# OPTIMAL PLACEMENT OF PMUS FOR RELIABLE OBSERVABILITY OF NETWORK UNDER PROBABILISTIC EVENTS USING BABC ALGORITHM 

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#### Abstract

This paper implements optimal PMU placement to make the network observable, and guarantees the observability of network under probabilistic events. With regards to the high cost of PMU units, another objective of the paper is minimizing of PMUs' installation cost. To demonstrate the effectiveness of the propose method, it is applied to IEEE 14-bus test system using a binary artificial bee colony algorithm (BABC) and compared with genetic algorithm (GA).


## INTRODUCTION

The appropriate operation of power system requires accurate observation of network's operation states. The measurements are obtained by the Remote Terminal Units (RTUs) located on the substations. They include active and reactive powers flowing through the lines, and the injected powers into the buses. Recently the measuring units use the synchronizing satellite signal of Global Positioning System (GPS), and they measure the positive sequences of voltages and currents [1]. Regarding that the PMUs can measure the currents of lines leading to the bus, by appropriate placement of PMUs, all the lines’ currents and buses' voltages can be obtained [2]. In [3], the graph theory and Simulated Annealing (SA) have been employed for solving the presented optimization problem of optimal PMU placement. In [4], for optimizing the number and location of PMUs, the genetic algorithm has been used. Ref. [5] which has proposed a new method for estimating the location, size and the type of harmonic loads, has assumed that the measurements have been implemented on a few points of network; the paper has not pointed to the optimality and the way of obtaining these points. In present paper, a new approach is proposed in which firstly, the nature of network is not changed with the change of topology. Secondly, the accuracy of results is maintained. Thirdly, there is the possibility of modulation of previous measurement results with the new installed units. These measurement results include the flowing and injected powers in network. The fourth advantage of the proposed method is that in the optimal placement of PMUs, to insure of network observability in the case of probabilistic events, the probabilistic events have also been considered.

## PROBLEM FORMULATION

The placement of PMUs is tried to be performed by employing the minimum number of them. In this paper, the considered problem is solved using BABC. For an n-bus network, the optimal placement problem is formulated as follows:

$$
\begin{equation*}
\min \sum_{i}^{n} w_{i} \cdot x_{i} \tag{1}
\end{equation*}
$$

st. $f(X) \geq u$
where: $w_{i}$ is the installation cost of PMU on bus $i$; this has been considered 1 for all buses i.e. $\left(w_{i}=1 \quad, \quad i=1, \cdots, n\right) \cdot u$ is the base vector which all of its inputs are 1. $f(X)$ is a vector function, if the voltage of an specific bus can be obtained from the measurement, its value is non-zero, otherwise it is zero. Here, the binary matrix, A , is defined as below:
$A_{h, d}= \begin{cases}1 & \text { If the two nodes } \mathrm{h} \text { and } \mathrm{d} \text { are connected to each other } \\ \text { through a branch } \\ 1 & \text { If there is no relation between } \mathrm{h} \text { and } \mathrm{d} \\ 0 & \text { If the two nodes are namesake }(\mathrm{h}=\mathrm{d})\end{cases}$
In this case, the vector function $f(X)$ is determined as following:

$$
\begin{equation*}
f(X)=A_{n \times n} \cdot X_{n \times 1} \tag{3}
\end{equation*}
$$

$X$ is a variable binary vector which its inputs are defined as follows:

$$
x_{i}= \begin{cases}1 & \text { if } P M U \text { is installed on bus } i  \tag{4}\\ 0 & \text { ow }\end{cases}
$$

The constraints of optimization problem can be determined based on the location and type of existing measurements. It is assumed that by installing one PMU on a specific bus, the voltage and current phasors of lines leading to that bus are determinable. Therefore, the voltages of all buses can be obtained. The way of determining of the constraints is expressed in three different conditions which are as follows:
a) In the case that only the PMUs exist, i.e. it has not been installed any other measuring device on the network.
b) In addition to phasor measurements, it is possible that the injected powers (the buses without power injection fall into this group) of some buses be measured by means of measurement devices which have been
previously installed.
c) Further to phaseor measurements and when the injected powers of some buses are known, the flowing powers through some lines may be known.
The way that these three conditions influence the optimal PMU placement is described in section 4. In this paper, for reliable observability of probabilistic events, the voltage stability method on the basis of classifying the probabilistic events based on Reactive Violation Index ( $R V I$ ) and estimation of cape point has been used [6], this method is explained as following: The way of calculating, where $\lambda$ is the loading coefficient of system. The operating point, $A$, shows the cape point of safe system, and the point $B$ is the probabilistic position of system operating point after probabilistic event with the same value for loading parameter used in point $A$. The reactive power violation index for probabilistic event of $i$ is defined as (5):

$$
\begin{equation*}
R V I_{i}=\sum_{j=1}^{N_{q}}\left(Q_{j i}^{n o}-Q_{j}^{\max }\right) \tag{5}
\end{equation*}
$$

$Q_{j}{ }^{\text {max }}$ : Maximum reactive power limitation of source $j$
$\mathrm{Q}_{\mathrm{ji}}{ }^{\text {no }}$ : The reactive generation of $j^{\text {th }}$ source with unlimited $Q$ which follows from the $i^{\text {th }}$ probabilistic event (with the same value for loading parameter in point $A$ ).
$\mathrm{N}_{\mathrm{q}}$ : the number of reactive power sources violating the $Q$ limitation.

## BABC ALGORITHM

In the ABC algorithm proposed by Karaboga, the position of a food source represents a possible solution to the optimization problem, and the nectar amount of a food source corresponds to the profitability (fitness) of the associated solution. Each food source is exploited by only one employed bee. In other words, the number of employed bees is equal to the number of food sources existing around the hive (number of solutions in the population). The employed bee whose food source has been abandoned becomes a scout. Using the analogy between emergent intelligence in foraging of bees and the ABC algorithm, the units of the basic ABC algorithm can be explained as follows:

## Producing initial food source sites

If the search space is considered to be the environment of the hive that contains the food source sites, the algorithm starts with randomly producing food source sites that correspond to the solutions in the search space. Initial food sources are produced randomly within the range of the boundaries of the parameters.

$$
\begin{equation*}
x_{i j}=x_{j}{ }^{\min }+\operatorname{rand}(0,1)\left(x_{j}{ }^{\max }-x_{j}{ }^{\min }\right) \tag{6}
\end{equation*}
$$

Where $i=1 .$. SN, $j=1$. . D. SN is the number of food sources and $D$ is the number of optimization parameters. In addition, counters which store the numbers of trials of solutions are reset to 0 in this phase. After initialization, the population of the food sources (solutions) is subjected to
repeat cycles of the search processes of the employed bees, the onlooker bees and the scout bees. Termination criteria for the ABC algorithm might be reaching a maximum cycle number (MCN) or meeting an error tolerance.

## Sending employed bees to the food source sites

As mentioned earlier, each employed bee is associated with only one food source site. Hence, the number of food source sites is equal to the number of employed bees. An employed bee produces a modification on the position of the food source (solution) in her memory depending on local information (visual information) and finds a neighboring food source, and then evaluates its quality. In ABC , finding a neighboring food source is defined by (7).

$$
\begin{equation*}
v_{i j}=x_{i j}+\varphi_{i j}\left(x_{i j}-x_{k j}\right) \tag{7}
\end{equation*}
$$

Within the neigbourhood of every food source site represented by xi, a food source ti is determined by changing one parameter of xi. In Eq. (7), j is a random integer in the range $[1, \mathrm{D}]$ and $\mathrm{k} 2\{1,2 \ldots \mathrm{SN}\}$ is a randomly chosen index that has to be different from $\mathrm{i} . \mathrm{ij}$ is a uniformly distributed real random number in the range $[-1$, 1]. As can be seen from Eq. (7), as the difference between the parameters of the $\mathrm{x}_{\mathrm{i}, \mathrm{j}}$ and $\mathrm{x}_{\mathrm{k}, \mathrm{j}}$ decreases, the perturbation on the position $\mathrm{x}_{\mathrm{i}, \mathrm{j}}$ decreases. Thus, as the search approaches to the optimal solution in the search space, the step length is adaptively reduced. If a parameter value produced by this operation exceeds its predetermined boundaries, the parameter can be set to an acceptable value. In this work, the value of the parameter exceeding its boundary is set to its boundaries. If $x_{i}>x_{\text {max }}$ then $x_{i} 1 / 4 x_{\text {max }}$ i, If $x_{i}<x_{\text {min }}$ then $x_{i} 1 / 4 x_{\min }$. After producing $t_{i}$ within the boundaries, a fitness value for a minimization problem can be assigned to the solution $\mathrm{t}_{\mathrm{i}}$ by (8).

$$
\text { fitness }_{i}=\left\{\begin{array}{ccc}
1 /\left(1+f_{i}\right) & \text { if } & f_{i} \geq 0  \tag{8}\\
1+a b s\left(f_{i}\right) & \text { if } & f_{i} \prec 0
\end{array}\right\}
$$

Where $f_{i}$ is the cost value of the solution $v_{i}$. For maximization problems, the cost function can be directly used as a fitness function. A greedy selection is applied between $\mathrm{x}_{\mathrm{i}}$ and $v_{i}$; then the better one is selected depending on fitness values representing the nectar amount of the food sources at xi and $v_{i}$. If the source at $v_{i}$ is superior to that of xi in terms of profitability, the employed bee memorizes the new position and forgets the old one. Otherwise the previous position is kept in memory. If xi cannot be improved, its counter holding the number of trials is incremented by 1 , otherwise, the counter is reset to 0 .

## Calculating probability values involved in probabilistic selection

After all employed bees complete their searches, they share their information related to the nectar amounts and the positions of their sources with the onlooker bees on the dance area. This is the multiple interaction features of the artificial bees of ABC. An onlooker bee evaluates the nectar information taken from all employed bees and chooses a food source site with a probability related to its nectar
amount. This probabilistic selection depends on the fitness values of the solutions in the population. A fitness-based selection scheme might be a roulette wheel, ranking based, stochastic universal sampling, tournament selection or another selection scheme. In basic ABC , roulette wheel selection scheme in which each slice is proportional in size to the fitness value is employed (9):

$$
\begin{equation*}
p_{i}=\frac{\text { fitness }_{i}}{\sum_{i=1}^{S N} \text { fitness }_{i}} \tag{9}
\end{equation*}
$$

In this probabilistic selection scheme, as the nectar amount of food sources (the fitness of solutions) increases, the number of onlookers visiting them increases, too. This is the positive feedback feature of ABC .

## Food source site selection by onlookers based on the information provided by employed bees

In the ABC algorithm, a random real number within the range $[0,1]$ is generated for each source. If the probability value (pi in Eq. (9)) associated with that source is greater than this random number then the onlooker bee produces a modification on the position of this food source site by using Eq. (7) as in the case of the employed bee. After the source is evaluated, greedy selection is applied and the onlooker bee either memorizes the new position by forgetting the old one or keeps the old one. If solution xi cannot be improved, its counter holding trials is incremented by 1 ; otherwise, the counter is reset to 0 . This process is repeated until all onlookers are distributed onto food source sites.

## Abandonment criteria: Limit and scout production

In a cycle, after all employed bees and onlooker bees complete their searches, the algorithm checks to see if there is any exhausted source to be abandoned. In order to decide if a source is to be abandoned, the counters which have been updated during search are used. If the value of the counter is greater than the control parameter of the ABC algorithm, known as the "limit", then the source associated with this counter is assumed to be exhausted and is abandoned. The food source abandoned by its bee is replaced with a new food source discovered by the scout, which represents the negative feedback mechanism and fluctuation property in the self-organization of ABC. This is simulated by producing a site position randomly and replacing it with the abandoned one. Assume that the abandoned source is xi, and then the scout randomly discovers a new food source to be replaced with xi. This operation can be defined as in Eq. (6). In basic $A B C$, it is assumed that only one source can be exhausted in each cycle, and only one employed bee can be a scout. If more than one counter exceeds the "limit" value, one of the maximum ones might be chosen programmatically [7]. Regarding the fact that parameters of the proposed problem are discrete time type and the performance of standard ABC is based on real numbers, this algorithm can not be used
directly for optimal placement of PMUs. Thus, in order to overcome this drawback a binary based ABC (BABC) algorithm is used for solution of the PMU placement problem. So, the Eq. (6) is modified as Eq. (10).
$x_{i j}=x_{j}{ }^{\min }+\operatorname{round}\left(\operatorname{rand}(0,1)\left(x_{j}{ }^{\max }-x_{j}{ }^{\text {min }}\right)\right)$
Where, $\left(0 \leq x_{i j}(\right.$ new $\left.) \leq 1\right)$.

## SIMULATION RESULTS

The proposed method for optimal placement of PMUs has been applied on the IEEE 14-bus test system (Fig. 1). All the effective signals of interrupted lines have been regarded as probabilistic events, and the results of gradation of probabilistic events using $R V I$ method and binary search have been obtained [6]. For the IEEE 14bus system, three critical probabilistic events have been considered (Table 1). These three numbers are optional and can be selected adequately for a system.


Fig. 1: IEEE 14-bus system
The following experiments are performed on this system.
Sate 1: The system is without any previous measurements, or the bus is without any injection
In this case, the binary matrix $A$ will be as $A_{1}$.
$A_{1}=\left[\begin{array}{llllllllllllll}1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1\end{array}\right]$

The output of BABC algorithm in state 1 , where the system
is safe, is as $\{2,7,11,13\}$. Now, with the outage of line 12 , as the first probabilistic event, the connection of buses 1 and 2 will be interrupted and the bus 1 will not be observable anymore. The binary matrix, $A$, related to this condition is modified as $A_{2}$ in which the elements $(1,2)$ and $(2,1)$ are changed from 1 to 0 . In state 1 , with the outage of lines 2-3 and 5-6, the results of BABC algorithm for each of these states are as $\{3,5,7,11,13\}$. Table 1 shows the optimal locations of PMUs for state 1 .

Table 1: Optimal Location of PMUs for state 1

| Number of PMUs | Optimal Locations | Situation |
| :---: | :---: | :---: |
| 4 | $\{2,7,11,13\}$ | safe |
| 5 | $\{3,5,7,11,13\}$ | Outage of line 1-2 |
| 5 | $\{3,5,7,11,13\}$ | Outage of line 2-3 |
| 5 | $\{3,5,7,11,13\}$ | Outage of line 5-6 |

Sate 2: The system is without any previous measurements, or the bus is without any injection
The buses with the known injected powers and those without the injected powers are assumed similar. In the IEEE 14-bus system, shown in Fig. 1, suppose that the bus 3 has a known injected power. In this case, if the phasors of two buses out of three buses 2, 3, 4 are known, then the voltage phasor of third bus can be determined using the Kirchhoff's Current Law (KCL). If so, we can transfer the bus having known injected power onto one of adjacent buses, and change the topology of network to make it simpler. If bus 3 have a known injected power, by transferring it to bus 4 , the binary matrix $A$ is modified to $A_{3}$ in which on the row 3 and column 4 the "OR" operand has been applied and the new row of 4 is created and the row 3 is eliminated, and all of the elements of column 3 become 0 ; The optimal locations of PMUs for state 2 are listed in Table 2.

Table 2: Optimal Location of PMUs for state 2

| Number of PMUs | Optimal Locations | Situation |
| :---: | :---: | :---: |
| 4 | $\{2,8,10,13\}$ | safe |
| 4 | $\{5,8,10,13\}$ | Outage of line 1-2 |
| 4 | $\{5,8,10,13\}$ | Outage of line 2-3 |
| 4 | $\{5,8,10,13\}$ | Outage of line 5-6 |

State 3: System is considered with the known injected and flowing powers in some of buses
In IEEE 14-bus system of Fig. 1, suppose that in addition to that bus 3 has known injected power, the flowing power ( $P$, Q) through the line between buses 7 and 8 is known. According to Kirchhoff's voltage law ( $K V L$ ), having the flowing power through a line and the voltage phasor of one of starting or ending bus of that line, the voltage phasor of bus at the other side can be calculated. For setting up such conditions, in binary matrix of $A$, the buses which the flowing power is known in their connecting lines, the corresponding rows of the two buses is being "OR"; the two rows are eliminated and the new row is substituted. For
example, if further to bus 3 , the flowing powers between buses 7 and 8 is known, by putting the rows 7 and 8 under "OR" operand, the new row of 8 is obtained and the new binary matrix of $A$ is modified as $A_{4}$. The results of optimal locations related to this state are presented in Table 3.

Table 3: Optimal Location of PMUs for state 3

| Number of PMUs | Optimal Locations | Situation |
| :---: | :---: | :---: |
| 3 | $\{2,6,9\}$ | safe |
| 3 | $\{5,6,9\}$ | Outage of line 1-2 |
| 3 | $\{5,6,9\}$ | Outage of line 2-3 |
| 3 | $\{5,6,9\}$ | Outage of line 5-6 |

## CONCLUSION

In optimal placement of PMUs for the aim of network complete observability, the probabilistic events must be considered. This paper proposes a new method for optimal PMU placement to observe the network topology completely and reliably under safe and probabilistic events using BABC algorithm. Also, in this paper, the possibility of merging the results of previous measurements including the flowing and injected powers in network with the new installed units has been considered. The proposed approach is applied to IEEE 14-bus system.

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