SUPER DECENTRALIZED CONTROL FOR DISTRIBUTION VOLTAGE REGULATION ROBUST AGAINST IMPERFECT POWER FLOW DATA COLLECTION

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ABSTRACT

Super Decentralized Control is a cooperative control method for voltage control devices on distribution networks. Each controller has a state estimation function and an optimal control function. The state estimation function estimates distribution voltage and current from imperfect power flow observations collected via limited communication lines. The optimal control function optimizes control of each voltage control device on the basis of a comparison between estimations and target voltage. By superposition of voltage controls, the distribution voltage is regulated as a decentralized feedback system. We designed a mathematical model of Super Decentralized Control and confirmed the capability of the model through power flow simulation.

INTRODUCTION

Future distribution management systems optimize voltage-control devices such as the Step Voltage Regulator (SVR) and the Static Var Compensator (SVC) on the basis of power flow measurements collected via communication lines for distribution networks [1]. However, these communication lines cannot always collect the data perfectly because of bandwidth shortages or communication problems. Management systems must therefore be robust against imperfect power flow data collection.

In response to this need, we are developing a decentralized cooperative controller for voltage regulation. As shown in Figure 1, a distribution network is equipped with plural controllers, each of which consists of a state estimation function and an optimal control function. We assume that every node that has a load, SVR, and SVC also has a sensor that measures local voltage and local current but that the controller can collect remote measurements from only limited nodes through the communication line.

In this paper, we propose a mathematical model for Super Decentralized Control as a decentralized state feedback system in which estimations and control signals can differ for each controller. We also present power flow simulation results that show cooperative behaviour of decentralized controllers and confirm the system can regulate the distribution voltage.

MATHEMATICAL MODEL

Network Topology

Figure 2 shows an example of a tree-shaped distribution network topology, which is described by a neighbouring matrix and a connection matrix in the following definition.

- Upstream neighbouring matrix $\boldsymbol{U}$: $u(p)$ is a node number of the upstream neighbouring node of node *p*.
- Downstream neighbouring matrix $\boldsymbol{D}$: $d(n,p)$ is a node number of the downstream neighbouring node of node *p* on the path to node *n*.
- Connection matrix $\boldsymbol{C}_{\mathrm{U}}$, $\boldsymbol{C}_{\mathrm{D}}$, $\boldsymbol{C}_{\mathrm{O}}$, $\boldsymbol{C}_{\mathrm{E}}$:

$$C_{\mathrm{U}}(n,p) = \begin{cases} 1: & \text{node } n \text{ is in upstream side of node } p \\ 0: & \text{the others} \end{cases} \quad (1)$$

$$C_{\mathrm{D}}(n,p) = \begin{cases} 1: & \text{node } n \text{ is in downstream side of node } p \\ 0: & \text{the others} \end{cases} \quad (2)$$

$$C_{\mathrm{O}}(n,p) = \begin{cases} 1: & \text{node } n \text{ is node } p\ (n=p) \\ 0: & \text{the others} \end{cases} \quad (3)$$

$$C_{\mathrm{E}}(n,p) = \begin{cases} 1: & \text{node } n \text{ is in parallel path of node } p \\ 0: & \text{the others} \end{cases} \quad (4)$$

Power Flow

Impedance

A branch from node $u(p)$ to node p is described as $u(p) \to p$, whose impedance is known parameter $r_{u(p)\to p} + jx_{u(p)\to p}$.

Voltage and current

Because of the limitation of power flow data collection, voltage and current observations can differ for different pairs of observing and observed nodes. The following definition gives a matrix description in which $\dot{V}_{ip}$ and $\dot{I}_{ip}$ is local voltage and local current of node *p* as the internal state of node *i*.

$$\dot{\boldsymbol{V}} = \begin{bmatrix} \dot{V}_{11} & \cdots & \dot{V}_{1N} \\ \vdots & \ddots & \vdots \\ \dot{V}_{N1} & \cdots & \dot{V}_{NN} \end{bmatrix} \quad (5)$$

$$\dot{\boldsymbol{I}} = \begin{bmatrix} \dot{I}_{11} & \cdots & \dot{I}_{1N} \\ \vdots & \ddots & \vdots \\ \dot{I}_{N1} & \cdots & \dot{I}_{NN} \end{bmatrix} \quad (6)$$

Power equation

The power equation of the branch $u(p) \to p$ is given by the following equation, which is common for the nodes with load, SVR, and SVC. Here, τ_{ip} is tap position of node *p* observed from node *i*. $\dot{I}'_{in}(p)$ is passing current of downstream node *n* through node *p*.

$$\dot{V}_{ip} = \tau_{ip}\dot{V}_{iu(p)} - \left(r_{u(p)\to p} + jx_{u(p)\to p}\right)\left(\dot{I}_{ip} + \sum_{n=1}^{N} C_{\mathrm{D}}(n,p)\,\dot{I}'_{in}(p)\right) \quad (7)$$

$$\dot{I}'_{in}(p) = \left(\tau_{id(n,p)} \times \tau_{id(n,d(n,p))} \times \cdots \times \tau_{in}\right)\dot{I}_{in} \quad (8)$$

Control sensitivity

The amount of small voltage change in node *n* caused by the amount of small control change in node *p* is defined as control sensitivity $K(i)_{np}$ in the following matrix description observed from node *i*.

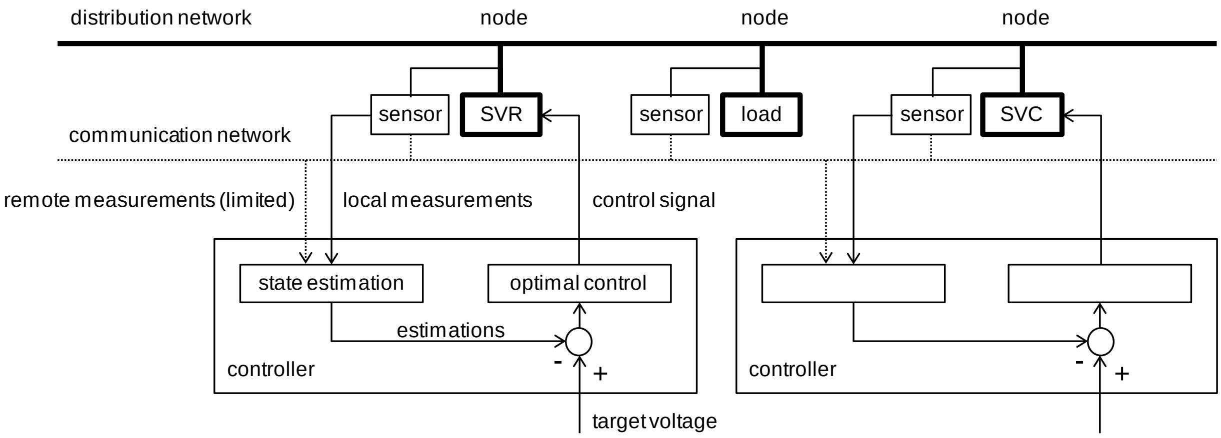

Figure 1. Structure of distribution network, communication line, and decentralized controllers.

$$\boldsymbol{K}(i)=\begin{bmatrix}K(i)_{11} & \cdots & K(i)_{1N}\\ \vdots & \ddots & \vdots\\ K(i)_{N1} & \cdots & K(i)_{NN}\end{bmatrix} \quad (9)$$

Sensitivity matrix indicates $K(i)_{np}=\partial\dot{V}_{in}/\partial\tau_{ip}$ for SVR and $K(i)_{np}=\partial\dot{V}_{in}/\partial\dot{I}_{ip}$ for SVC. For the nodes without a control device, $K(i)_{np}=0$.

State Estimation

For the power equation of voltage and current, linear equation (7) is true for every pair of neighbouring nodes. The number of equation is N-1 for the number of nodes N, which is summarized by the following matrix equation.

$$\boldsymbol{A}(i)\begin{bmatrix}\dot{V}_{i1}\\ \vdots\\ \dot{V}_{iN}\\ \dot{I}_{i1}\\ \vdots\\ \dot{I}_{iN}\end{bmatrix}=\begin{bmatrix}\dot{V}_{\mathrm{S}}\\ \mathbf{0}_{N-1}\end{bmatrix} \quad (10)$$

Here, $\boldsymbol{A}(i)$ is a coefficient matrix that consists of impedance $r_{u(p)\to p}$ and $x_{u(p)\to p}$, tap position τ_{ip}, and connection matrix $C_{\mathrm{D}}(n,p)$. $\dot{V}_{\mathrm{S}}$ is a stable feeder voltage of the substation and $\dot{V}_{i1}=\dot{V}_{\mathrm{S}}$ is a fixed condition. In this matrix equation, since we have 2N variables for N equations, the size of $\boldsymbol{A}(i)$ is N×2N.

For the observation equation, the following matrix equation is given for imperfect observations $\tilde{V}_{ip}$ and $\tilde{I}_{ip}$ of node p observed from node i.

$$\boldsymbol{I}_{2N}\begin{bmatrix}\dot{V}_{i1}\\ \vdots\\ \dot{V}_{iN}\\ \dot{I}_{i1}\\ \vdots\\ \dot{I}_{iN}\end{bmatrix}=\begin{bmatrix}\tilde{V}_{i1}\\ \vdots\\ \tilde{V}_{iN}\\ \tilde{I}_{i1}\\ \vdots\\ \tilde{I}_{iN}\end{bmatrix} \quad (11)$$

By solving equations (10) and (11) simultaneously as follows, we can obtain an approximation voltage and current that minimize the square error of the power equation and the square error of the observation equation.

$$\begin{bmatrix}\boldsymbol{A}(i)\\ \boldsymbol{I}_{2N}\end{bmatrix}\begin{bmatrix}\dot{V}_{i1}\\ \vdots\\ \dot{V}_{iN}\\ \dot{I}_{i1}\\ \vdots\\ \dot{I}_{iN}\end{bmatrix}=\begin{bmatrix}\dot{V}_{\mathrm{S}}\\ \mathbf{0}_{N-1}\\ \tilde{V}_{i1}\\ \vdots\\ \tilde{V}_{iN}\\ \tilde{I}_{i1}\\ \vdots\\ \tilde{I}_{iN}\end{bmatrix} \quad (12)$$

There are 2N variables for 3N equations; thus, this is solved as an over constrained problem. However, since the reliability of the observations differ for different pairs of observing and observed nodes, equation (12) should be weighted by following reliability matrix for more accurate approximation.

$$\boldsymbol{W}=\begin{bmatrix}W_{11} & \cdots & W_{1N}\\ \vdots & \ddots & \vdots\\ W_{N1} & \cdots & W_{NN}\end{bmatrix} \quad (13)$$

A member of reliability matrix W_{ip} indicates reliability of node p observed from node i, which takes a continuous value between 0 (unobservable) and 1 (observable without any error). By using reliability matrix $\boldsymbol{W}$, equation (12) is weighted as following.

$$\boldsymbol{H}(i)=\begin{bmatrix}\boldsymbol{I}_N & & \mathbf{0}\\ & \mathrm{diag}([W_{i1}\ \cdots\ W_{iN}]) & \\ \mathbf{0} & & \mathrm{diag}([W_{i1}\ \cdots\ W_{iN}])\end{bmatrix} \quad (14)$$

$$\boldsymbol{H}(i)\begin{bmatrix}\boldsymbol{A}(i)\\ \boldsymbol{I}_{2N}\end{bmatrix}\begin{bmatrix}\dot{V}_{i1}\\ \vdots\\ \dot{V}_{iN}\\ \dot{I}_{i1}\\ \vdots\\ \dot{I}_{iN}\end{bmatrix}=\boldsymbol{H}(i)\begin{bmatrix}\dot{V}_{\mathrm{S}}\\ \mathbf{0}_{N-1}\\ \tilde{V}_{i1}\\ \vdots\\ \tilde{V}_{iN}\\ \tilde{I}_{i1}\\ \vdots\\ \tilde{I}_{iN}\end{bmatrix} \quad (15)$$

The answer to this equation is given by the following, which is voltage and current at individual nodes on the network estimated by decentralized controller i.

$$\begin{bmatrix}\dot{V}_{i1}\\ \vdots\\ \dot{V}_{iN}\\ \dot{I}_{i1}\\ \vdots\\ \dot{I}_{iN}\end{bmatrix}=\left(\begin{bmatrix}\boldsymbol{A}(i)\\ \boldsymbol{I}_{2N}\end{bmatrix}^{\mathrm{T}}\boldsymbol{H}(i)^2\begin{bmatrix}\boldsymbol{A}(i)\\ \boldsymbol{I}_{2N}\end{bmatrix}\right)^{-1}\begin{bmatrix}\boldsymbol{A}(i)\\ \boldsymbol{I}_{2N}\end{bmatrix}^{\mathrm{T}}\boldsymbol{H}(i)^2\begin{bmatrix}\dot{V}_{\mathrm{S}}\\ \mathbf{0}_{N-1}\\ \tilde{V}_{i1}\\ \vdots\\ \tilde{V}_{iN}\\ \tilde{I}_{i1}\\ \vdots\\ \tilde{I}_{iN}\end{bmatrix} \quad (16)$$

Optimal Control

For the difference equation of estimation voltage and target voltage, the following equation is given with the voltage difference $\Delta\dot{V}_{in}$, the amount of control change Δf_{ip}, and sensitivity matrix $\dot{\boldsymbol{K}}(i)$.

$$\begin{bmatrix}\Delta\dot{V}_{i1}\\ \vdots\\ \Delta\dot{V}_{iN}\end{bmatrix}=\boldsymbol{K}(i)\begin{bmatrix}\Delta f_{i1}\\ \vdots\\ \Delta f_{iN}\end{bmatrix} \tag{17}$$

For the limitation equation, the amount of control change Δf_{ip} of SVR or SVC is limited by limitation $\Delta f_{\mathrm{ref}\,p}$ as in the following equation.

$$\boldsymbol{I}_N\begin{bmatrix}\Delta f_{i1}\\ \vdots\\ \Delta f_{iN}\end{bmatrix}=\begin{bmatrix}\Delta f_{\mathrm{ref}\,1}\\ \vdots\\ \Delta f_{\mathrm{ref}\,N}\end{bmatrix} \tag{18}$$

By solving equations (17) and (18) simultaneously as follows, we can obtain optimal controls that minimize the square error of the difference equation and the square error of the limitation equation.

$$\begin{bmatrix}\boldsymbol{K}(i)\\ \boldsymbol{I}_N\end{bmatrix}\begin{bmatrix}\Delta f_{i1}\\ \vdots\\ \Delta f_{iN}\end{bmatrix}=\begin{bmatrix}\Delta\dot{V}_{i1}\\ \vdots\\ \Delta\dot{V}_{iN}\\ \Delta f_{\mathrm{ref}\,1}\\ \vdots\\ \Delta f_{\mathrm{ref}\,N}\end{bmatrix} \tag{19}$$

There are *N* variables for 2*N* equations; thus, this is solved as an over constrained problem. However, since the priority of voltage regulation may differ for different nodes and the strength of limitation may differ for different control devices, equation (19) should be weighted by the following priority matrix and limitation matrix for more efficient control.

$$\boldsymbol{M}=\begin{bmatrix}M_{11} & \cdots & M_{1N}\\ \vdots & \ddots & \vdots\\ M_{N1} & \cdots & M_{NN}\end{bmatrix} \tag{20}$$

A member of priority matrix M_{ip} indicates voltage regulation priority of node *p* manipulated from node *i*, which takes a continuous value between 0 (uncontrolled) and 1 (controlled).

$$\boldsymbol{L}=\begin{bmatrix}L_{11} & \cdots & L_{1N}\\ \vdots & \ddots & \vdots\\ L_{N1} & \cdots & L_{NN}\end{bmatrix} \tag{21}$$

A member of limitation matrix L_{ip} indicates limitation of control device *p* manipulated from node *i*, which takes a continuous value between 0 (without limitation) and 1 (with limitation). By using priority matrix $\boldsymbol{M}$ and limitation matrix $\boldsymbol{L}$, equation (19) is weighted as follows.

$$\boldsymbol{G}(i)=\begin{bmatrix}\mathrm{diag}([M_{i1} \ \cdots \ M_{iN}]) & \boldsymbol{0}\\ \boldsymbol{0} & \mathrm{diag}([L_{i1} \ \cdots \ L_{iN}])\end{bmatrix} \tag{22}$$

$$\boldsymbol{G}(i)\begin{bmatrix}\boldsymbol{K}(i)\\ \boldsymbol{I}_N\end{bmatrix}\begin{bmatrix}\Delta f_{i1}\\ \vdots\\ \Delta f_{iN}\end{bmatrix}=\boldsymbol{G}(i)\begin{bmatrix}\Delta\dot{V}_{i1}\\ \vdots\\ \Delta\dot{V}_{iN}\\ \Delta f_{\mathrm{ref}\,1}\\ \vdots\\ \Delta f_{\mathrm{ref}\,N}\end{bmatrix} \tag{23}$$

The answer to this equation is given by the following, which is optimally allocated control for every control device by decentralized controller *i*.

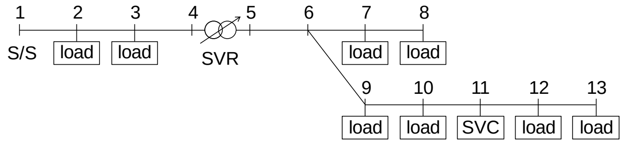

Figure 2. Example of a distribution network.

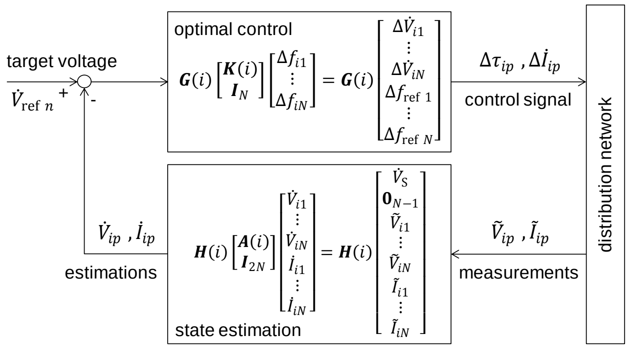

Figure 3. Block diagram of a decentralized controller.

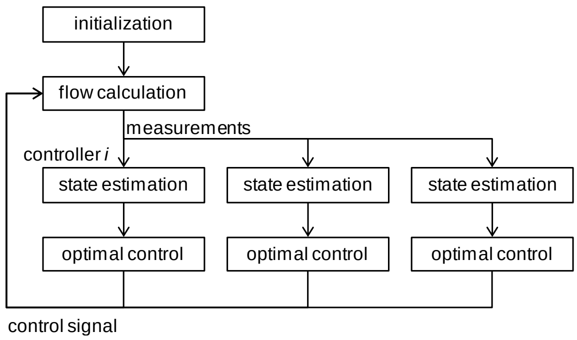

Figure 4. Simulation of decentralized control.

$$\begin{bmatrix}\Delta f_{i1}\\ \vdots\\ \Delta f_{iN}\end{bmatrix}=\left(\begin{bmatrix}\boldsymbol{K}(i)\\ \boldsymbol{I}_N\end{bmatrix}^{\mathrm{T}}\boldsymbol{G}(i)^2\begin{bmatrix}\boldsymbol{K}(i)\\ \boldsymbol{I}_N\end{bmatrix}\right)^{-1}\begin{bmatrix}\boldsymbol{K}(i)\\ \boldsymbol{I}_N\end{bmatrix}^{\mathrm{T}}\boldsymbol{G}(i)^2\begin{bmatrix}\Delta\dot{V}_{i1}\\ \vdots\\ \Delta\dot{V}_{iN}\\ \Delta f_{\mathrm{ref}\,1}\\ \vdots\\ \Delta f_{\mathrm{ref}\,N}\end{bmatrix} \tag{24}$$

In this way, as summarized in Figure 3, each decentralized controller optimizes local control on the basis of allocation with remote controls. Though the control signal is given into only a local control device, the distribution voltage is regulated by superposition of the effects from individual control devices.

POWER FLOW SIMULATION

Simulation Model

We developed a power flow simulation including the mathematical model of Super Decentralized Control as shown in Figure 4. First, in the initial condition, no control device has any output. On the basis of feeder voltage and load amount, true measurements of distribution voltage and current are given by backward-forward power flow calculation. Second, decentralized controllers obtain parts of the measurements as imperfect observations. The controllers estimate voltage and current at individual nodes on the network as internal state. Third, on the basis of internal state, they optimally allocate controls and output control signals to the local control devices. After that, the simulation process returns to power flow calculation and true measurements are updated. In this way, time sequence behaviour of the distribution network is simulated through cyclic power flow calculation and feedback control.

Simulation Results

We conducted a simulation on the distribution network in Figure 2 and evaluated the capability of Super Decentralized Control for voltage regulation. The parameters for the network are as follows.

- Feeder voltage: 6600 [V]
- Active power of load: 160 [kW]
- Reactive power of load: 120 [kVar]
- Branch impedance: 0.8+0.3j [Ω]

The decentralized controllers are equipped for SVR at node 5 and SVC at node 11. Controllers 5 and 11 observe nodes 2 to 8 and nodes 9 to 13, respectively. The reliability is set to 1.0 for those observations. The other nodes are assumed to have alternative observations stable at 6600 [V] and 30 [A], whose reliability is set to 0.01. The control signals for SVR and SVC have gain 0.2 against the answer to equation (24).

Figure 5 shows the distribution voltage in initial condition of the simulation. The upper and lower graphs are the voltages at nodes 1 to 8 and nodes 9 to 13, respectively. Break line indicates appropriate voltage, ± 5% of target voltage 6600 [V]. In the initial condition, the distribution voltage was far lower than appropriate voltage in downstream nodes. In contrast, Figure 6 shows the distribution voltage regulated by decentralized controllers. The distribution voltage was boosted by SVR at node 5 and SVC at node 11 and kept an appropriate voltage.

Figure 7 shows time sequence of node voltage from time section in Figure 5 to time section in Figure 6. The horizontal axis indicates a number of feedback loops. There was no voltage oscillation for any nodes, and there was uniform convergence around the target voltage. After ten feedback loops, tap position of SVR at node 5 was 1.03 and power output of SVC at node 11 was 200 [kVar].

CONCLUSION

We designed a mathematical model of Super Decentralized Control and confirmed it can regulate the distribution voltage by a power flow simulation. In the simulation, the decentralized controllers uniformly converged the distribution voltage around target voltage on the basis of local observations and limited remote observations. For future work, we plan to apply the modern control theory for Super Decentralized Control in order to find control parameters optimized for network topology, control device availability, and communication capacity.

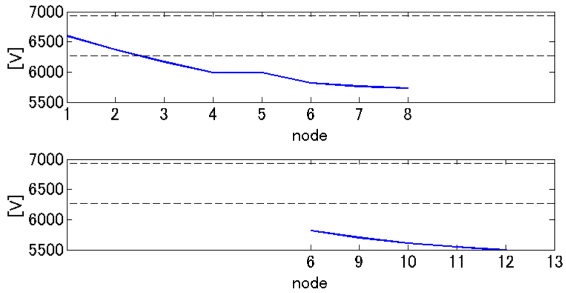

Figure 5. Distribution voltage (initial condition).

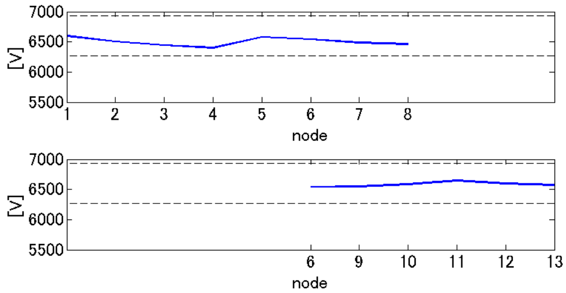

Figure 6. Distribution voltage (regulated condition).

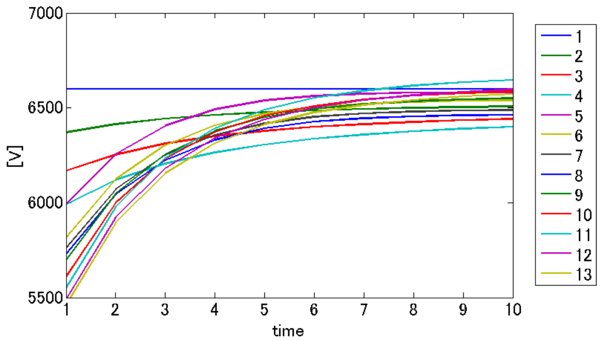

Figure 7. Time series of distribution voltage.

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[1] S. Grenard, A. Queric, and O. Carre, 2011, "Technical and Economic Assessment of Centralised Voltage Control Functions in Presence of DG in the French MV Network", *Proceedings of 21st International Conference on Electricity Distribution*, Frankfurt, Paper 0208.