AN EVOLUTIONARY ALGORITHM BASED TECHNIQUE TO DETERMINE RATIONAL APPROXIMATION OF FREQUENCY DOMAIN RESPONSES

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ABSTRACT

The aim of this paper is to present an alternative methodology for obtaining rational functions that suitably approximate the frequency response of power system networks. Through this methodology, part of the system under analysis can be replaced by a Frequency Dependant Network Equivalent (FDNE), which simplifies system representation and makes possible the accurate evaluation of large power networks.

The methodology is based on Evolutionary Algorithms (EAs). Basically, different set of values for the rational function parameters are simultaneously assessed. The EA suitably changes the values for the function parameters, decreasing the RMS error between the system's original frequency response and the response provided by the rational function.

The results obtained through the proposed methodology were compared with the ones obtained through the Vector Fitting methodology, which is the common approach used for solving similar problems.

INTRODUCTION

It is widely known that the frequency domain approach is commonly applied in the assessment of power systems due to its lower computation burden. Despite of the better accuracy for the network models, time domain approaches are restricted to evaluate only small portions of a system. Thus, when assessing large power networks, only a frequency domain approach is considered. However, even in such scenario, the performance of the solution process may be considerably affected depending on the size of the linear system involved.

The common approach to overcome such problem consists on using network equivalents. So, part of the system under study is replaced by a simple equivalent circuit, and only the buses and branches that are the focus of the study are maintained intact. The equivalent circuit is composed by only a few network elements, decreasing the size of the corresponding linear system and enabling the achievement of a solution.

Nevertheless, when executing complex studies, such as harmonic assessment, the equivalent circuit should provide responses similar to the ones provided by the original network at any frequency range. Consequently, the task of designing a suitable equivalent circuit becomes not a trivial one.

Basically, designing the FDNE consists on determining a rational function which approximates the frequency response data (admittance data) from the part of the system that is not the focus of the analysis. Through this rational function, a much simpler circuit could be derived according to the type and the number of poles. Thus, the number of network elements used to represent the whole system can be considerably reduced, enabling the execution of power system simulation for very complex networks.

The admittance data used to find the rational function can be obtained through a frequency scan simulation in the complete network representation. Then, in order to simplify the following simulations and improve its computation performance, the equivalent circuit can be used to replace the corresponding part of the system.

There are a few methodologies used to determine rational functions to approximate the frequency responses [1] [2]. The most promising one is the Vector-Fitting methodology [1]. It is a widely accepted methodology, due to its accuracy and performance. Several other papers have been published containing improvements made on the original methodology. Vector-Fitting implementation is also very complex, leading its authors to freely distribute a toolbox.

This paper proposes an alternative methodology, based on EAs, which defines the values for the parameters of a rational function that approximates the frequency response from the part of the system under study that should be replaced by the FDNE.

The use of EAs in power system topics has become very common in the past few years. It was initially used to optimize the solution of load flow problems. For example, in [4] the authors use the EA to define the optimal generation dispatch. Another example in using EA for solving load flow problems is presented in [3]. In this paper, the EA was used to define the most suitable set of injected harmonic currents, in order to estimate the harmonic voltage distortion at the buses of a power network.

EAs are also used for parameter estimation in power system studies. In [5], the authors use an EA to determine the best values for the parameters of a motor model. The convergence of this approach was verified through the torque characteristics comparison between the estimated model and the one provided in the manufacturer manual.

Following a similar approach, an EA was used in the present paper to estimate the parameters of a rational function that can represent the frequency response behavior of a power system network. Further details about the methodology are presented in the following section.

METHODOLOGY

Evolutionary Algorithms

A brief description of the EAs and their fundamental aspects are provided in this section, since the configuration of their parameters directly affects the results of the parameter estimation problem.

Evolutionary Algorithms are a branch of Evolutionary Computation, which is a generic population-based metaheurist optimization process [15].

The EAs correspond to an iterative process. Making an analogy with the natural selection process, each iteration is also called *generation*. Each *generation* is composed by a population of *individuals*, which correspond to a set of possible solutions. Each *individual* is composed by a *gene*, which stores the value for a parameter of the solution.

Initially, the *population* for the first *generation* is randomly selected. Then, all the corresponding *individuals* are evaluated. At the evaluation stage, the *fitness* of each individual is evaluated through a function, which indicates how satisfactory the solution is. The *fittest individuals* tend to obtain higher evaluations. After evaluating the first *population*, the EA starts its iterative process. The EA operators (*mutation*, *recombination* and *reproduction*) are executed, in order to compose the next *population*. The first two operators modify the *genes* of the *individuals* from the previous *generations*, in order to create different solution alternatives. The reproduction operator selects the fittest individuals, in order to compose the population for the next generation.

The EA Applied to the Rational Function Estimation Problem

The basic idea of the present work is to define the values for the parameters of a rational function, similar to the one presented in Equation (1), which can suitably represent the frequency response behavior of a power system network.

$$G(s) = \sum_{n=1}^{N} \frac{c_n}{s - p_n} + d + se \quad (1)$$

So, in the same way that happens with the Vector-Fitting methodology, a maximum number of poles N is previously defined.

Solution Codification

Only the values for the poles of the rational function were considered in the design of the EA's *individual*, in order to reduce the amount of parameters to be estimated directly through the EA and help the problem convergence. The values for the zeros (d) and the residues (se) of the rational function were estimated though the least squares method during the evaluation of each *individual*.

Two different codifications for the *individuals* were assessed in this paper. The simplest case is illustrated by Figure 1. Such codification considers that the rational function is composed by real poles only.

	Pole #1		*Pole #N*
Value	γ_1	•••	γ_N
Deviation	Δ_1		Δ_N

Figure 1 – Codification for Real Poles

The second codification is illustrated in Figure 2, and considers that the rational function is composed only by complex poles. As complex roots come in complex conjugate pairs, the number of poles that is actually directly considered in the codification is *N/2*. The corresponding complex conjugate pair of each pole is considered during the evaluation process. However, the number of values to be estimated (number of genes) continues to be *N*, due to the real and imaginary part of each pole.

	Pole #1			*Pole #N/2*	
	Real Part	*Imag. Part*		*Real Part*	*Imag. Part*
Value	α_1	β_1	•••	$\alpha_{N/2}$	$\beta_{N/2}$
Deviation	δ_1	Δ_1		$\delta_{N/2}$	$\Delta_{N/2}$

Figure 2 – Codification for Complex Poles

Fitness Function – Solution Evaluation

Initially, it is verified if each solution alternative leads to a passive network equivalent, i.e., if the real part of the frequency response at every frequency value is positive.

Then, if the solution alternative is passive, its corresponding rational function is evaluated through the inverse of the RMS error between the original frequency response data from the power system network and the frequency response provided by the corresponding rational function.

Initial Population

In order to generate the initial population, the frequency range from the original frequency response data is arbitrarily divided into 100 partitions. Then, in order to define the initial value for each pole, a partition is randomly selected. The pole value corresponds to a frequency value randomly selected within the partition limits. The pole deviation corresponds to half of the difference between the partition's higher and lower limits.

When using the codification for complex poles, the imaginary part value and its deviation are defined according to the procedure described before. The real part value and its deviation are defined by dividing the value and the deviation of its corresponding imaginary part by 100.

Reproduction

A (μ, k, λ) evolutionary strategy was used in the present work. It means that λ *individuals* were generated from a population of μ *individuals*. Such strategy also defines a maximum number of *generations k* that an *individual* can

remain during the EA convergence process. Through this approach, *individuals* that reach the limit of *k generations* are replaced by new ones (which are created through the process described for composing the initial population), giving the opportunity to test different solution alternatives, that might contribute favorably for the problem solution.
Following the evaluation stage, the first 25% of the fittest *individuals* are select to compose the next *generation*. The other *individuals* that compose the population of the next *generation* are created through the recombination and mutation operators.

Recombination

The recombination is the EA operator that combines the characteristics of two solution alternatives into a new one. For the recombination operator execution, a probability value is randomly selected for each *individual* from the previous *generation*. If such value is below a probability rate previously defined on the EA configuration, the recombination operator is applied into the selected *individual*, in order to create a new one.
In this paper, the recombination was made by calculating the average of the values and deviations. Figure 4 illustrates the recombination process for the codification defined for real poles rational functions.

γ_1	γ_2	γ_3	γ_4	γ_5
Δ_1	Δ_2	Δ_3	Δ_4	Δ_5

(a) *Individual* #1

γ_6	γ_7	γ_8	γ_9	γ_{10}
Δ_6	Δ_7	Δ_8	Δ_9	Δ_{10}

(b) *Individual* #2

$(\gamma_1+\gamma_6)/2$	$(\gamma_2+\gamma_7)/2$	$(\gamma_3+\gamma_8)/2$	$(\gamma_4+\gamma_9)/2$	$(\gamma_5+\gamma_{10})/2$
$(\Delta_1+\Delta_6)/2$	$(\Delta_2+\Delta_7)/2$	$(\Delta_3+\Delta_8)/2$	$(\Delta_4+\Delta_9)/2$	$(\Delta_5+\Delta_{10})/2$

(c) New *individual*

Figure 4 – Recombination operator

Mutation

The mutation operator is applied for each *individual* from the previous *generation* several times (according to a specific number of mutations per *individual* previously defined on the EA configuration), in order to create new ones.
The mutation operator defines the value and the deviation for the new *individual*'s *genes* according to the values and deviations from the selected *individual*. An auto-adaptive approach was used, as illustrated in Equations (3) and (4) for the codification used for real poles. Through this approach, the EA is capable of making a better verification of the region around each pole for a better solution. So, the chance of disturbing the process convergence is also reduced.

$$\Delta_n^{new} = \Delta_n \cdot e^{\left(\tau' \cdot \mathrm{N}(0,1) + \tau \cdot \mathrm{N}_n(0,1)\right)} \quad (3)$$

$$\gamma_n^{new} = \gamma_n + \Delta_n^{new} \cdot \mathrm{N}_n(0,1) \quad (4)$$

Where:

- $\mathrm{N}(0,1)$: is a random number selected from a normal distribution with mean value 0 and standard deviation 1for every *individual*.
- $\mathrm{N}_n(0,1)$: is a random number selected from a normal distribution with mean value 0 and standard deviation 1for every *gene*.
- τ': *individual*'s learning rate
- τ: *gene*'s learning rate

RESULTS

The data from two different frequency responses were used in this paper to assess the proposed methodology. The results were compared with those provided by the Vector-Fitting methodology.
The EA-based methodology applied the codification with real poles for Case #1 and the codification with complex poles for Case # 2. The approximation considered 2 poles for Case #1 and 6 poles for Case #2.
In Figures 5 and 6 one can visually compare the approximations determined by the EA-based methodology and by the Vector Fitting methodology with the original frequency responses. There is no visual difference between the approximations.
In Tables I and II, one can observe the rational function parameters defined by each methodology. The total RMS error of each approximation is also presented.
In Case #1, the simulation considered 100 *generations*, 20 *individuals* per *generation*, 100% for the recombination probability and 10 mutations per *individual*. In order to make a fair comparison, the Vector Fitting methodology was executed 2,000 times.
In Case #2, the simulation considered 100 *generations*, 200 *individuals* per *generation*, 100% for the recombination probability and 10 mutations per *individual*. Also here, in order to make a fair comparison, the Vector Fitting methodology was executed 20,000 times.

CONCLUSIONS

A new methodology for determining a rational function approximation of frequency response data was introduced. Although the approximation made through the Vector-Fitting methodology presented smaller errors, the EA-based methodology also presented good approximation results. The errors are in the magnitude order as those obtained through the Vector-Fitting methodology.
An important aspect to highlight is that through the EA-based methodology does not require any passitivity enforcement procedure to reach a passive equivalent. As a result, the authors of this paper intend to execute further investigations over this topic, evaluating the methodology for other frequency responses.

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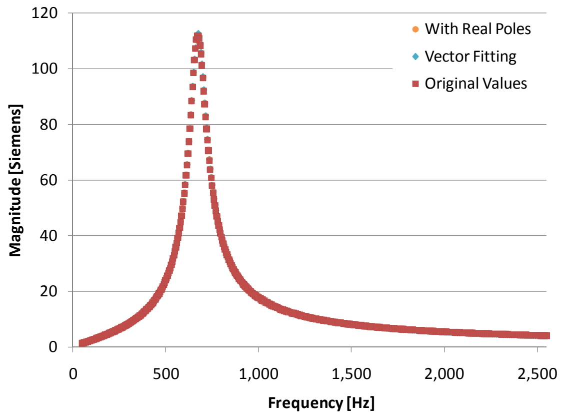
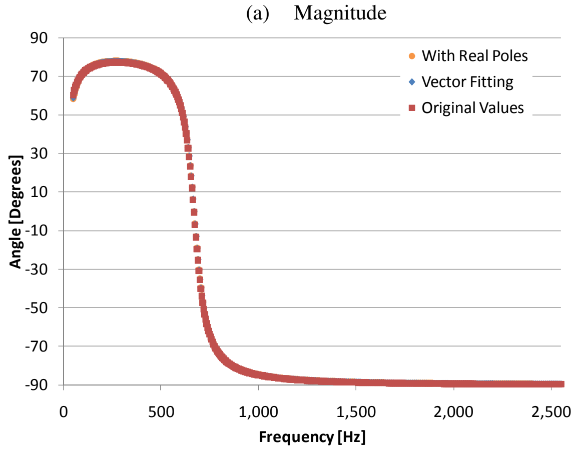
Table I – Rational Function Parameters for Case #1

	Codification with Real Poles	Vector Fitting
Zero #1	$4.18e^{+01}$	$2.67e^{+02}$
Pole #1	$-3.40e^{+03}$	$-1.81e^{-02}$
Zero #2	$2.69e^{+02}$	$4.03e^{+01}$
Pole #2	$-1.85e^{+02}$	$-2.92e^{+03}$
“d”	$1.27e^{-03}$	$1.69e^{-03}$
“e”	$1.66e^{-05}$	$1.66e^{-05}$
Error %	*0.94*	*0.74*



(a) Magnitude

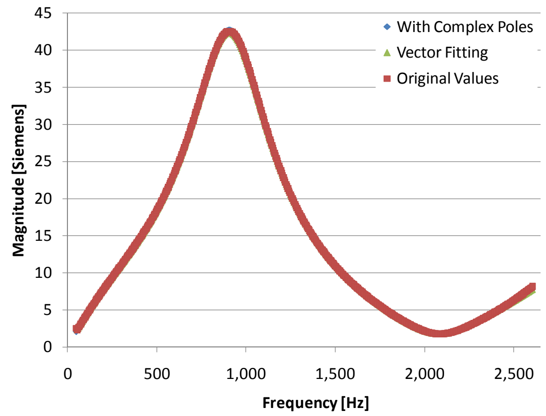


(b) Angle

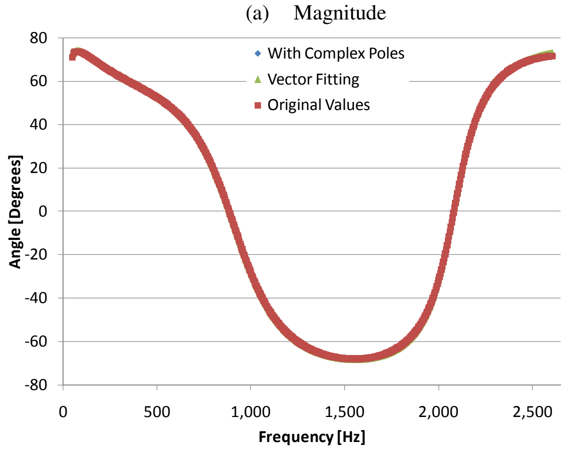
Figure 5 – Frequency Response for Case #1

Table II – Rational Function Parameters for Case #2

	Codification with Complex Poles	Vector Fitting
Residue #1	$2.09e^{+01}-j5.31e^{+04}$	$1.51e^{+02}$
Pole #1	$-6.51e^{+03}+j9.38$	$-6.14e^{+01}$
Residue #2	$2.09e^{+01}+j5.31e^{+04}$	$1.56e^{+02}$
Pole #2	$-6.51e^{+03}-j9.38$	$-3.96e^{+03}$
Residue #3	$4.30e^{+02}+j8.81e^{-01}$	$-3.71e^{-01}-j4.37$
Pole #3	$-7.73e^{+02}+j1.31e^{+04}$	$-4.67-j3.11e^{+02}$
Residue #4	$4.30e^{+02}-j8.81e^{-01}$	$-3.71e^{-01}+j4.37$
Pole #4	$-7.73e^{+02}-j1.31e^{+04}$	$-4.67+j3.11e^{+02}$
Residue #5	$7.54e^{+01}-j1.49e^{-10}$	$4.34e^{+02}-j4.37$
Pole #5	$-7.32e^{+01}$	$-7.82e^{+02}-j1.31e^{+04}$
Residue #6	$7.54e^{+01}+j1.49e^{-10}$	$4.34e^{+02}+j4.37$
Pole #6	$-7.32e^{+01}$	$-7.82e^{+02}+j1.31e^{+04}$
Parameter “d”	$1.08e^{-02}$	$4.07e^{-03}$
Parameter “e”	$1.82e^{-06}$	$2.10e^{-06}$
RMS Error %	*1.55*	*1.09*



(a) Magnitude



(b) Angle

Figure 6 – Frequency Response for Case #2

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