MODEL PREDICTIVE CONTROL OF CONGESTION AND VOLTAGE PROBLEMS IN ACTIVE DISTRIBUTION NETWORKS

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ABSTRACT
This paper presents a centralized control scheme, inspired of Model Predictive Control (MPC), to manage thermal overloads and correct abnormal voltages in real-time. The control scheme is able to smoothly bring the system within the desired limits, taking into account its near-future evolution. The control method effectiveness is illustrated on a 20-kV, 32-bus network hosting four distributed generation units.

INTRODUCTION
The progressive penetration of renewable energy sources connected to Medium-Voltage (MV) distribution systems is expected to create new operational problems. Over- or under-voltages as well as thermal overloads of some branches (cables, lines and transformers), referred to as congestions, are the main issues raised by Distributed Generation (DG) units and the prevailing load patterns. While voltages can somewhat exceed their limits for a limited time, the tripping of overloaded branches by protection relays makes congestion management more constraining. In the framework of active distribution networks, load tap changers, shunt capacitors, flexible loads and DG units are the main controls available in real-time.

Advances in communication technology and progress in Smart Grids make it realistic to devise a centralized controller to mitigate the above two problems. Although it may require investments in terms of communication infrastructure, it is considerably less expensive than reinforcing the network for coping with problems that take place for limited periods of time.

The two above mentioned issues can be dealt with through separate control schemes as in [1,2] for voltage, and [3,4] for thermal problems, or through a combined control scheme as in [5]. References [1] proposed a centralized scheme, inspired of MPC [6][6], inherently able to compensate for modelling inaccuracies and measurement noise, a key feature missing in many control schemes of the literature. The control actions, calculated from a multi-step optimization, are updated and corrected by real-time measurements. The proposed controller uses a sensitivity model to predict the behaviour of the system and the multi-step optimization entails solving a quadratic programming problem. In [3], the “last-in first-off” principle was considered for the centralized control of congestions caused by DG units, while maximizing the generation capacity yielded. In [5] a decentralized approach was proposed to manage voltage and thermal problems using a sensitivity-based model of the network.

This paper presents an extension of the approach introduced in [1], incorporating congestion management, for the combined corrective control of both problems. The focus is on the constraints added to the multi-step optimization problem, and the computation of the corresponding sensitivities.

PREDICTIVE CONTROL SCHEME
The proposed controller estimates the system behaviour at the future \( N_p \) time steps using a sensitivity model. Thus, at a given time step \( k \), an optimal sequence of control actions \( \Delta u(k+i) (i = 0,\ldots,N_c - 1) \) is calculated for the \( N_c \) future time steps, with the objective of bringing bus voltages and branch currents within the desired limits. In accordance with MPC principle, only the first component \( \Delta u(k) \) of the calculated sequence is applied at time \( k \). At the next time step, based on the new measurements received, the whole procedure is repeated.

In this paper, the focus is on DG units and:

\[
\Delta u(k) = u(k) - u(k-1) = [\Delta P_g(k)^T, \Delta Q_g(k)^T]^T
\]

where \( \Delta P_g \) (resp. \( \Delta Q_g \)) is the vector of active (resp. reactive) power changes of the DG units, and \( T \) denotes array transposition. The multi-step optimization involves the quadratic objective:

\[
\min \sum_{i=0}^{N_c-1} ||\Delta u(k+i)||_R^2 + ||\epsilon||_W^2
\]

where the first term minimizes the total (i.e. multi-step) control effort, weighted by the diagonal matrix \( R \). By assigning proper weights to the various control actions, the controller favours the “cheap” ones. The vector \( \epsilon = [\epsilon_1, \epsilon_2, \epsilon_3]^T \) includes variables aimed at relaxing the operational constraints in case of infeasibility. Nonzero values are heavily penalized by the diagonal matrix \( W \). The constraints are as follows:

\[
-\epsilon_1 1 + V_{\text{min}}(k+i) \leq V(k+i|k)
\]

\[
V(k+i|k) \leq V_{\text{max}}(k+i) + \epsilon_2 1
\]

\[
V(k+i|k) = V(k+i-1|k) + \frac{\partial V}{\partial u} \Delta u(k+i-1)
\]

\[
I(k+i|k) \leq I_{\text{max}}(k+i) + \epsilon_3 1
\]
\[ I(k+i) = I(k+i-1) + \frac{\partial I}{\partial u} \Delta u(k+i-1) \quad (2d) \]

for \( i = 0, 1, ..., N_C - 1 : \)
\[ u_{\text{min}}^i \leq u(k+i) \leq u_{\text{max}}^i \quad (2e) \]
\[ \Delta u_{\text{min}}^i \leq \Delta u(k+i) \leq \Delta u_{\text{max}}^i \quad (2f) \]

The limits \( u_{\text{min}}^i, u_{\text{max}}^i, \Delta u_{\text{min}}^i \) and \( \Delta u_{\text{max}}^i \) relate to DG unit capabilities and acceptable rates of change. \( V(k+i|k), I(k+i|k) \) are vectors of predicted bus voltages and branch current magnitudes (given the measurements at time \( k \)), \( \frac{\partial V}{\partial u} \) and \( \frac{\partial I}{\partial u} \) are sensitivity matrices of bus voltages and branch currents to control variables, and \( 1 \) denotes a unit vector.

The voltage limits \( V_{\text{min}}(k+i) \) and \( V_{\text{max}}(k+i) \) at the \( i \)-th prediction step are progressively tightened as described in [1, 2]. A similar procedure is followed for the current limit \( I_{\text{max}}(k+i) \). This is illustrated in Fig. 1 showing the limit relative to a particular current, progressively tightened over the prediction horizon (using parameter \( \rho \)) to meet the target value \( I_{\text{max}} \) after \( N_p \) steps. Note that the latter value is set conservatively below the effective thermal capability monitored by the branch protection.

**Figure 1:** Limit imposed on current in (2c)

**SENSITIVITY COMPUTATION**

An accurate sensitivity matrix, to be used for predicting the system behaviour, should incorporate the variation of load powers with voltage, the actual network impedances and the actual system operation point. Unfortunately, this information is not known accurately in practice and some approximations are required.

As regards sensitivities of bus voltages with respect to the generated powers, they can be obtained from the inverse of the Jacobian matrix extracted from an off-line power flow calculation. They can also be extracted from the solutions of two power flow calculations with a different generated power, by computing the ratio of variation of the monitored bus voltage to the variation of generated power. Owing to the capability of MPC to compensate for modelling inaccuracies, these sensitivities can be updated infrequently.

As regards sensitivities of currents, Eq. (2d) relative to the \( j \)-th branch current can be rewritten as:
\[ j(k+i) = j(k) + \sum_{i=1}^{N_g} \frac{\partial j}{\partial P_{gi}}(k)\Delta P_{gi}(k) + \frac{\partial j}{\partial Q_{gi}}(k)\Delta Q_{gi}(k) \quad (3) \]

where \( j \) is the current magnitude, \( P_{gi}, Q_{gi} \) are the active and reactive powers generated by the \( i \)-th DG unit, and \( N_g \) is number of DG units. If the branch is not on the path from the \( i \)-th DG unit to the source substation, the partial derivatives in (3) can be set to zero. Otherwise, from the expression of the current:
\[ j(k+i) = \frac{S_j}{V_i} = \sqrt{P_j^2 + Q_j^2} / V_i \quad (4) \]

where \( P_j \) (resp. \( Q_j \)) is the active (resp. reactive) power flow, \( S_j \) is the apparent power and \( V_i \) the bus voltage magnitude, the sensitivities can be approximated by:
\[ \frac{\partial j}{\partial P_{gi}} \approx \frac{P_j}{V_i S_j} \frac{\partial P_{gi}}{S_j} \approx \frac{P_j}{S_j} \]
\[ \frac{\partial j}{\partial Q_{gi}} \approx \frac{Q_j}{V_i S_j} \frac{\partial Q_{gi}}{S_j} \approx \frac{Q_j}{S_j} \quad (5) \]

where it is assumed that \( P_j \) (resp. \( Q_j \)) does not change significantly when \( Q_{gi} \) (resp. \( P_{gi} \)) is varied, and, the change of \( P_j \) (resp. \( Q_j \)) is equal to the change in \( P_{gi} \) (resp. \( Q_{gi} \)). The bus voltage is also assumed constant and equal to 1 pu.

It is assumed that active and reactive power flows are measured in the monitored branch, in which case these measurements are used in (5) to update the sensitivities. The simplest solution consists of computing the sequence of corrective actions \( \Delta u(k+i) \) (\( i = 0, ..., N_C - 1 \)) using the sensitivities evaluated at step \( k \). However, these sensitivities may change significantly with the operating point; in particular, they change sign in case of power flow reversal. This may lead to over- or under-estimating the system response, especially when the active or reactive power flow crosses zero, in which case power flow oscillation might take place. To deal with this issue, the following alternative schemes were contemplated:

- when the power flow approaches zero, the corresponding sensitivities are set to zero, which leads the optimization (2) to automatically rely on other control actions;
- using the sensitivities evaluated from measurements collected at time \( k \), a first sequence of corrective actions is computed. At the resulting predicted states, new sensitivities are recomputed and the average between the original and the recomputed values is used to solve, for a second time, the
optimization problem (2). The so recomputed control actions are applied to the system (the intermediate ones are ignored).

SIMULATION RESULTS

The 32-bus, 20-kV distribution network shown in Fig. 2, was used to test the proposed corrective control. The network is connected to the external grid through two 5-MVA HV/MV transformers. Both transformers are in operation in the initial operating conditions.

The network hosts three 4.5 MVA synchronous generators driven by hydro turbines and one 3.33-MVA doubly fed induction generator driven by wind turbine. It feeds 12 loads modelled as constant current (resp. impedance) for active (resp. reactive) power, and three induction motor loads.

![Figure 2: One-line diagram of the test system](image)

The following measurements are collected and transmitted to the controller: active and reactive power and voltage magnitude at the terminals of the four DG units, active and reactive power flows in the HV/MV transformers, and voltages at load buses 7, 11, 19, 28, 29 and 31. Measurements are simulated by adding a Gaussian noise $N(0, \sigma)$ with $\sigma = 0.0033$ pu to the corresponding values obtained from detailed time simulation.

The transformer thermal capabilities are set to 5 MVA, and the same value is taken as conservative thermal limit. Furthermore, a congestion scenario is simulated by tripping one transformer, which leads to overloading the other one.

The sequences of corrective actions, including reactive (more prioritized controls) and active (less prioritized controls) power production corrections, are computed and applied to the DG units. The weights in matrix $R$ are set to 50 times bigger values for active power changes than for reactive power changes. The weight assigned to the slack variables $\varepsilon_1, \varepsilon_2$ (resp. $\varepsilon_3$) is 1000 (resp. 10000) times larger than that assigned to reactive power corrections. The progressive tightening is tuned with $\rho = 5$ for voltages and $\rho = 10$ for the current while the prediction and control horizon were set to $N_p = N_c = 3$.

Case 1

In the first test case, the sensitivities of current to DG unit outputs are computed from (5) at the time power measurements are received, and kept at this value at all steps of the multi-step optimization (2).

At the initial operating point, the direction of power flows is from distribution to transmission. Therefore, after the transformer outage, the controller reduces the power flow by decreasing the reactive power outputs of the DG units, as shown in Fig.s 3 and 4. When solving the optimization problem (2), it is found that the sole reactive power reduction cannot alleviate the congestion; hence, although more penalized, a reduction of active powers is computed and applied, as seen from Fig. 5.

![Figure 3: Case 1: power flows in the remaining transformer](image)

Figure 6 shows the time evolution of the monitored load voltages. The initial increase, at $t = 100$ s, is due to the power flowing from the MV to the HV bus in the impedance made larger by the transformer outage. Under the effect of DG unit reactive power reduction, the voltages start decreasing at $t = 110$ s. In any case, they remain inside the specified range of $[0.98, 1.03]$ p.u, and the original congestion is not aggravated by a voltage problem.

An oscillation can be observed in the branch reactive power flow around zero, as well as in the bus voltages...
around their steady-state values. This is caused by the use of constant sensitivities, which leads to wrongly estimating the system evolution when the reactive power flow crosses zero.

To compensate for this, the optimization resorts to active power curtailment in order to bring the current below the specified limit.

The overall system response is a little less oscillatory, as confirmed by Fig. 8 for the variations of reactive powers.

Case 2
In this case, a variant suggested in the previous section is considered, with the sensitivities \( \frac{\partial F_j}{\partial P_{Gi}} \) (resp. \( \frac{\partial F_j}{\partial Q_{Gi}} \)) set to zero whenever the active (resp. reactive) power flow in the \( j \)-th branch becomes smaller than a tolerance. The latter has been set to 0.5 MW (Mvar).

The same initial operating point and disturbance are considered as in Case 1.

Figures 7, 8 and 9 show that, as expected, the reactive powers of DG units are no longer decreased after the reactive power flow falls in the range [-0.5 +0.5] Mvar.

Case 3
In this case the advantages of the previous two approaches are combined as follows:

- If the measured and predicted values of the active (resp. reactive) power flow have the same sign, the controller uses sensitivities computed from (5);
- if the active (or reactive) power flow has a predicted value of opposite sign compared to the measured value, the sensitivities are corrected as the average of the values stemming from measurement and prediction, and a new optimization is performed.
The results are shown in Figs. 10 to 12. The reactive power flow decreases to zero without oscillations.

![Figure 10: Case 3 : power flows in the remaining transformer](image)

![Figure 11: Case 3 : DG unit reactive powers](image)

![Figure 12: Case 3 : DG unit active powers](image)

Furthermore, Table I provides the final values of the active and reactive power flows in the remaining transformer, after applying the corrective actions. As can be seen, in Case 2, the controller relied a little more on DG unit active power curtailment to bring the current below its limit, while in other two cases the controller resorted more to the “cheap” reactive power controls.

<table>
<thead>
<tr>
<th>Case</th>
<th>Active power (MW)</th>
<th>Reactive power (Mvar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case1</td>
<td>5.00</td>
<td>0.020</td>
</tr>
<tr>
<td>Case2</td>
<td>4.985</td>
<td>-0.374</td>
</tr>
<tr>
<td>Case3</td>
<td>5.00</td>
<td>0.021</td>
</tr>
</tbody>
</table>

**CONCLUSION**

In this paper an extension of the approach introduced in [1,2] has been proposed for automatic, centralized corrective control of both thermal and voltage problems.

The control scheme computes a sequence of corrective actions smoothly applied to DG unit active and reactive powers. It relies on easy to compute sensitivities.

It has been tested on a 32-bus test system with detailed dynamic model of generation units. The simulations suggest using average sensitivities whenever the currently measured and the future predicted power flows have opposite signs. This yields smoother system responses. Furthermore, the compensation of modelling inaccuracies, a key advantage of the MPC approach, is demonstrated when sensitivities are infrequently updated during the simulation.

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**REFERENCES**


