

## A NOVEL TECHNIQUE FOR MODELING AGGREGATED HARMONIC-PRODUCING LOADS

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### ABSTRACT

*Through this discussion, a Harmonic Coupled Norton Equivalent (HCNE) model is pointed out as the most suitable one to represent the behavior of harmonic producing loads. A methodology for obtaining the parameters of HCNE model using measurements is presented on this paper.*

*Two different simulations are carried out using a simple power network, in order to assess the impact of the load modeling. One considers the typical current source model, in order to represent the harmonic producing load, and the other considers the HCNE model. The results show considerable differences on the harmonic indices, indicating that the load modeling may considerably affect the harmonic assessment of power networks.*

### INTRODUCTION

Nowadays, it is widely known that harmonic distortions levels are steadily increasing in distribution networks due to the proliferation of power electronic-based home appliances. This increasing trend represents a concern for planning engineers on utility companies, due to the distributed and random nature of these non-linear loads. In order to adequately prepare for the future, utilities must be able to predict the harmonic distortions injected by these non-linear loads and evaluate the ability of the existing power networks to accommodate these distortions within the limits established by the national regulations and international standards.

Nevertheless, as the number and variety of power electronic products available continue to grow rapidly, the task of providing a reliable and distortion free service becomes even more complicated for the utility companies. In this new environment, improved assessment methods and models of power network components are being required, in order to evaluate the collective impact of these types of load. These new methods and models should not only allow evaluating the level of disturbances in the power networks. They should also be able to indicate how the disturbance levels vary during the day, among different locations, and according to the type and size of the connected load.

However, it is difficult to measure the impact these simplifications would have over the results, while a methodology that considers the distributed and the random of the harmonic-producing loads simultaneously

does not became available.

The work presented on this paper continues the work presented in [6]. The methodology was extended, in order to deal with real measurements from power distribution transformers. A comparative evaluation was carried out on a real power distribution feeder.

### HARMONIC LOAD MODELING

#### Common Practice – The Current Source Model

Due to the lack of models that accurately represent the behavior of harmonic producing loads, authors tend to over simplify the loads' representation and make use of current source models to represent the harmonic content injected in the power networks.

Commonly, the approach used to model harmonic producing loads is based on a set of current sources, one for every harmonic frequency being considered in the analysis.

According to this modeling, the distortion injected into the system by each current source is represented through its corresponding magnitude and phase-angle. These parameters are normally supplied with respect to the phasor of the fundamental current and considering the response of the harmonic producing load when a 1pu purely sinusoidal voltage is applied to it. Table I illustrates the parameters for the current sources used to represent the non-linear loads in such modeling.

Table I – Parameters for the current source model

Harmonic Order	Magnitude	Angle
<i>fundamental</i>	$ i_{spectrum}^{fund} $	$\theta_{spectrum}^{fund}$
3	$ i_{spectrum}^3 $	$\theta_{spectrum}^3$
5	$ i_{spectrum}^5 $	$\theta_{spectrum}^5$
7	$ i_{spectrum}^7 $	$\theta_{spectrum}^7$
⋮	⋮	⋮

Depending on the bus of the system where the non-linear load is connected and the behavior of the harmonic producing load at the fundamental frequency, the fundamental current will assume a specific value. This value can be obtained through regular load flow. After the convergence this process, equations (1) and (2) are used in order to define the parameters for the other harmonic currents.

$$\begin{cases} |i^h| = |i^{fund}| \times \frac{|i_{spectrum}^h|}{|i_{spectrum}^{fund}|} & (1) \\ \theta^h = \theta_{spectrum}^h + h \times (\theta^{fund} - \theta_{spectrum}^{fund}) & (2) \end{cases}$$

From these equations one can note that the magnitude of

the harmonic currents are simply scaled up with respect to the magnitude of the fundamental current; and the angle is shifted with respect to the angle of the fundamental current. So, the pattern of the harmonic spectrum of the distortion injected into the system by the non-linear load is constant and proportional the power consumption of the harmonic producing load at the fundamental frequency. Thus, according to this modeling, the harmonic injection into the power network does not vary according to the voltage applied at the bus where the load is connected.

Such assumption is valid only when the distortion of the voltage does not change considerably through the power network, and has several negative impacts on the harmonic assessment of power systems.

One of the main issues regarding this modeling is related to accurate representation of harmonic producing loads. Actually, the harmonic content injected into the power network by the non-linear loads does depend on the voltage waveform applied to them. For example, on uncontrolled AC/DC converters, the voltage waveform influences the switching characteristics of their electronic components directly, which will define the harmonic content produced by such kind of non-linear loads. Thus, as there are different distortion levels at different points of the network, it is not reasonable to assume that non-linear loads will inject into the system the same harmonic currents according the same spectrum pattern.

The other issue is related to the influence one harmonic producing load may have over the operation of another harmonic producing load. This problem is illustrated through Fig. 1. This figure presents a simplified representation of a distribution feeder containing distributed harmonic producing loads. The harmonic voltage at a specific bus  $k$  can be calculated through equation (3). Through this equation, one can clearly notice that the harmonic distortion of the voltage at a specific bus depends on the contribution of the other harmonics sources.

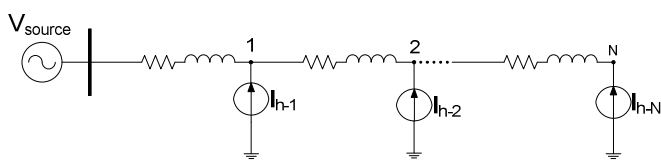


Fig. 1 – Distribution system with distributed harmonic producing loads

$$\dot{V}_{at\ bus\ k}^h = \sum_{i=1}^N \dot{Z}_{k-i}^h \times \dot{I}_i^h \quad (3)$$

So, the influence of the distortions generated by one harmonic producing load over the operation of other harmonic producing loads cannot be efficiently evaluated when the current source model is used to represent them. This approach tend to be too conservative and simulated results normally present higher values for the voltage a current distortions than the ones actually found when real measurements are taken from system in study [1].

### Typical Norton Equivalent Model

In order to overcome the limitations of such modeling, the authors of [2] have proposed an alternative approach for the harmonic load models. The approach was based on Norton equivalent circuits.

Through this approach, the current injected into the system by the load would depend on the voltage applied over the load. The main challenge of this research was to apply the modeling in real networks, once the value of the admittance in the Norton equivalent circuit should be determined through a change in the operation condition of the system.

Such modeling has partially overcome the issue regarding the non-dependence of the voltage by the non-linear loads in harmonic analysis of power systems. In this approach, the harmonic content injected at each specific frequency is affected only by the harmonic voltage of the respective frequency. Basically, at each specific frequency, a particular Norton equivalent circuit is used to represent the load behavior. Depending on the harmonic voltage applied at the load bus, part of the harmonic content inject by the current source will flow through the admittance of the Norton equivalent circuit, and part will be definitely injected into the system.

Equation (4) represents the mathematical behavior of the model. The disadvantage of this model is related with the fact that harmonic current injected into the system at a specific frequency depends only on the harmonic voltage of the same frequency.

In reality, the harmonic current injected into the system at each specific frequency depends on the voltage waveform, once the switching condition of the semiconductor components inside the non-linear load is related to it. Therefore, one can say that the harmonic current at a specific frequency injected into the system by non-linear load depends on all harmonic voltages.

$$\begin{bmatrix} \dot{I}_{injected}^{fund} \\ \dot{I}_{injected}^{3rd} \\ \dot{I}_{injected}^{5th} \\ \vdots \\ \dot{I}_{injected}^{Nth} \end{bmatrix} = \begin{bmatrix} \dot{I}_{Norton}^{fund} \\ \dot{I}_{Norton}^{3rd} \\ \dot{I}_{Norton}^{5th} \\ \vdots \\ \dot{I}_{Norton}^{Nth} \end{bmatrix} - \begin{bmatrix} \dot{Y}_{Norton}^{fund} & 0 & 0 & \dots & 0 \\ 0 & \dot{Y}_{Norton}^{3rd} & 0 & \dots & 0 \\ 0 & 0 & \dot{Y}_{Norton}^{5th} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \dot{Y}_{Norton}^{Nth} \end{bmatrix} \times \begin{bmatrix} \dot{V}^{fund} \\ \dot{V}^{3rd} \\ \dot{V}^{5th} \\ \vdots \\ \dot{V}^{Nth} \end{bmatrix} \quad (4)$$

### Harmonic Coupled Norton Equivalent Model

The drawbacks that were pointed out for the previous modeling can be solved when one uses a harmonic coupled admittance matrix in the Norton equivalent circuit of the non-linear load. Such kind of approach has been introduced in [3], in order to model arc furnaces. In [4] the same authors used this approach to model controlled and uncontrolled three-phase AC/DC converters. In [5], the authors have extended the work for single-phase AC/DC converters. All models were based on the theoretical operating conditions of the modeled

equipments and presented a good concordance with time-domain results, indicating its accuracy. In [3], the authors have tried to extend this type of modeling for home appliances using the measured response of such equipments. Unfortunately, due to numerical difficulties, the attempt failed and the authors ended up using the typical Norton equivalent approach described in the previous section.

Basically, the HCNE model consists of a non symmetrical full admittance matrix. For a particular row, each value in this matrix reflects how each harmonic voltage affects the corresponding harmonic current. The values in the diagonal indicate how the harmonic current is affected by the harmonic voltage of the corresponding frequency, and the off-diagonal values indicate how the harmonic current is affected by the harmonic voltages of the other frequencies being considered in the analysis.

Equation (5) represents the mathematical behavior of the model.

$$\begin{bmatrix} j_{injected}^{fund} \\ j_{injected}^{3rd} \\ j_{injected}^{5th} \\ \vdots \\ j_{injected}^{Nth} \end{bmatrix} = \begin{bmatrix} j_{Norton}^{fund} \\ j_{Norton}^{3rd} \\ j_{Norton}^{5th} \\ \vdots \\ j_{Norton}^{Nth} \end{bmatrix} - \begin{bmatrix} \bar{Y}_{Norton}^{fund,fund} & \bar{Y}_{Norton}^{fund,3rd} & \bar{Y}_{Norton}^{fund,5th} & \dots & \bar{Y}_{Norton}^{fund,Nth} \\ \bar{Y}_{Norton}^{3rd,fund} & \bar{Y}_{Norton}^{3rd,3rd} & \bar{Y}_{Norton}^{3rd,5th} & \dots & \bar{Y}_{Norton}^{3rd,Nth} \\ \bar{Y}_{Norton}^{5th,fund} & \bar{Y}_{Norton}^{5th,3rd} & \bar{Y}_{Norton}^{5th,5th} & \dots & \bar{Y}_{Norton}^{5th,Nth} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \bar{Y}_{Norton}^{Nth,fund} & \bar{Y}_{Norton}^{Nth,3rd} & \bar{Y}_{Norton}^{Nth,5th} & \dots & \bar{Y}_{Norton}^{Nth,Nth} \end{bmatrix} \times \begin{bmatrix} \hat{V}^{fund} \\ \hat{V}^{3rd} \\ \hat{V}^{5th} \\ \vdots \\ \hat{V}^{Nth} \end{bmatrix} \quad (5)$$

### OBTAINING THE HARMONIC COUPLED NORTON EQUIVALENT MODEL

In order to obtain the HCNE model for a non-linear load, a set of suitable measurements at the point of common coupling (PCC) between the load and the system should be taken. Each measurement should represent a possible operation condition of the non-linear load. Fig. 2 illustrates the equivalent electric circuit, indicating the PCC where the measurements should be taken. The system is represented through its Thévenin equivalent circuit, and the non-linear load is represented through the HCNE model.

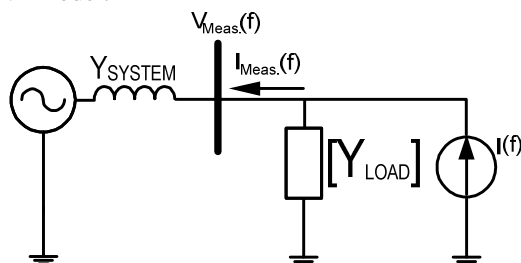


Fig. 2 – Equivalent electric circuit used to obtain the parameters of the HCNE model.

The HCNE model is obtained by solving several linear systems, one for each frequency being considered on harmonic analysis study. Each linear system is built up using the set of suitable measurements taken at the PCC.

Assuming that the behavior of a non-linear load can be represented only by the odd harmonics, Equation (8) shows the linear system that should be solved in order to obtain the first line of the HCNE model. This particular system provides the values for the admittances and the current source that define the behavior of the non-linear load at the fundamental frequency.

The values for the admittances and current source that define the behavior of the non-linear load at any other frequency can be obtained in a similar way, by solving different linear systems. Basically, the matrix containing the measured voltage values would remain the same. Only the vector containing the measured current values would be changed accordingly the frequency of the parameters being calculated, i.e., in order to calculate the parameters that define the behavior of the non-linear at the M<sup>th</sup> harmonic order, the measured current values at the M<sup>th</sup> harmonic order should be used in a linear system similar with the one shown by Equation (8). Therefore, in order to represent the load by N harmonic frequencies, (N+1) sets of measurements will be needed. For the case illustrated in Equation (8), as only the odd harmonics are being considered, the non-linear load would be modeled up to the (2xN-1)<sup>th</sup> harmonic order.

It is important to highlight that the set of measurements used to obtain the HCNE model should represent different operation conditions of the non-linear load being modeled, in order to guarantee that the linear system would be linearly independent.

$$\begin{bmatrix} j_{meas.\#1}^{fund} \\ j_{meas.\#2}^{fund} \\ j_{meas.\#3}^{fund} \\ \vdots \\ j_{meas.\#N}^{fund} \\ j_{meas.\#(N+1)}^{fund} \end{bmatrix} = \begin{bmatrix} \hat{V}_{meas.\#1}^{3rd} & \hat{V}_{meas.\#1}^{5rd} & \dots & \hat{V}_{meas.\#1}^{Mth} & 1 \\ \hat{V}_{meas.\#2}^{3rd} & \hat{V}_{meas.\#2}^{5rd} & \dots & \hat{V}_{meas.\#2}^{Mth} & 1 \\ \hat{V}_{meas.\#3}^{3rd} & \hat{V}_{meas.\#3}^{5rd} & \dots & \hat{V}_{meas.\#3}^{Mth} & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \hat{V}_{meas.\#N}^{3rd} & \hat{V}_{meas.\#N}^{5rd} & \dots & \hat{V}_{meas.\#N}^{Mth} & 1 \\ \hat{V}_{meas.\#(N+1)}^{3rd} & \hat{V}_{meas.\#(N+1)}^{5rd} & \dots & \hat{V}_{meas.\#(N+1)}^{Mth} & 1 \end{bmatrix} \times \begin{bmatrix} \bar{Y}_{fund} \\ \bar{Y}_{3rd} \\ \bar{Y}_{5th} \\ \vdots \\ \bar{Y}_{Mth} \\ j_{fund} \end{bmatrix} \quad (6)$$

The process of obtaining the can be extended, in order to use real measurements. When using real measurements, the number of measurements is considerably larger than the number of equations for the linear system represented in Equation (6). In such cases, the HCNE model can be obtained through the least squares method.

### RESULTS

In order to evaluate the impact of the different model, ten different single-phase transformers were measured from a 25kV power distribution feeder located in a residential area. All transformers measured had the same rated power: 37.5kVA. The measuring protocol consisted on recording a 6-cycles snapshot at every 3 seconds. The sampling rate used was 256 samples/cycle.

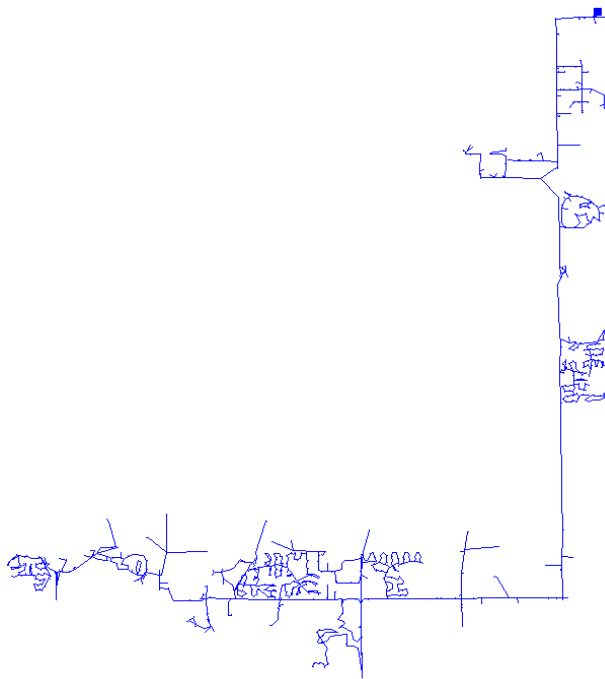


Fig. 3 – Power Distribution Feeder used for the Simulations

The current source model, the Norton equivalent model and the HCNE model were obtained for each of the ten transformers for the same time of the day. Only weekdays were considered. As each transformer were measured for approximately three weekdays days, around 3,600 snapshots were used for obtaining the models for each transformer.

Then, three multi-phase harmonic loads flow simulations were executed, one for each type of model. The models obtained for each of the ten transformers were used to represent all single-phase transformers with a rated power equal to or lower than 37.5kVA (corresponding to approximately 40% of feeders installed power). The other loads were modeled as constant power loads at the fundamental frequency only.

So, each single-phase transformer with rated power equal to or lower than 37.5kVA was randomly linked to all three models obtained for one of the ten measured transformers. A compensation based on the fundamental power demanded from the measured transformers and the demand calculated for each power transformer was carried out; in order bring the results closer to what could be found in the real system.

The voltage and current THDs were measured at the substation bus. Such values were used to compare the results from the three simulations. The results are summarized in Table II.

## CONCLUSIONS

This paper presented a discussion about the approaches commonly used to model harmonic-producing loads. The advantages and disadvantages of each approach were pointed out, and the HCNE model was considered as the

one with higher chances of modeling harmonic-producing loads accurately. A methodology for obtaining the parameters for the HCNE model from measurements was introduced, and it was used to model real power distribution transformers.

Comparing the THD results, one can notice considerable differences for the voltage and current distortions at the load bus, indicating that the influence of the load modeling in the harmonic load flow results is significantly. As a result, the authors of this paper intend to execute further investigations over this topic, improving the system representation and trying to extend the load modeling to all non-linear loads connected to the network.

Table II – Model Comparison

	THD[%]		
	Current Source	Norton Equivalent	HCNE
Voltage Phase A	1.378	1.119	0.458
Voltage Phase B	1.358	1.147	0.343
Voltage Phase C	1.300	1.066	0.397
Current Phase A	7.288	8.610	11.162
Current Phase B	10.050	9.193	7.519
Current Phase C	17.573	8.745	9.311

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