ABSTRACT

Inline Voltage Regulators such as tap changers have been widely deployed to correct voltage drop across a typical distribution feeder. Their optimal placements (e.g., to minimize worst-case voltage drop) are well studied problems. However, with the advent of variable generation (solar, wind, etc.), some feeders can exhibit reverse power flow and consequently voltage rise. Moreover, by nature, solar and wind power output can change quickly, thereby causing wild swings in the voltage profile in a matter of seconds. This paper addresses the problem of how to place an inline Voltage Regulator to simultaneously address BOTH voltage drop and voltage rise on the same feeder. We present a fast graphical method that not only determines the optimal placement (minimizing worst-case voltage excursion), but also gives valuable insights into the limiting factors and the effects of perturbing various parameters of the given feeder. We also applied the method to two generic German feeders.

INTRODUCTION

High penetration of photovoltaics (PV) on distribution feeders can cause voltage regulation problems. In particular, in high irradiance scenarios reverse power flow can occur and voltage can rise above accepted limits. Since irradiance (and therefore PV real power output) can change rapidly on a partly cloudy day, this leads to rapid voltage fluctuations on a timescale of seconds [1]. Traditional solutions such as load tap-changers and switched capacitor banks operate in coarse increments and with slow time constants on the order of tens of seconds to minutes, making them ineffective in combating rapid voltage fluctuations. In addition, these traditional devices are often limited by the total number of actuations in each device’s lifetime. In contrast, recent cost reduction in power electronics has enabled the creation of idealized Voltage Regulators (VR) [2] which react fast (timescale of cycles), regulate perfectly (to e.g. 1.0 pu) and have no lifetime limits on the total number of actuations.

This paper considers the optimal placement of idealized VR along a linear secondary distribution feeder. We optimize the worst-case voltage excursion from nominal, but unlike traditional methods, we account for the dramatically different voltage profiles during both worst-case voltage rise (low-load, high PV generation) and worst-case voltage drop (high-load, no PV generation) scenarios. Our results show that idealized VR can be an effective solution to the problem of rapid, unpredictable voltage fluctuation.

PROBLEM SETTING & PREVIOUS WORK

Our VR placement method takes the following inputs:
1. a feeder’s worst-case voltage-drop profile V_{io}(x) (typically high load, no PV output scenario), and
2. the same feeder’s worst-case voltage-rise profile V_{hi}(x) (typically low load, high PV output scenario).

The voltage profiles describe how (rms) voltage varies as a function of location x, and can come from actual voltage measurements, or standard load-flow studies (based on measured or presumed loading and PV generation). The location x can represent actual physical distance, pole or node (bus) numbering, or some other proxy. What constitutes “worst cases” is entirely up to the utility engineer/planner.

In the traditional problem setting, only the voltage-drop profile V_{io}(x) is considered, and various graphical methods are known (e.g. [3]). Figure 1 illustrates perhaps the simplest such method. Let ΔV (= 8.6%) be the original worst-case voltage drop, which happens at the end of the feeder. Then the optimal VR placement is at the point (x = 0.4) where the voltage drop is ΔV/2 (= 4.3%), so that in the new profile, worst-case drop before VR (i.e. between source and VR) = worst-case drop after VR (i.e. between VR and end of feeder) = worst-case drop along entire feeder = ΔV/2 (≈ 4.3%). The optimal VR location is usually not the midpoint of the feeder, unless the profile V_{io}(x) happens to be a straight line. This simple method can be adapted for variations in the problem setting – e.g. when source voltage ≠ 1 p.u., when the VR output ≠ 1 p.u., and when load-drop compensation is employed.
For each VR placement location $x$, define:

- $F_{hi}(x) = \text{worst-case voltage-rise (along entire feeder) as a function of VR placement } x$.
- $F_{lo}(x) = \text{worst-case voltage-drop (along entire feeder) as a function of VR placement } x$.
- $F_{both}(x) = \max(|F_{hi}(x)|, |F_{lo}(x)|) = \text{worst-case voltage-exursion (along entire feeder) as a function of VR placement } x$.

All three are measured against some nominal voltage value e.g. $V_{\text{nominal}} = 1 \text{ p.u.}$

In the traditional setting which ignores voltage-rise, optimality is often defined as minimizing $|F_{lo}(x)|$. For this paper, we consider optimality as minimizing worst-case voltage excursion $F_{both}(x)$. In other words, we do not make a-priori judgements about whether the real voltage is best represented by the voltage-drop profile or the voltage-rise profile, or some (algebraic or probabilistic) combination of the two; instead we simply minimize the worst-case excursion possible in either profile.

Note that if there is a finite number of placement options (e.g. pole locations), a brute-force approach can iterate through all $N$ options, and for each option $x_j (j = 1, 2, ..., N)$ calculate $F_{hi}(x_j)$ and $F_{lo}(x_j)$ by solving two separate power-flow problems. This requires solving $2N$ power-flow problems. Instead, our graphical method requires only the 2 original profiles (which may be obtained by solving 2 power-flow problems).

**GRAPHICAL METHOD**

Our graphical method is based on a series of observations. First, by having the “constant-current” assumption in traditional methods, the voltage rise and drop profiles for a given placement $x$ would be piecewise-vertically-shifted versions of the original rise and drop profiles. Figure 2 shows an example where source voltage $= 1.03 \text{ p.u.}$ and VR is placed at $x = 0.4$ with VR output $= 1 \text{ p.u.}$ (Note that in this example, the voltage-rise profiles (both original and new) have an “inflection point” near $x = 0.6$, representing e.g. a large PV plant.)

Second, we can define:

- $F_{hi}^{\text{before}}(x) = \text{worst-case voltage-rise before VR (i.e. between source and VR)},$
- $F_{hi}^{\text{after}}(x) = \text{worst-case voltage-rise after VR (i.e. between VR and feeder end)}$

Then the worst-case voltage-rise along the whole feeder is simply the worse of the two:

$$F_{hi}(x) = \max\left(F_{hi}^{\text{before}}(x), F_{hi}^{\text{after}}(x)\right)$$
Third, assuming \( V_{hi}(x) \) is monotonically increasing (i.e., a strictly rising profile with no dips), then the worst-case before-VR voltage-rise happens right before the VR, i.e. the last “unregulated” point:

\[
P_{hi}^{\text{before}}(x) = V_{hi}(x) - V_{\text{nominal}}
\]

Fourth, again assuming \( V_{hi}(x) \) is monotonically increasing, then the worst-case after-VR voltage-rise happens at the end of the feeder, \( x = x_{\text{end}} \). By using the constant-current assumption, the new profile (after VR) is a vertically shifted version of the original profile, and therefore:

\[
P_{hi}^{\text{after}}(x) = V_{hi}(x_{\text{end}}) - V_{hi}(x) + V_{\text{reg}} - V_{\text{nominal}}
\]

where \( V_{\text{reg}} \) is the VR’s output voltage. Graphically, \( -V_{hi}(x) \) is the original voltage-rise profile reflected across the horizontal axis, and adding the constant term \( V_{hi}(x_{\text{end}}) + V_{\text{reg}} - V_{\text{nominal}} \) simply shifts the reflected profile vertically. In other words, we can obtain the \( P_{hi}^{\text{after}}(x) \) curve by flipping (reflecting) the original \( V_{hi}(x) \) curve across the horizontal axis, and then shifting it up so that \( P_{hi}^{\text{after}}(x_{\text{end}}) = V_{\text{reg}} - V_{\text{nominal}} \).

Figure 3 illustrates how the above techniques can be used to obtain \( P_{hi}^{\text{before}}(x) \) and \( P_{hi}^{\text{after}}(x) \), and then \( P_{hi}(x) \) is simply the upper envelope of the two.

Obviously, the same techniques can be applied to obtain the \( P_{lo}^{\text{before}}(x) \), \( P_{lo}^{\text{after}}(x) \) and \( P_{lo}(x) \) curves. (This time we need to assume the original profile \( V_{lo}(x) \) is monotonically decreasing, i.e. a strictly drooping profile.) Finally, \( P_{hi}(x) \) and \( P_{lo}(x) \) can be combined into the \( P_{both}(x) \) curve, as illustrated in Figure 4.

\[
P_{hi}^{\text{before}}(x) = V_{hi}(x) - V_{\text{nominal}}
\]

\[
P_{hi}^{\text{after}}(x) = V_{hi}(x_{\text{end}}) - V_{hi}(x) + V_{\text{reg}} - V_{\text{nominal}}
\]

where \( V_{\text{reg}} \) is the VR’s output voltage. Graphically, \( -V_{hi}(x) \) is the original voltage-rise profile reflected across the horizontal axis, and adding the constant term \( V_{hi}(x_{\text{end}}) + V_{\text{reg}} - V_{\text{nominal}} \) simply shifts the reflected profile vertically. In other words, we can obtain the \( P_{hi}^{\text{after}}(x) \) curve by flipping (reflecting) the original \( V_{hi}(x) \) curve across the horizontal axis, and then shifting it up so that \( P_{hi}^{\text{after}}(x_{\text{end}}) = V_{\text{reg}} - V_{\text{nominal}} \).

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Figure 3 illustrates how the above techniques can be used to obtain \( P_{hi}^{\text{before}}(x) \) and \( P_{hi}^{\text{after}}(x) \), and then \( P_{hi}(x) \) is simply the upper envelope of the two.

Figure 4 also shows three optimal points:

- \( x_{hi}^* (\approx 0.2) \) minimizes worst-case voltage-rise \( |F_{hi}(x)| \) to \( \approx 5\% \), but worst-case voltage-drop \( |F_{lo}(x)| \) to \( \approx 7\% \). This is the optimal placement prescribed by the voltage-rise profile alone.
- \( x_{lo}^* (\approx 0.6) \) minimizes \( |F_{lo}(x)| \) to \( \approx 3\% \), but \( F_{hi}(x) \) to \( \approx 9\% \). This is the optimal placement prescribed by the voltage-drop profile alone (i.e., traditional method, adapted for source voltage \( > 1 \)).
- \( x_{both}^* (\approx 0.3) \) minimizes \( F_{both}(x) \) to \( \approx 6\% \). As expected, \( x_{both}^* \) lies between \( x_{hi}^* \) and \( x_{lo}^* \), and represents a balance between worst-case voltage drop and rise. (It is also possible for \( x_{both}^* \) to be equal to \( x_{hi}^* \) or \( x_{lo}^* \), as we will show below.)
The precise location of $x_{both}$ gives insight into the limiting factors. In this example, $x_{both}$ is at the intersection of the $F_{hi}^{before}(x)$ and $F_{lo}^{after}(x)$ curves. Therefore, the 6% excursion takes the form of:

(i) +6% voltage rise just before VR ($F_{hi}^{before}$), and

(ii) −6% voltage drop at end of feeder ($F_{lo}^{after}$).

These limits in turn suggest that the worst-case excursion can be further optimized (reduced) by:

(i) lowering source voltage (to combat before-VR voltage rise), and/or

(ii) raising $V_{reg}$ (to combat after-VR voltage drop).

Figure 5 shows option (ii) at work: Raising $V_{reg}$ by 1% shifts the $F_{hi}^{after}(x)$ and $F_{lo}^{after}(x)$ curves vertically up by 1%, which in turn improves the optimal $F_{both}(x)$ value from 6% to ≈ 5.5%. In the new setting, $x_{both} = x_{hi} ≈ 0.25$, and raising $V_{reg}$ further would not help.

CASE STUDIES

In this section, we apply the graphical method to two real-life generic German secondary feeders (Table 1 and Figure 6). In both feeders the source voltage is 1 p.u. on 400 VLL base.

<table>
<thead>
<tr>
<th>Feeder</th>
<th>Total loads</th>
<th>Total PV</th>
<th>Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>99.7 kW</td>
<td>86.0 kW</td>
<td>3294 ft</td>
</tr>
<tr>
<td>B</td>
<td>37.1 kW</td>
<td>282.2 kW</td>
<td>1717 ft</td>
</tr>
</tbody>
</table>

Table 1: Characteristics of two feeders.

The voltage-rise and voltage-drop profiles for both Feeders A and B are solved using load-flow studies with a commercial package. The voltage-drop profiles correspond to 100% loading and no PV (e.g. cloudy day or evening after sunset), while the voltage-rise profiles correspond to 40% loading and 100% PV (e.g. sunny afternoon). These are then transformed into the various $F(x)$ curves. The curves for Feeders A and B are shown in Figures 7 and 8 respectively. The markers in the plots correspond to the locations of actual loads.

Figure 5: Raising $V_{reg}$ by 1%, i.e. setting $F_{lo}^{after}(x_{end}) = F_{hi}^{after}(x_{end}) = 1\%$, improves optimal $F_{both}(x)$ value from 6% to 5.5%.

Figure 6: One-line diagrams for Feeder A (left) and Feeder B (right) used in case studies.

Figure 7: The various $F(x)$ curves for Feeder A. Source voltage = $V_{reg} = 1$. To avoid excessive clutter, $F_{both}$ is not shown explicitly – it is the upper envelope of all the other curves.
Feeder A (Figure 7) has roughly as much PV as peak load. Consequently, without VR, it exhibits roughly equal worst-case voltage rise (≈ 8%) and drop (≈ 6%). Using the traditional method which only considers voltage drop, the VR would be placed at $x_{hi}^*$ to halve the voltage drop to ≈ 3%. Similarly, placing the VR at $x_{hi}^*$ would halve the voltage rise to ≈ 4%. To mitigate both voltage rise and drop, the optimal placement is at $x_{both}^*$ and after regulation the worst-case voltage excursion ≈ 4.5%. At this point, the limiting factors are (i) $p_{before}^{lo}$ (voltage drop just before VR) and (ii) $p_{after}^{hi}$ (voltage rise at end of feeder). Therefore, (i) raising source voltage and/or (ii) lowering $V_{reg}$ should improve excursion. Indeed, we found that worst-case excursion can be improved to ≈ 4%. (Relevant plots are omitted due to space limitation.)

Feeder B (Figure 8) has two unusual features. First, the nearest load of interest is located ≈1000 feet from the source, with a long conductor in between. Second, it has a huge amount of PV – about 7.6 times its peak load. As a result, without VR, voltage rise (6%) is much more significant than voltage drop (1%). Mainly for this reason, in this example, $p_{hi}^{before}(x)$ is the limiting curve, i.e. $F_{both}(x) = F_{hi}(x) = p_{hi}^{before}(x)$. Thus, minimizing excursion is the same as minimizing voltage rise, and the VR should simply be placed at the first load, achieving worst-case voltage excursion ≈ 3%. One way to achieve smaller worst-case excursion in this feeder is to lower source voltage. E.g., Figure 9 shows the case when source voltage = 0.985 p.u. ($V_{reg}$ remains at 1), and $x_{both}^*$ has shifted a bit and the worst-case excursion is reduced to ≈ 2.3%.

**CONCLUSION & FUTURE WORK**

This paper introduces a fast graphical method for finding the optimal placement for an idealized VR in a linear feeder. The method optimizes the worst-case voltage excursion, accounting for the different voltage profiles for both worst-case voltage rise and worst-case voltage drop scenarios, in order to combat the problem of rapid, unpredictable voltage fluctuations caused by varying PV power output and loads. The graphical method is intuitive yet mathematically rigorous. Further, its graphical nature gives insight into the limiting factors, and the effects of perturbing various parameters of the given feeder (such as source voltage and regulator output voltage).

Several extensions of this work could make it more practically useful, e.g., optimal joint placement of multiple VRs, different optimality criteria (e.g. minimizing weighted average excursion), handling general radial feeders (not just linear feeders) and load-drop compensation.

**REFERENCES**

