A NOVEL ALGORITHM TO THE MULTI-STAGE GRID EXPANSION PROBLEM TAKING INTO ACCOUNT GRID TOPOLOGY MODIFICATIONS AND STORAGE DEVICES

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ABSTRACT

In this paper an algorithm is introduced that solves the multi-stage grid expansion problem, which is extended by the following issues: Grid topology modifications in the form of dynamically added busses and electrical storage devices. The adapted mathematical formulation and the methodology to extend the conventional expansion problem as well as the algorithm to solve the resulting non-convex problem are presented. The computational results by drawing on test systems demonstrate the optimality of the solutions. The closing discussion illustrates the potentials and limitations of the methods developed.

INTRODUCTION

Many power systems are faced by a continuously increasing capacity of electric generating units from renewable energy sources. For their integration network expansion is necessary. The grid expansion resulting from the transformation of production and consumption patterns should best be optimal with regard to the investment costs. Therefore, the network expansion planning has to consider the complete technological portfolio – including storage devices as well as topological modifications (see Figure 1) – and should be optimally scheduled.

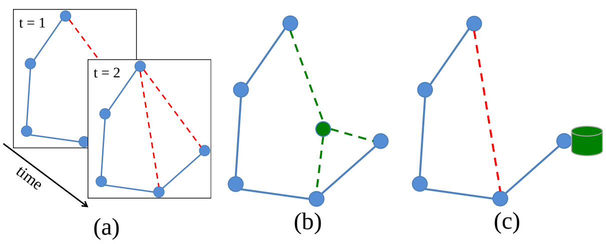

Figure 1: Example for (a) conventional multi-stage grid expansion, (b) topological changes and (c) storage devices.

PROBLEM FORMULATION

The need for simultaneous consideration of various technologies (power lines and storage systems) over a period of multiple stages requires an extension of traditional approaches to the problem of network expansion planning. The proposed mathematical model (equations 1 to 17) is based on the single-stage hybrid formulation according to [1], whereas the formulation of multiple stages – apart from the investment restriction (equation 12) – was done likewise in [2].

The following nomenclature is used:

Indices:

i, j, k	nodes (busses)
t	stages

Sets:

W	set of all busses
W_{old}	set of busses without topological modifications
W_{new}	set of busses added by topological modifications
T	set of all stages

Variables:

K	total expansion costs (objective value)
$c_{ij,t}$	cost of a candidate circuit in branch i-j in stage t
$c_{k,t}$	cost of a candidate storage unit in node k in stage t
$n_{ij,t}$	number of circuits in branch i-j in stage t
$n^0_{ij,t}$	number of circuits in branch i-j in in the current topology
n_t	vector of all decision variables $n_{ij,t}$ in stage t
$\bar{n}$	vector of the maximum number of branches
$s_{k,t}$	number of storage units in node k in stage t
s_t	vector of all decision variables $s_{k,t}$ in stage t
$\bar{s}$	vector of the maximum number of storage units
s^g_t	vector of the actual usage of the storage power in stage t
s^0_t	vector of the storage units in stage t in the current topology
z_t	discount factor in stage t
$\bar{C}_t$	maximum investment costs in stage t
S_t	$N \times M$ node-branch incidence matrix in stage t (N: number of nodes, M: number of branches)
$f_{ij,t}$	total active power flow on branch i-j in stage t
$\bar{f}_{ij}$	maximum power flow of a circuit on branch i-j
f_t	vector of all active power flows $f_{ij,t}$ in stage t
$q_{i,t}$	voltage angle at bus i in stage t
g_{ij}	susceptance of a circuit on branch i-j
g_t	vector of actual generation at all busses in stage t
$\bar{g}$	vector of maximum generation at all busses
d_t	vector of demand at all busses in stage t
m_t	number of years in stage t
r_t	market interest rate in stage t

The integrated formulation can be stated as follows:

(1) $\min_{n_{ij,t}, s_{k,t}} \rightarrow K = \sum_{(t\in T)} \left(\sum_{(i,j\in\Omega)} c_{ij,t}\cdot\left(n_{ij,t}-n_{ij,t-1}\right) + \sum_{(k\in\Omega)} c_{k,t}\cdot\left(s_{k,t}-s_{k,t-1}\right) \right)$

(2) with $c_{ij,t}=c_{ij}z_t=\frac{c_{ij}}{\left(1+r_t\right)^{t\cdot m_t}}$ and $c_{k,t}=c_k z_t=\frac{c_k}{\left(1+r_t\right)^{t\cdot m_t}}$

s.t.

(3) $S_t f_t + g_t + s_t^g = d_t \ \forall t\in T$

(4) $f_{ij,t} - g_{ij}\cdot n^0_{ij,t}\cdot\left(q_{i,t}-q_{j,t}\right) = 0, \ \forall (i,j)\in\Omega$ and $\forall t\in T$

(5) $\left|f_{ij,t}\right| \le (n^0_{ij,t}+n_{ij,t})\cdot \bar f_{ij}, \ \forall (i,j)\in\Omega$ and $\forall t\in T$

(6) $0\le g_t\le \bar g \ \forall t\in T$

(7) $0\le n_t \le \bar n - n_t^0 \ \forall t \in T$

(8) $0\le s_t\le \bar s_t \ \forall t\in T$

(9) $0 \le s_t^g \le s_t + s_t^0 \ \forall t\in T$

(10) $n_{ij,t}\ge n_{ij,t-1} \ \forall (i,j)\in\Omega$ and $\forall t\in T$

(11) $s_{k,t}\ge s_{k,t-1} \ \forall k\in\Omega$ and $\forall t\in T$

(12) $\sum_{(i,j\in\Omega)} c_{ij,t}\cdot(n_{ij,t}-n_{ij,t-1}) + \sum_{(k\in\Omega)} c_{k,t}\cdot(s_{k,t}-s_{k,t-1}) \le \bar C_t \ \forall t\in T$

(13) $\bar C_t, c_{ij,t}, c_{k,t}, g_{ij,t}, \bar f_{ij}, g_t, \bar g, d_t, s_t^g \in \mathbb{R}_0^+$

(14) $q_{i,t}, f_t \in \mathbb{R}$

(15) $n_t, \bar n, n^0_{ij,t}, s_t, \bar s, s_t^0 \in \mathbb{N}_0$

(16) $\bar C_t, c_{ij,t}, c_{k,t}, \bar f_{ij}, \bar g, \bar n, \bar s, d_t, z_t, r_t, m_t = const$

(17) $i,j,k \in \Omega = \Omega_{old}\cup\Omega_{new}$ with: $\Omega_{old}\cap\Omega_{new}=\varnothing$

METHODOLOGY

The technologies added to the traditional expansion planning are discussed in the following.

Topological Changes

To find new branch candidates under incorporation of topological information the option of adding complete new nodes has been given to the solution procedure (see Figure 1b). By means of a fast marching algorithm (e.g. see [3]) a pre-analysis identifies line candidates:

1. Definition of the cost map: See Figure 2 (e.g. for each colour a different cost value is allocated).
2. Extraction of the distance map by applying the fast marching algorithm for each node of the base topology: See Figure 3.
3. Summation of the distance maps in all possible combination: See Figure 4. The point with the minimal value in the resulting distance map equals the point that connects the considered busses with the least effort (costs). This point can be considered as a new bus candidate.
4. Extraction of the shortest paths (see Figure 3) between all bus candidates and calculation of the shortest distances (see Figure 2) between the busses. Thereby, the reactance of the branch candidates can be calculated according to the to the present voltage level and according to the technology applied.
5. Add new busses (Ω_{new}) and new branch candidates.

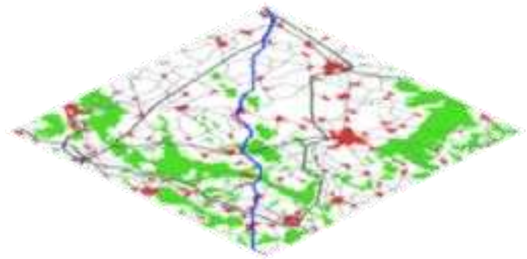

Figure 2: Example of a cost map (blue line: shortest distance between upper and lower point).

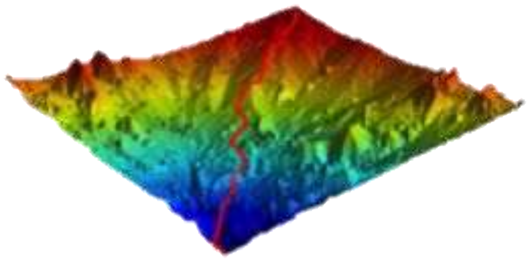

Figure 3: Example of a distance map for the lower point (red line: shortest path from the upper to the lower point).

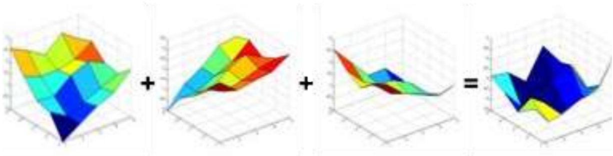

Figure 4: Example of the summation of three distance maps and its resulting distance map.

Storage Devices

Electrical power storage systems can be regarded as a competitive technical solution to conventional grid expansion (see Figure 1c). Therefore, the multi-stage hybrid formulation has been modified by adding the decision variables *s* and s^g (see equations 1, 3, 8, 9 and 11): Whereas the integer *s* represents the monetary investment decision, the linear variable s^g corresponds to the active power "generation" (discharge) in the considered point in time. The actual discharge power of the storage device (s^g) may never exceed the installed maximum power (*s*) of the storage unit (see equation 9). The separation between a technical and a monetary investment variable allows finding the power-related optimal solution at a point in time. Furthermore, the energy-related feasibility has to be considered: For this purpose the following process verifies each solution that is found energetically:

1. Reformulation of the problem:
   (1) replaced by: $\min \rightarrow K = \sum_{(k\in\Omega)} c_k^{sg}\cdot s_k^{sg}$
   with $c_k^{sg}>0$, $c_k^{sg}=const \ \forall k\in\Omega$
   (7) replaced by: $0\le n\le 0$
   (8) replaced by: $0\le s\le 0$
   (9) replaced by: $-s^0\le s^g\le s^0 \ \forall s_k^0>0$ with $s^g\in\mathbb{R}$

2. Optimize the linearized and reformulated problem for a certain number of points in time within one or more predefined demand sets (load profiles).
3. Extract the maximum discharge power ($s^g > 0$) and charge power ($s^g < 0$) of the installed storage devices out of the solution vector.
4. Integrate the discharge and charge power over time to validate the energetic capacity of the storage devices in the current topology. If the energy of one or more storage devices is taking values below zero, the current solution is infeasible.

ALGORITHM

The proposed solution algorithm to solve the mixed integer linear programming is based on a heuristic branch and cut method (B&C) that was proposed in [4] and is named "Split and Stint Algorithm" (S&S) by the authors: The simultaneous consideration of power lines and storage facilities during the solution process leads to a constant change of the problem's constraints in the decision tree, thus resulting in a constantly changing solution space. Accordingly, convexity is no longer given in general. Classic B&C approaches do not guarantee the optimal solution in this case. The new method meets the problem of non-convexity by the partition of the non-convex solution space into convex partial solution spaces (see Figure 5).

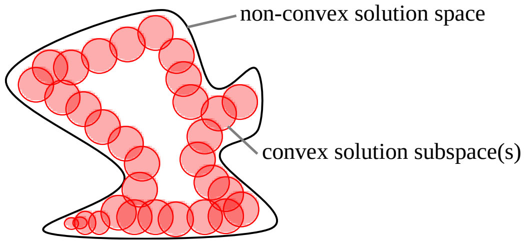

Figure 5: Partitioning of a non-convex solution space into convex solution subspaces.

The algorithm searches from these subspaces for valid solutions. The optimal solution can be reached, if the search covers the whole (non-convex) solution space. This prerequisite is fulfilled by a modification of the B&C algorithm: Whereas in the classical approach the decision variables can only increase ("Branching"), the S&S algorithm also allows a decrease of their value by means of the new "Splitting" method (see Figure 6).

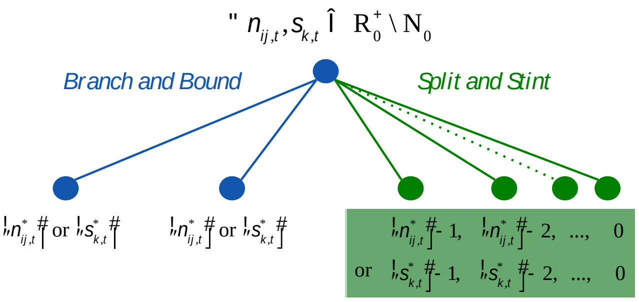

Figure 6: "Branching" (B&C) versus "Splitting" (S&S).

By means of so-called "Stints" – similar to the "Cuts" of the B&C method – an efficient pruning of the decision tree can be achieved, thus avoiding the entire enumeration of solution space. Classical "Cuts" – e.g. if it is foreseeable that no improvement in the value of the objective function can be realized – are extended in the form of the following "Stints" to cut the decision tree if one of the following conditions is met:

1. Feasibility: The current solution is mathematically or – in case of storage systems – energetically not feasible.
2. Prescheduling: $n_{ij} = 0$ and $s_k = 0 \ \forall \ i,j,k \in \Omega$
3. Pruning: $K + \min\left(c_{ij}, c_k \ \forall \ i,j,k \in \Omega\right) \geq boundary$

CASE STUDIES

The robustness of the algorithm was tested on the basis of the following test systems: The "Garver 6 Bus System" [5], the "IEEE Reliability Test System" (24 busses) in version 1 [6] and 2 [7] and the "Southern Brazilian 46 Bus System" [8].[1]
The convention for the annotation is as follows: **W**ith/**N**o (generation) **R**escheduling **W**ith/**N**o (initial) **N**etwork.
In the conventional case of single-stage, without topological changes and without storage systems the algorithm found the same optimal solutions discussed in the papers above. Furthermore, the algorithm found for the first time – to the knowledge of the authors – a better solution for the "Southern Brazilian 46 Bus System", which is presented in Table I. The reference solution is based on the transportation formulation [8], thus equation (4) is being neglected.

Reference System	Reference Solution	Solution of "Split and Stint Algorithm"
46-Bus NRWN	127,21 with: $n_{5-11}=2,\ n_{46-11}=1,$ $n_{14-22}=1,\ n_{20-21}=2,$ $n_{28-31}=1,\ n_{31-32}=1,$ $n_{24-25}=2,\ n_{25-32}=1,$ $n_{42-43}=2$	121,16 with: $n_{20-21}=2,\ n_{42-43}=2,$ $n_{5-11}=2,\ n_{25-32}=1,$ $n_{31-32}=1,\ n_{28-31}=1,$ $n_{46-11}=1,\ n_{24-25}=2$

Table I: New solution found for the "46 Bus System" for the conventional grid expansion problem.

For the "6 Bus System" an artificial cost map was created (see Figure 2) and the busses were allocated according to the branch lengths (costs) of the reference system. However, the topological changes had no impact on the optimal solution.
The solutions of the multi-stage problem (ten stages) for the "6 Bus System" and "24 Bus System" (versions 1 and

1 For the test system data as well as the data assumed for topological changes, storage systems and multi-stage analysis please contact the authors.

2) are shown in Table II.

Reference System	Present Value	Expansion Plan
6-Bus NRWN	153,8	$n_{2-6,1}=2,\ n_{4-6,1}=1,\ n_{3-5,2}=1,$ $n_{4-6,3}=2,\ n_{4-6,5}=3,\ n_{4-6,10}=4$
6-Bus WRWN	75,1	$n_{2-3,1}=1,\ n_{3-5,2}=1,\ n_{4-6,4}=1,$ $n_{4-6,9}=2,\ n_{4-6,10}=3$
6-Bus NRNN	229,5	$n_{2-3,1}=1,\ n_{3-5,2}=1,\ n_{4-6,4}=1,$ $n_{4-6,9}=2,\ n_{4-6,10}=3$
6-Bus WRNN	152,8	$n_{1-5,1}=1,\ n_{2-3,1}=2,\ n_{3-5,1}=1,$ $n_{4-6,1}=1,\ n_{4-6,3}=2,\ n_{3-5,5}=2,$ $n_{2-6,9}=1$
24-Bus-V1 WRWN	61,5	$n_{6-10,5}=1,\ n_{10-12,5}=1,\ n_{14-36,9}=1,$ $n_{7-8,10}=2$
24-Bus-V2 WRWN	67,8	$n_{6-10,4}=1,\ n_{1-13,5}=1,\ n_{10-12,9}=1,$ $n_{20-23,9}=1,\ n_{7-8,10}=2$

Table II: Solutions for the "6 Bus System" and "24 Bus System" for the multi-stage analysis.

The solutions for the single-stage problem including storage systems for the "6 Bus System" and "24 Bus System" (versions 1 and 2) are shown in Table III.

Reference System	Present Value	Expansion Plan
6-Bus NRWN	180	$n_{4-6}=1,\ s_1=3,\ s_2=1,\ s_5=1$
6-Bus WRWN	95	$n_{2-3}=1,\ s_1=1,\ s_2=1,\ s_5=1$
6-Bus NRNN	210	$n_{1-5}=1,\ n_{2-3}=1,\ n_{3-5}=1,$ $s_2=2,\ s_4=2,\ s_5=2$
6-Bus WRNN	175	$n_{1-5}=1,\ n_{2-3}=3,\ n_{3-5}=1,$ $s_4=2,\ s_5=1$
24-Bus-V1 WRWN	66	$n_{7-8}=1,\ s_6=1,\ s_{11}=1$
24-Bus-V2 WRWN	85,03	$n_{11-13}=1,\ n_{20-23}=1,\ s_5=1,$ $s_6=1$

Table III: Solutions for the "6 Bus System" and "24 Bus System" including storage devices.

CONCLUSION AND OUTLOOK

A novel method to solve the multi-stage network expansion problem is presented, which simultaneously considers power lines and storage expansion. Firstly, the approach searches for new branch candidates on the basis of essential topological features. Furthermore, storage devices are taken into consideration. Secondly, the developed algorithm optimally solves the resulting mixed integer linear programming problem with a non-convex solution space.

The comparative evaluation between reference results of conventional problems shows the robustness of the developed solution method and its solution quality.

The following limitations of the proposed methods are in the focus of further research:

1. Enhancement of the algorithm to solve bigger problems (e.g. in the form of further theorized "Stints").
2. Extension of the energetic verification for storage units (e.g. mutual charging is not regarded in the current verification process).
3. Simultaneous consideration of various power line- and storage-technologies.
4. Implementation of stochastic variables such as electric generating units from renewable energy sources.

REFERENCES

[1] R. Romero, A. Monticelli, A. Garcia and S. Haffner, 2002, "Test systems and mathematical models for transmission network expansion planning", *IEE Proceedings – Generation, Transmission and Distribution*, vol. 149, no. 1, 27-36.

[2] H. Zhang, V. Vittal, G. T. Heydt and J. Quintero, 2012, "A Mixed-Integer Linear Programming Approach for Multi-Stage Security-Constrained Transmission Expansion Planning", *IEEE Transactions on Power Systems*, vol. 27, no. 2, 1125-1133.

[3] J. A. Sethian, 1999, *Level Set Methods and Fast Marching Methods*, Cambridge University Press, Berkeley, USA.

[4] A. S. Sousa, N. Asada, 2011, "A Heuristic Method Based on the Branch and Cut Algorithm to the Transmission System Expansion Planning", *IEEE*.

[5] L. L. Garver, 1970, "Transmission Network Estimation Using Linear Programming", *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-89, no. 7, 1688-1697.

[6] IEEE System Task Force "IEEE Reliability Test System-1996", *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-98, no. 6, 2047-2054.

[7] N. Alguacil, A. L. Motto and A. J. Conejo „Transmission Expansion Planning: A Mixed-Integer LP Approach", *IEEE Transactions on Power Systems*, vol. 18, no. 3, 1070-1077.

[8] S. Haffner, A. Monticelli, A. Garcia, J. Mantovani and R. Romero, 2000, "Branch and bound algorithm for transmission system expansion planning using a transportation model", IEE *Proceedings – Generation, Transmission and Distribution*, vol. 147, no. 3, 149-156.