratory, is used. The Newton-Raphson based three-phase current injection method (NR_T CIM) is used to solve the load flow. Fig.3 shows the diagram of the new physically-based load modelling for system analysis. In each residential house, through the communication network supported by measurements, the total load complex power consumption \( \bar{S}_{\text{total}} \) can be obtained at desired time points \( \tau_s \) (s = 1, 2, ..., N) with variable time-step of resolution \( \Delta T \).

![Figure 2. The status of power usage and critical parameters of typical water heater in an individual household](image)

The TCL electricity load can be expressed by
\[
P_{\text{TCL}} = n_{\text{TCL}} P_{\text{TCL}}
\]
where \( n_{\text{TCL}} \) is the energy efficiency ratio, and \( P_{\text{TCL}} \) is the heating power. The total electrical power needed by the TCL can be treated as a voltage sensitive ZIP load model,
\[
\begin{align*}
\bar{S}_{\text{TCL}} &= P_{\text{TCL}} + jQ_{\text{TCL}} = S_p,\text{TCL} + S_I,\text{TCL}\phi_\text{f} + Y_{\text{TCL}}\phi_\text{O}^2 \\
\bar{S}_P,\text{TCL} &= \phi_p,\text{TCL}[1 + j(\phi_\text{O}^2 - 1)^{1/2}] \\
\bar{S}_I,\text{TCL} &= \phi_I,\text{TCL}[1 + j(\phi_\text{O}^2 - 1)^{1/2}] \\
Y_{\text{TCL}} &= \phi_\text{O},\text{TCL}[1 + j(\phi_\text{O}^2 - 1)^{1/2}] 
\end{align*}
\]
(2) where \( \phi_\text{p} \) is the power factor; \( \phi_\text{f}, \phi_\text{O} \) are the power fraction of ZIP load; and \( \phi_\text{O} \) is the load voltage factor.

(3) Lights
As a typical non-TCL, the light model includes an installed capacity \( P_{\text{installed}} \) that represents the total consumption. The scheduling of the light model (ON/OFF) is determined by the demand multiplier \( \eta_{\text{demand}} \) (p.u.), which is stochastically modeled based on common human behaviours and is also related to the furnished floor area of a house. The energy consumption is calculated as,
\[
P_{\text{light}} = P_{\text{installed}} \cdot \eta_{\text{demand}}
\]
(3) Each light object used in a house will have a power factor \( \phi_{\text{light}} \) based on different lighting types [3]; several examples of these are: (1) Incandescent lighting: 1.00; (2) Fluorescent lighting: 0.95; (3) Compact fluorescent lighting (CFL): 0.92; (4) Solid-state lighting (SSL): 0.75;

![Figure 3. The schematic diagram physically-based residential load modelling in load flow calculation](image)
are the expectation and variance, which means we can find the critical point for a given load level. As described in (8), for a general load level $A$ in the transmission system, the SVSR calculations considering load probability density impact TTC of the system operating changes from point $O$ to $A$, just as the dark part $OAC$, shown in Fig.5.

\begin{equation}
S_o = \{P_{org}, C_{org} \mid P_{org} \in \Phi, C_{org} \in \Sigma_T\} \tag{7}
\end{equation}

The $P_{org}$ is a possible point in $\Phi$ whose critical point is $C_{org}$ in a proportionally increasing mode. Thus, the surface of a hyper-cone could be used to include $S_o$, just as the dark part $OAC$, shown in Fig.5.

**TRANSMISSION-LEVEL**

**Static Voltage Stability Region Analysis based on hiper-cone-like probabilistic nodal load modelling**

With the new PNLM, we could get a more accurate load probability distribution density. In order to analyze the how load probability density impact TTC of the transmission system, the SVSR calculations considering probabilistic factors is utilized. Fig.5. $A$ is the current operating point and $\Sigma_T$ is the SVSR boundary in full power injection space. If a direction like $AC_0$ is specified, the critical point $C_0$ can be obtained by some saddle node bifurcation (SNB) points calculation methods such as CPF. We can construct a hyper-tangent plane $\Sigma$ over $C_0$ to approximate $\Sigma_T$ locally; more hyper-tangent planes may be necessary due to the curvature of $\Sigma_T$ like $\Sigma_\alpha$ and $\Sigma_\beta$ in Fig.6. Vector quantification clustering analysis [1] can be used to determine how many hyper-planes should be used.

The load demand can be assumed to be subject to a normal distribution $f(t; \bar{S}_{node})$. By using the Monte Carlo sampling method, we can get a set of possible operating points $\Phi$ around $A$, as shown in Fig.5. For static voltage security assessment in full power injection space, it is common to increase the load demand and generator output proportionally. Their trajectory can be represented by a line $\overline{OAC_0}$ from point $O$ through $A$ to $C_0$. As the operating point varies, all possible trajectories could be described as follow:

\begin{equation}
S_o = \{P_{org}, C_{org} \mid P_{org} \in \Phi, C_{org} \in \Sigma_T\} \tag{7}
\end{equation}

The $P_{org}$ is a possible point in $\Phi$ whose critical point is $C_{org}$ in a proportionally increasing mode. Thus, the surface of a hyper-cone could be used to include $S_o$, just as the dark part $OAC$, shown in Fig.5.

**Figure 5. Hyper-cone-like probabilistic loading model in power injections space**

The length of $AC_0$ is proportional to the value of the TTC. In real systems, the prediction of where the potential operating point changes direction is the key point for probabilistic TTC calculations, especially for load change directions which are not dispatchable. What we are actually concerned with, is the possible variations in direction of the current operating point in the transmission system, rather than the uncertainty of the current operating point. So, here we treat all possible directions of system operating changes from point $O$ to $A$, means we can map $S_o$ to a new set $S_A$ to represent all possible trajectories from point $A$. As described in (8), for a possible trajectory in $S_o$ denoted as $OP_{org}C_{org}$, we can find a new critical point $C_{sub}$ in $\Sigma_T$ to let $AC_{sub}$ in parallel with $OP_{org}C_{org}$, then $AC_{sub}$ becomes one trajectory in $S_A$.

\begin{equation}
S_A = \{AC_{sub} \mid \exists! \overline{AC_{sub}} // OP_{org}C_{org}, \overline{OP_{org}C_{org}} \subseteq S_o, C_{sub} \in \Sigma_s\} \tag{8}
\end{equation}

Thus the $AC_{sub}$ associated with $A$ is easier to obtain.

**Case analysis**

The test case system consists of a networked sub-
transmission system modified from the IEEE 118 test case at 100 KV as shown in Fig.6. The sub-transmission system is divided into two main parts named receiving power and supplying power (shown as a dashed line).

Figure 6. The IEEE 118 bus test system
For comparison, the probabilistic load model is considered in the two following cases:
Case-1: as reported in [1], with means one of unity and standard variances of 20% of the means;
Case-2: The normal distribution model, which has the same means as Case-1, but with a standard deviation of 0.0023 due to the different load level of individual nodes.

As shown earlier in Table 1, when considering generation output and load fluctuation, Case 1 needs 19 hyper-planes to form the intersection of the sub-hyper-cone and the SVSR boundary whereas Case 2 only needs 10. Thus, the computing time of the F.C. method also changes accordingly; Case 1 requires twice the computational runtime of Case 2. Meanwhile $E_{TTC}^{FC}$ of both cases are very close, that is to say changes in $(\sigma_{TTC}^{FC})^2$ has little impact on $E_{TTC}^{FC}$, but the situation is quite different to the distribution shape of $f_{TTC}(t;x)$. The distribution shape variation of $f_{TTC}(t;x)$ becomes much smaller when decreasing $(\sigma_{TTC}^{FC})^2$, as showed in Fig.7.

Table 1 Results comparison considering load and generator output uncertainties

<table>
<thead>
<tr>
<th>k</th>
<th>$N_{cw}$</th>
<th>$\theta_{ab}$ (°)</th>
<th>Time (s)</th>
<th>$E_{TTC}^{FC}$ (MW)</th>
<th>$(\sigma_{TTC}^{FC})^2$</th>
<th>$\text{error}$(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>19</td>
<td>39.76</td>
<td>1.547</td>
<td>877.9509</td>
<td>33.8513</td>
<td>0.0795</td>
</tr>
<tr>
<td>0.0267--</td>
<td>10</td>
<td>39.31</td>
<td>0.781</td>
<td>877.8485</td>
<td>0.7196</td>
<td>0.0033</td>
</tr>
</tbody>
</table>

Standard Monte Carlo (M.C.) method

<table>
<thead>
<tr>
<th>k</th>
<th>Time(s)</th>
<th>$E_{TTC}^{MC}$ (MW)</th>
<th>$(\sigma_{TTC}^{MC})^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>630.172</td>
<td>877.2537</td>
<td>26.2865</td>
</tr>
<tr>
<td>0.0267--</td>
<td>660.125</td>
<td>877.8193</td>
<td>0.2469</td>
</tr>
</tbody>
</table>

Figure 7. Probability density of TTC of case 1 & 2
A new SVSR calculation method was developed in conjunction with a probabilistic nodal loading model based on physically-based residential load models. A hyper-cone-like model was adapted to achieve fast calculations of the SVSR boundary. Case studies were presented and the effectiveness of the proposed approach was verified by comparison with Monte-Carlo method and the conventional rough PNLM.

REFERENCES


ACKNOWLEDGMENTS

The authors would like to thank the Pacific Northwest National Laboratory, and David Chassin in particular, for their extensive support in using GridLAB-D. Financial support from the Pacific Institute for Climate Solutions (PICS) is gratefully acknowledged. Thank are also due to Mr. Simon Parkinson for assistance with GridLAB-D.