MULTI-OBJECTIVE MAINTENANCE OPTIMIZATION

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ABSTRACT

The conventional approach to maintenance is based on the observation of only one goal in the process of individual project prioritization. However, it is clear that current maintenance management problem is multi-objective in nature and requires the determination of the optimal maintenance strategy that achieves the best trade-off between multiple conflicting goals such as: (1) to minimize the cost of maintenance activities, (2) to maximize system reliability and component condition, (3) to minimize operation costs, (4) to maintain the facilities with the highest number of priority risks, (5) to maintain facilities with the worst condition of equipment, (6) to maintain component which failure affect the largest number of customers and result in the largest electricity outage, etc. This paper will present a systematic approach to the problem of distribution network maintenance management in the area of multi-objective optimization. In other words we have to prioritize individual maintenance activities nominee (replacement, repair and overhaul). Prioritization concept is based on the simultaneous satisfaction of several different and mutually contradictory objectives: minimum value of the component health index, minimum cost of maintenance activities and the minimum number of component priority risk. The maintenance management problem (i.e. making of optimal admissions list with prioritized maintenance activities) will be solved by optimization techniques which use vector objective function. According to the specific problem of maintenance management optimization we made a program routine.

INTRODUCTION

Pareto optimality is a concept in economics with applications in engineering. The term is named after Vilfredo Pareto (1848–1923). Pareto optimality is a situation that arises when the funds are allocated in such a way that no single source of funds can be improved without sacrificing at least one source of funds. In other words, an expression of Pareto optimality in economics is solution with multiple objectives [1]. Not Pareto optimal solutions can improve part of the system without worsening the remaining parts. Figure 1 shows 4 geometric examples of Pareto optimality. In these figures, the circles represent objectives that are satisfied best when the area of the circle is maximized. The constraints are that the circles may not overlap and must fit within the triangle.

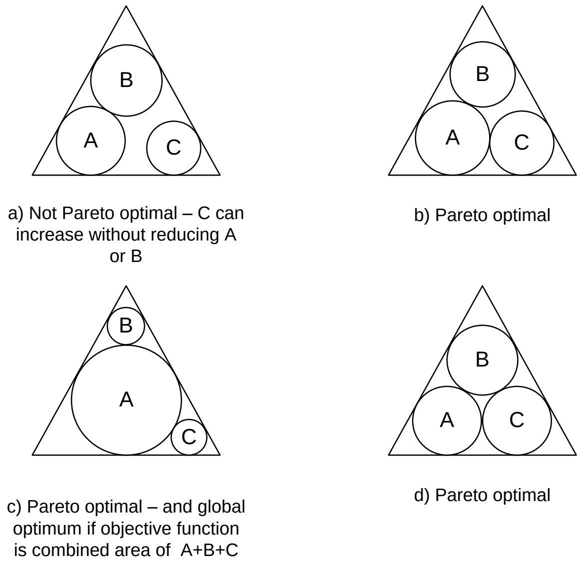

Figure 1: Geometric example of Pareto optimality [2]

We might further impose a global objective function in this case that is equal to the sum of the circle areas. Only one case of geometric example (c) is the global optimum whereas three of them are Pareto optimal. One case of geometric example (a) is not Pareto optimal because it can increase the area of a circuit without violating constraints. Unlike the optimization solution with a single goal, the problem with multiple objectives is more a concept than a definition. As a rule, there is no single global solution but it is often necessary to define a set of points that all fit into a predetermined definition of optimum [3].

1. SETTING OF OPTIMIZATION PROBLEM

The notion of optimality in optimization problem with a single objective is very well defined as the search for the minimum or maximum value of a given objective function. On the other hand the notion of optimality in optimization problem with a multi-objective optimization or vector optimization concept is not so obvious because of the presence of multiple, incomparable and conflicting goals. Generally speaking, there is no single optimal (or superior solution) that simultaneously achieves the minimum (or maximum) value of the objective function. The concept of Pareto optimality, which was originally developed in economics, was introduced into solving the problem of optimization of maintenance activities with more goals. As part of theoretical explanations, the solution x * is said to be Pareto optimal if and only if there is no other solution in the realizable domain that can make some improvement of the objective function without worsening at least one other objective function. Suppose that the components of the distribution system with health index less than or equal to a prescribed minimum (K_{min}) may be scheduled for maintenance activities that will be implemented and that the budget (B) is available for all maintenance activities that can be performed. The problem of optimization of maintenance activities with more goals can be described mathematically as follows:

$$\min_{x \in E^n} F(x) = \begin{bmatrix} F_1(x) \\ F_2(x) \\ \vdots \\ F_k(x) \end{bmatrix} = \min_{x \in E^n} F(x) = [F_1(x), F_2(x), ..., F_k(x)]^T \quad (1)$$

$$E^n = \{x \in N : K_x(t) \le K_{min} ; \sum B_x(t) \le B\} \quad (2)$$

where :

F (x) - Vector of objective function,
E_n - Subset of distribution system components which at time *t* have health index value not greater than 4 and are nominated in maintenance plan,
N - The entire set of all components nominated for the maintenance plan,
K_x – Health index of the individual component,
K_{min} – Appointed the minimum value of health index,
$\sum B_x(t)$ - The sum of the costs of maintenance activities for all nominated components
B – The appointed amount of the budget in the current fiscal year.

Principle of Pareto optimality, explained earlier in this chapter, can be expressed in mathematical terms, i.e. x * is Pareto optimal if:

$F_i(x) < F_i(x^*)$, i=1,2,…,k; for at least one function (3)

and $F(x) \le F(x^*)$ (4)

Generally speaking, at the optimization problem with multiple objectives there are several Pareto optimal solutions. Therefore the optimization problem could be solved by choosing of one solution that achieves the best compromise between all the conflicting objectives. In the literature that deals with the optimization with multiple objectives this solution is called a compromise formula [4]. The next chapter will deal with the procedure of finding a compromise solution.

2. DECISION MAKING WITH CONFLICTING GOALS

At the compromise programming the best or compromise solution will be selected by a distance. In the method of distance we minimize the distance away from the Pareto optimal set and a utopia point F^0. Utopia point F^0 is the point for which at same time applies both extreme values (maximum or **minimum**) of all targets. Such a point or the solution does not exist (it is not feasible), however, at compromise programming that solution is set as a goal that cannot be achieved, but that should be strived for or to which should be as close as possible. Because of all these, a utopian solution is associated with the following utopia vector of goals:

$$F^0(x) = [\min\ F_1(x) \min\ F_2(x) ... \min\ F_k(x)]^T \quad (5)$$

In this paper utopia vector represents the vector that simultaneously achieves the minimum maintenance cost, maximum operation security (reliability) due to the good condition of equipment and minimal risk of failure. In order to achieve these objectives, a utopian single component, which needs to be chosen between all the nominees, is the one with the minimum amount of cost of maintenance activity, the minimum health index of the equipment and the maximum risk of failure? Therefore, a compromise solution is a solution that minimizes the distance from the utopian or impossible solutions and the Pareto optimal set. The scalarization of this optimization technique with multiple objectives was performed using the weighted global criterion method with the weight coefficients in which all the objective functions combine and form into one. Norma of proximity, whose minimum value was requested by this optimization, follows as:

$$\min\ L_p(x) = \min \left[\sum_{i=1}^{k} w_i^p \left[F_i(x) - \mathrm{F_i}^0 \right]^p \right]^{\frac{1}{p}} \quad (6)$$

In order to obtain a dimensionless objective functions their transformation was performed. After this procedure, called normalization, the following relations was obtained:

$$\min\ L_p(x) = \min \left[\sum_{i=1}^{k} w_i^p \left[\frac{F_i(x) - F_i^0}{F_i^{\max} - F_i^0} \right]^p \right]^{\frac{1}{p}} \quad (7)$$

Knowing, by the definition, that $F_i^0 = \text{min imum } \{F_i(x) \mid x \in X\}$ the relation (7) can be written in the following way:

$$\min L_p(x) = \min \left[\sum_{i=1}^{k} w_i^p \left[\frac{F_i(x) - F_i^{\min}}{F_i^{\max} - F_i^{\min}} \right]^p \right]^{\frac{1}{p}} \qquad (1 \le p \le \infty) \tag{8}$$

It should be mentioned that the introduced weighting coefficients w_i will be placed such that $\sum_{i=1}^{k} w_i = 1$ and w> 0.
Setting the weight coefficients of the individual objectives of maintenance activities depends on engineering intuition and expert knowledge of the decision maker in the domain of risk management, economics, reliability and condition based maintenance of equipment, etc. This function of L_p norm indicates how close is a compromise solution to an utopian solution, where p denotes type of distance. If $p = 2$, all deviations from Utopian solutions are brought into direct connection with their amplitude which coincides with the so-called "collective benefit" [5]. If it is $p \ge 2$, the greater importance (value of weighting factor) is given to a higher deviation of function of a specific norm. L_2 refers to the Euclidean norm. If $p = \infty$ only the maximum deviation is taken into account. The function of norm L_∞ is called the Chebyshev norm or min-max criterion and it coincides with the so-called "single benefit" [5].
In this paper, function of Euclidean norm L_2 was chosen as an index of priorities in the process of establishing optimal classification of nominated components regarding the necessity of their maintenance, repair, or replacement and keeping in mind at the same time on costs, risk of failure and operation of the individual components. The proposed optimization of maintenance activities is a first pioneering step out from the usual practice of making plans of maintenance activities. Existing maintenance management in the power distribution system is primarily focused to the budget constraints in the planning process, and contained none or just a few other objectives and constraints.

3. THE EXAMPLE OF MAINTENANCE OPTIMIZATION

Multi-objective optimization, which is presented in this paper, has been implemented in order to achieve optimal maintenance management. In other words we made a prioritized classification of nominated activities. The Table I present the 12 maintenance activities nominees. Suppose we restrict ourselves to maintain those components that have the health index value less than or equal to 4 and the total allowable budget is limited to 1,000,000 € in the current fiscal year.

Table I: Prioritized classification of maintenance activities

Name (number) of maintenace activity	Health index of equipment (F_1)	Risk of failure (F_2)	Costs (F_3)	w_1	w_2	w_3	$F_1 - F_1^{min} / \Delta F_1$	$F_2^{max} - F_2 / \Delta F_2$	$F_3 - F_3^{min} / \Delta F_3$	Euclidean distance L_2	Sum of costs
12	2	415	45.000	0	1	0	0,333333333	0,649334946	0,060846561	0,034672	45.000
11	2	318	22.000	0	1	0	0,333333333	0,76662636	0	0,038331	67.000
1	1	235	100.000	0	1	0	0	0,866989117	0,206349206	0,04127	167.000
3	1	458	125.000	0	1	0	0	0,597339782	0,272486772	0,054497	292.000
5	1	878	125.000	0	1	0	0	0,089480048	0,272486772	0,054497	417.000
9	2	562	140.000	0	1	0	0,333333333	0,471584039	0,312169312	0,066738	557.000
7	2	588	145.000	0	1	0	0,333333333	0,440145103	0,325396825	0,0687	702.000
4	3	952	230.000	0	1	0	0,666666667	0	0,55026455	0,110053	932.000
2	4	333	65.000	0	1	0	1	0,748488513	0,113756614	0,114555	997.000
10	4	125	75.000	0	1	0	1	1	0,14021164	0,152599	1.072.000
8	3	477	300.000	0	1	0	0,666666667	0,574365175	0,735449735	0,157906	1.372.000
6	2	456	400.000	0	1	0	0,333333333	0,599758162	1	0,202236	1.772.000
deltaF=max	3	827	378.000								
maxF	4	952	400.000								
minF	1	125	22.000								
Utopia point	**1**	**952**	**22.000**								

In Table 1 are given all the information about the health index of the equipment, the risk of failure, maintenance cost, weight factors for each objective and Euclidean distances are calculated. Mathematical expression $\Delta F_i(x)$ in the formulas that appear in Table I present the difference $F_i^{\max}(x) - F_i^{\min}(x)$. Table II presents a procedure to determine the health index of the equipment in a scale of 1 to 10 depending on the total score of a single component which was assigned on evaluation of equipment condition and implementation of different diagnostic methods.

Table II: Health index of equipment

Health index of equipment	Description	Total score
1	Catastrophic	0-10
2	Very bad	11-20
3	Bad	21-30
4	Acceptable to bed	31-40
5	Acceptable	41-50
6	Acceptable to good	51-60
7	Good	61-70
8	Very good	71-80
9	Excellent	81-90
10	Perfect	91-100

The total cost of all the nominees of critical components is 1.772 million € which is 77.2% higher than a defined amount of the budget.
Reconciling all data of Table I, together with individual nominated maintenance activities the following utopia vector was determined $F^0(x) = [\min F_1(x) \max F_2(x) \min F3(x)]^T$ = [1 952 22.000]. This means that the component with the highest priority in the maintenance plan is the one which has minimal index of the equipment, the maximum risk of failure and the least cost of maintenance activity. It is obvious that such a component does not exist, but the goal of compromise programming is to maintain those components that are closest to utopia vector or so called „ideal solution."

Figure 2 shows the normalized deviation of all components from the utopia vector which is limited to the value 0 and 1. Normalized deviation value of 0 indicates that the component has the same target value as the value of utopia vector for a given single goal.

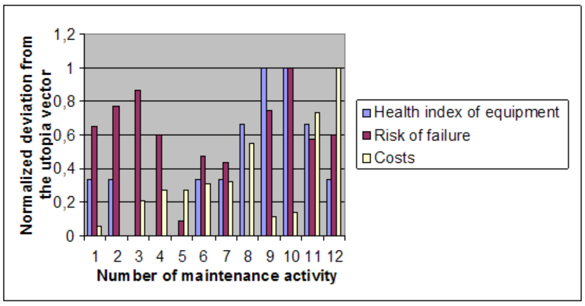

Figure 2: Normalized deviation from the utopia vector

A value of 1 indicates that the value of the goal is farthest from the "ideal solution." Using the Euclidean metric or relation (8) accompanied with selected weighting factors the compromise solution that achieves the minimum value of $L_2(x)$ is a component numbered by 12. Table I shows the discernible optimal classification of components in respect of nominated maintenance activities which optimal sequence is as follows: 12, 11, 1, 3, 5, 9, 7, 4, 2, 10, 8 and 6.

Figure 3 show that activity 12 has the highest priority versus lowest priority of activity 6 in the classifications of maintenance activities. The scheduled budget resources

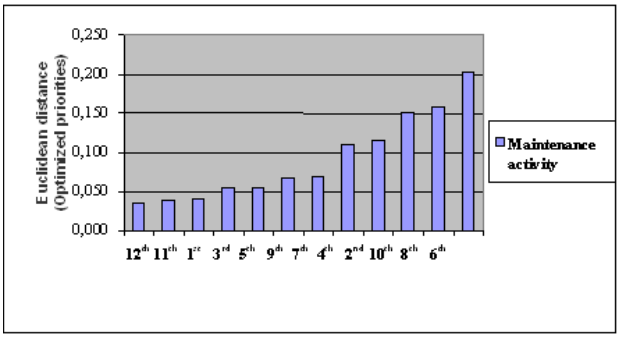

Figure3: Prioritization of maintenance activities

of 1,000,000 € in the current fiscal year should be allocated to the components 12, 11, 1, 3, 5, 9, 7, 4 and 2. Overall planned maintenance activities have a total expenditure of 997,000 €.

4. CONCLUSION

A method called maintenance activity optimization algorithm with existence of several opposite object was suggested in this paper. The proposed method presents systematic tool designed to help managing persons regarding decision making about problems with several objects like: company incomes and cost limits, the system failure probability named scheduled reliability level, asset condition etc. When this method of maintenance planning, based on Pareto optimality, is used in combination with monetary constraints the managing person is enabled to estimate how investments and maintenance strategy influence financials and asset condition from short term and long term perspective. The adopted methodology integrate asset condition, risk and costs in such way that enables decision maker to chose global optimal solution attuned to actual business goals by means of weighting factors.

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