THE USE OF RECURSIVE LEAST SQUARE TO DETERMINE THE MODEL PARAMETERS OF A TRANSFORMER IN DIFFERENT FREQUENCIES

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ABSTRACT

This paper presents an alternative approach for determining the parameters of single-phase two-winding transformer. The dynamic parameters of a transformer, including its inductances and resistances are estimated by the application of recursive least squares algorithm on the measured data taken from the terminal voltage and current. The results show that the resistances increase and the inductances decrease with the elevation of the frequency. The model properly represents the characteristics of the original transformer. The accuracy of the results is also verified by simulation tests.

INTRODUCTION

Due to the importance of transformers to power systems, this element must be studied in detail so that the developed models can, in fact, represent the actual operation of the equipment. In this way, frequency modelling is essential in the design of transformers and, thus, frequency response analysis technique is nowadays widely used in field application as a diagnostic tool.

The model should have two basic characteristics which are not always concordant: simple mathematical function and accurate results. This trade-off must be considered when formulating the model, since better numerical precision usually means sophistication, which increases the solution's computational cost.

Therefore, the frequency response analysis and parameter identification are topics that have been discussed in several surveys, either jointly or separately.

Studies related to the transformer operation in frequency sweep were made by [1], resulting in circuit models that correspond to the behaviour of equipment in different frequencies.

A method for calculating the leakage impedance in transformer windings was studied by [2]. The method is based on a formula which is derived from an exact solution of Maxwell's Equations for coils on ferromagnetic cores. The calculation is based on winding geometry and on the core magnetic permeability and effective electrical resistivity and the model takes account of the frequency dependent nature of short circuit and leakage impedances in transformers.

An alternative approach to conventional open and short-circuit test for determining the parameters of N-windings transformer operating at power frequency on an on-line mode was verified by [3]. This approach is based on linear least errors square (LES) algorithm. The algorithm can be considered as an alternative to the open and short-circuited test, which is normally used for transformer parameters identification.

The recursive least square method was used by [4] to estimate the dynamic equivalent circuit parameters of a transformer at its actual operating point.

A finite element magnetic model has been introduced into the reverse method of transformer design by [5]. The model was found to be more accurate than existing model, which was based on magnetic theory, though at the expense of complexity of programming.

A parameter identification method using the least squares based on orthogonal decomposition to identify the leakage inductances and resistances of transformer winding was made by [6]. The algorithm provided a good foundation for online identification of transformer parameters and offers great potential for practical application.

Some models and methods that describe the operation of a transformer at deep saturation were showed by [7]. Data processing of the voltage and current to obtain the transformer parameters, including non linear inductor, were presented. The non linear transformer model obtained from sum of exponentials curve fitting generates more reliable results than traditional linear model.

A method to diagnostic the transformer winding using short-circuit impedance was verified by [8]. The frequency response analysis was utilized to identify changing in the short circuit impedance.

The use of least squares method to estimate the transformer equivalent circuit parameters was made by [9], that provided experimental results of the transformer parameters as a function of frequency and determined optimal approximation polynomial functions for each parameter.

These researches show the importance to study methods that are used to identify the model parameters of a transformer. Thus, this paper presents an alternative approach for determining the parameters of single phase two winding transformer. The dynamic parameters of a transformer, including its inductances and resistances, are estimated by the application of recursive least squares routine on the measured terminal voltages and current. The technique can be used for different frequencies.

This paper first emphasises the characteristics of transformers and tests realized, as well as the method utilized to determine the model parameters. Then, are presented the results and conclusions.

THEORICAL FUNDAMENTATION

Open Circuit Test

The open circuit test is performed to determine the transformers excitation current and core impedance. This test is made applying voltage and frequency rating on the low voltage winding, while the other winding remains open [10].

Equivalent circuit of a Transformer

The equivalent circuit utilized to this paper is presented in the Figure 1.

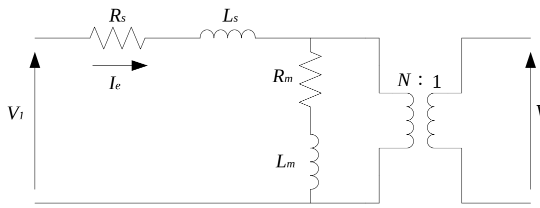


Fig. 1. Transformer equivalent circuit with modifications [3].

In Figure 1: V_1 and V_2 are the voltage in primary and secondary winding, respectively; R_s and L_s represents the leakage impedance; R_m and L_m the magnetization impedance, N is the transformation ratio and I_e corresponds to the excitation current. All variables are referred to the primary side.

Recursive Least Squares

The Recursive Least Squares (RLS) is a technique that estimates, iteratively, the unknown parameter. The method provides estimation monitoring, reaching the expected value before stopping criterion, which results in reduced computational effort [11].

In (1), (2) and (3) has a set of expressions which may be used to implement the algorithm.

$$\theta(k+1) = \theta(k) + K(k)\cdot[y_{k+1} - \varphi^T(k+1)\cdot\theta(k)] \qquad (1)$$

$$K(k) = P(k)\cdot\varphi(k+1)\cdot[\lambda + \varphi^T(k+1)\cdot P(k)\cdot\varphi(k+1)]^{-1} \qquad (2)$$

$$P(k+1) = \frac{[I - K(k)\cdot\varphi^T(k+1)]\cdot P(k)}{\lambda} \qquad (3)$$

In (1), (2) and (3): θ is unknown parameters vector, K is the gain matrix, P is proportional to the covariance matrix of θ, y represents measured values, I is the identity matrix, λ is the forgetting factor and φ corresponds to the approximation functions that represents the model.

The forgetting factor used in this paper was 1.

Transformer model

Figure 1 shows a double-winding single phase transformer model. According to the Kirchhoff's Law, the model of equivalent circuit of a transformer can be describe by (4) and (5).

$$V_1 - NV_2 = R_S\cdot I_e + L_S\cdot\frac{dI_e}{dt} \qquad (4)$$

$$NV_2 = R_m\cdot I_e + L_m\cdot\frac{dI_e}{dt} \qquad (5)$$

The RLS can be applied in different frequencies providing (6) and (7).

$$\begin{bmatrix} V_1^\omega(\Delta t_1) - NV_2^\omega(\Delta t_1) \\ V_1^\omega(\Delta t_2) - NV_2^\omega(\Delta t_2) \\ \vdots \\ V_1^\omega(\Delta t_{2500}) - NV_2^\omega(\Delta t_{2500}) \end{bmatrix} = R_S^\omega\cdot\begin{bmatrix} I_e^\omega(\Delta t_1) \\ I_e^\omega(\Delta t_2) \\ \vdots \\ I_e^\omega(\Delta t_{2500}) \end{bmatrix} + L_S^\omega\cdot\begin{bmatrix} \frac{dI_e^\omega}{dt}(\Delta t_1) \\ \frac{dI_e^\omega}{dt}(\Delta t_1) \\ \vdots \\ \frac{dI_e^\omega}{dt}(\Delta t_{2500}) \end{bmatrix} \qquad (6)$$

$$\begin{bmatrix} NV_2^\omega(\Delta t_1) \\ NV_2^\omega(\Delta t_2) \\ \vdots \\ NV_2^\omega(\Delta t_{2500}) \end{bmatrix} = R_m^\omega\cdot\begin{bmatrix} I_e^\omega(\Delta t_1) \\ I_e^\omega(\Delta t_2) \\ \vdots \\ I_e^\omega(\Delta t_{2500}) \end{bmatrix} + L_m^\omega\cdot\begin{bmatrix} \frac{dI_e^\omega}{dt}(\Delta t_1) \\ \frac{dI_e^\omega}{dt}(\Delta t_1) \\ \vdots \\ \frac{dI_e^\omega}{dt}(\Delta t_{2500}) \end{bmatrix} \qquad (7)$$

In (6) and (7): ω represents the fundamental frequency of supply voltage, Δt corresponds to the sample time.

Thus, the RLS estimates the leakage and magnetization impedances for each frequency.

MATERIAL AND METHODS

In this section, the transformers characteristics and details of the performed tests are presented.

Transformers

The transformers employed in this work are shown in Table I.

Table I
Transformers

Transformer	Potency (VA)	Transformation ratio	Core material
1	1000	220/110	FeSi
2	60	127/12	FeSi

The block diagram of the setup employed to determine the transformer parameters is presented in Figure 2.

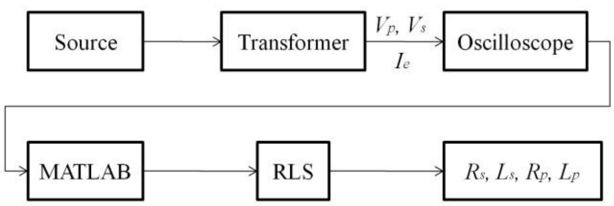


Fig. 2. Block diagram of the setup employed to determine the transformer parameters.

Experimental Procedure

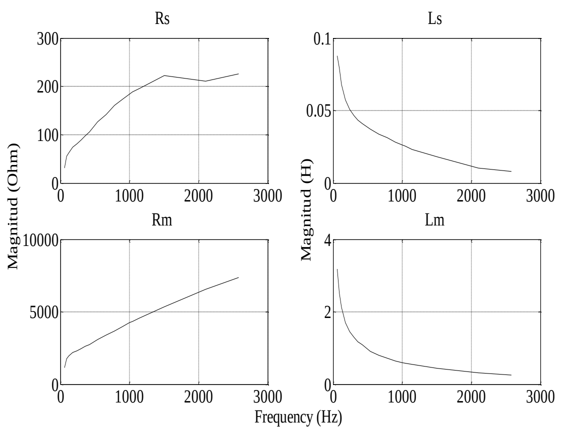
The open circuit test has been performed applying the nominal voltage in different frequencies. The source used to feed the transformer has the following specifications: zero to 400 V, 40 to 5000 Hz and 3 kVA of maximum power output.

The frequency range utilized in this work was 60 to 2580 HZ. This procedure was repeated with the two transformers tested.

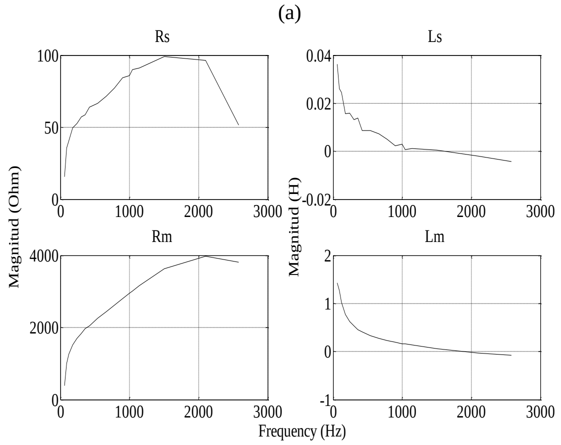
As the frequency was varied, the voltage signals and excitation current were measured and recorded on four-channelled oscilloscope (sampling rate 1 GS/s). The recorded signals were subsequently processed employing a routine developed in Matlab®.

RESULTS

The parameter frequency responses of the transformers are presented in Figure 3.



(a)



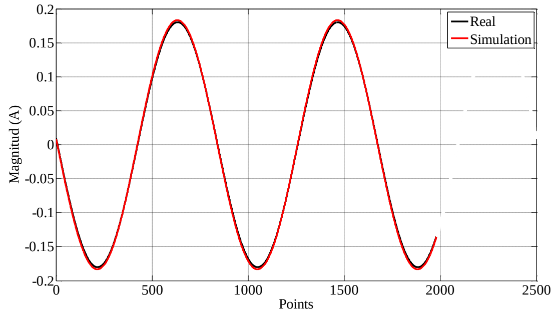
(b)

Fig. 3. Frequency response of the model elements: (a) Transformer 1; (b) Transformer 2.

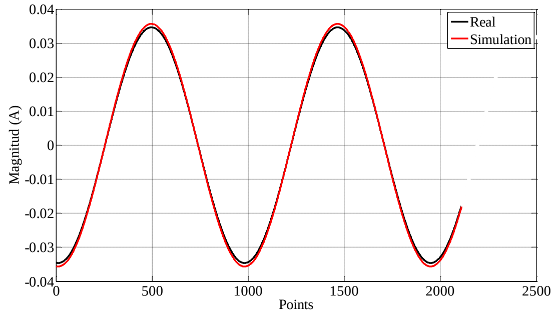
The frequency elevation reduces the area of current circulation, which increases the resistances. Furthermore, in high frequencies, the magnetization effect is not significant. The inductances decreases because of the proportional inversely ratio between frequency and inductance. Moreover, the effect of stray capacitances increases with the elevation of frequency, causing reduction in the magnetization inductances and growth of resistances.

The estimates represent optimal values that correspond to the behaviour of the transformers.

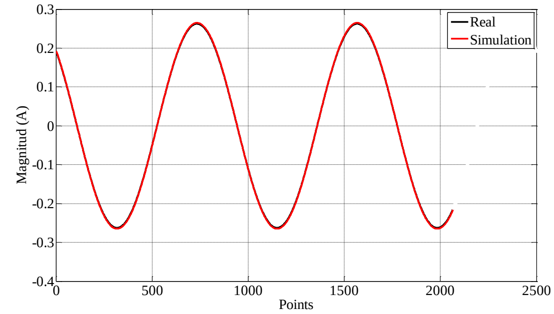
The results validation was achieved with the comparison between fundamental current and simulated current. The results for 60 Hz and 2580 Hz are showed in Figure 4.



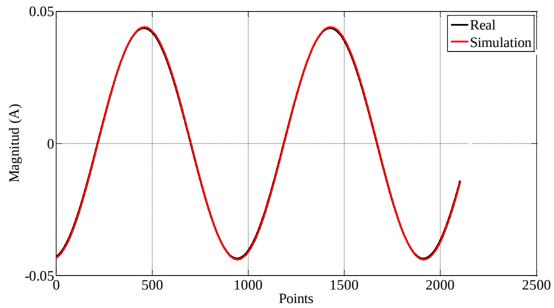
(a)



(b)



(c)



(d)

Fig. 4. Fundamental current and simulation to: transformer 1, (a) 60 and (b) 1, 2580 Hz; transformer 2, (c) 60 Hz and (d) 2, 2580 Hz.

In Figure 4 is observed that the fundamental current coincides with the simulated current. The R^2 for the comparisons were made and the lowest one was 0.9990, that represents a good approximation, since is very close to the unity.

Mathematical expressions were be obtained by the frequency response of the parameters. These expressions are presented in (8), (9), (10), (11), (12), (13), (14) and (15).

$$R_{S1}(f) = 86.03 \cdot f^{0.1977} - 164.5 \quad (8)$$

$$L_{S1}(f) = 0.3314 \cdot f^{-0.1619} - 0.0834 \quad (9)$$

$$R_{m1}(f) = 22.16 \cdot f^{0.7185} + 1100 \quad (10)$$

$$L_{m1}(f) = 25.22 \cdot f^{-0.4883} - 0.2644 \quad (11)$$

$$R_{S2}(f) = -4373 \cdot f^{-0.0053} + 4300 \quad (12)$$

$$L_{S2}(f) = 0.1923 \cdot f^{-0.3012} - 0.0214 \quad (13)$$

$$R_{m2}(f) = 2407 \cdot f^{0.151} - 38565 \quad (14)$$

$$L_{m2}(f) = 9.326 \cdot f^{-0.3722} - 0.563 \quad (15)$$

The index 1 and 2 corresponds to the transformers 1 and 2, respectively. The potential curve presented the best result to R^2, so that the lowest one was 0.9641.

CONCLUSION

This paper presented the parameter of equivalent circuit of two transformers as a function of the frequency.

The technique allows determining optimal values for the leakage and magnetization impedance, in different frequencies, using only the open circuit test. Furthermore, the model presents simple mathematical functions to identify the transformer equivalent circuit.

The elevation of frequency increases the resistances and decreases the inductances.

The validation of results are achieved by the comparison between the fundamental current and simulated current, obtaining a R^2 minimum of 0.9990, resulting in a good modelling of behaviour.

Mathematical expressions could be obtained by the curves of frequency response. The curves determined provide a good approximation, because the R^2 are very close to the unity.

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