ABSTRACT
Considering the majority of the power losses occur in distribution systems, it is worthwhile to investigate the use of reactive power compensation (RPC) from wind turbines installed in the distribution system for loss reduction. Therefore, this paper analyses the effectiveness of the RPC for loss reduction under different system conditions. These include power factor of the system load, X/R ratio of the cables, and the electrical location of the wind turbines. The result of the analysis shows that the effectiveness of RPC for loss minimization mainly depends on the power factor of the load followed by the location of the wind turbine in the system. In this regard, in a system where the average load power factor is around unity, e.g. 0.98, RPC is not attractive for loss reduction. However, when the average power factor of the load is around 0.90, RPC is able to decrease the system loss by around 20%. Though the 2/3 rule can be applied to site wind turbine for maximum loss reduction through the use of RPC, the overall sitting problem of the wind turbine is more likely to be dependent on its active power output rather than RPC. The X/R ratio of the cable, on the other hand, has very little impact on the effectiveness of the approach.

INTRODUCTION
The ongoing increase in the introduction of wind power into the distribution system has presented distribution network operators (DNOs) with a number of challenges and opportunities. The challenges relate to the effect of wind power on the power quality and reliability of the system. These effects are widely investigated [1], [2] and mitigation solutions are also proposed to increase the wind power hosting capacity of distribution systems [3], [4].

On the opportunities side one can mention the possible decrease in system power losses. However it is widely known that wind power, distributed generation (DG), in general, can decrease the power losses in a given distribution system depending on its capacity and location in the distribution system [5], [6]. Thus, the sizing and sitting of DG to achieve maximum loss reduction has been the subject of numerous papers [6]–[8].

Beside the loss reduction through the provision of active power locally, wind turbines present the DNO with flexible reactive power to improve the voltage profile of the system and to reduce the system loss. In this regard, considerable research efforts have been devoted to deal with optimizing reactive power output of a wind power plant to minimize the power losses and improve the voltage profile of the system [5], [9], [10]. These papers, however, are interested in either with optimization approach or the algorithm of the optimization model. In contrast, this paper analyzes under what power system conditions reactive power compensation (RPC) can be an attractive solution for loss reduction.

LOSS REDUCTION USING REACTIVE POWER COMPENSATION IN A DISTRIBUTION SYSTEM
RPC in a distribution system can contributes to power losses reduction by providing the reactive power demand in the system locally, which would have been supplied from the external grid through the substation transformer. This will reduce the current flow through the cables in the system, and reduce the system power losses. This section investigates the impact of the following three parameters of the system on the effectiveness of RPC for loss reduction:

- Power factor of the loads in the system
- X/R ratio of the cables connecting the reactive power source (RPS) to the substation
- The location of the RPS in the feeder

The size of the RPS is also an important factor. However, since the sizing of the RPS depends on the power factor of the load and the location of the RPS, it is not investigated as a separate factor.

To carry out the investigation, the simple radial distribution system shown in Fig. 1 is used.

![Figure 1 A simple radial feeder](image)

For each combination of power factor of the load, X/R of the cable, and location of the RPS, the optimal level of reactive power injection and the resulting decrease in power losses is calculated using an optimal power flow program. It is assumed that the loads in the system are of constant power type. The maximum feeder load is chosen to be 3.7 MVA. The total impedance of the cable \((Z_{tot} = Z_1 + Z_2 + Z_3)\) is chosen to be 1.5 Ω so as to make sure that the voltage at the end of the feeder
is always above 0.95 p.u. The results of the investigation are provided in the following figures.

Figure 2 loss reduction using RPC for different power factor of the load in the line

![Figure 2](image)

Figure 3 loss reduction using RPC for different positions of the reactive power source between Bus 2 and 4

![Figure 3](image)

**Power factor of the load and X/R ratio of the cable**

The results in Fig. 2 are obtained by placing the RPS at Bus 3 (with $Z_1 = Z_2 = Z_3 = 1/3 Z_{tot}$) while the 3.7 MVA load is equally divided between Bus 2 and Bus 4 with the power factor of the load varying from zero lagging to zero leading. Two X/R ratio values are considered for the total impedance, $Z_{tot}$.

The figure shows that the power factor of the load in the system influences greatly the effectiveness of RPC for loss reduction. This is expected as more inductive or capacitive the load is the more the RPS is capable of supplying the demand locally. However, if no reactive power is being consumed by the load in the system, there is nothing RPC can do to reduce the power losses in the system. Considering that constant power load types are being investigated, one may expect the injection of reactive power to increase the voltage level in the system thereby reducing the current flow, hence the power losses, in the system. But this is not the case, as RPC itself introduces current and will be counterproductive in this case.

Fig. 2 shows almost no difference in the percentage of loss reduction due to RPC when the X/R ratio of the cable changes. Obviously, the magnitude of loss reduction ($\Delta P_{losses}$) will be inversely proportional to the X/R ratio of the cable for a given impedance ($Z_{tot}$) as can be seen in (1).

$$\Delta P_{losses} = (I_2^2 - I_1^2)R$$

But, the percentage change in power losses ($\%\Delta P_{losses}$) will be independent of the resistance (R), hence the X/R of the cable, for a given load as given in (2).

$$\%\Delta P_{losses} = \frac{(I_2^2 - I_1^2)}{I_1^2}$$

Where $I_1$ and $I_2$ are the currents before and after RPC.

**The location of the reactive power source in the network**

Fig. 3 shows the amount of loss reduction achieved and the amount of reactive power required when the RPS is located at varying distances between Bus 2 and Bus 4. That is, $Z_2 = 2/3 \times k \times Z_{tot}$ and $Z_3 = 2/3 \times (1-k) \times Z_{tot}$ where $k$ is the percentage that varies from 5% to 95%. The total load in the system is kept constant at 3.7 MVA with power factor of 0.8 inductive, equally divided between Bus 2 and 4.

Fig. 3 shows that RPC is more effective in loss minimization when the turbines are located relatively further away from the station. This can be seen from both the perspective of loss minimization as well as the amount of reactive power required to achieve a given loss minimization. That is, as the RPS is moved closer to Bus 4 higher loss reduction is achieved with lower reactive power injection.

**ANALYTICAL ANALYSIS OF LOSS REDUCTION THROUGH REACTIVE POWER COMPENSATION**

In the previous section, it is seen that placing the RPS at the end point of the feeder provides a maximum loss reduction with minimum reactive power requirement. However, this does not mean that the end point of a feeder is always the optimal location of a RPS for loss minimization. In fact, there is a rule which states that in a uniformly loaded line, 2/3 of the reactive power demand of the feeder placed at 2/3 of the electrical distance from the substation minimizes the feeder power losses [11]. But, the example network here is not a uniformly loaded line, hence the result is different.

Assume that the RPS is a wind turbine. The 2/3 rule is derived based on only reactive power consideration. Wind turbines produce active power as well. Taking both into account, the placement of wind turbines on a uniformly loaded line can be searched for using the same approach as in [11]. As
shown in Fig. 4, a wind turbine injects current \( \tilde{I}_w \) at a distance \( l_w \) from the substation in a uniformly loaded line. Then, it is required to determine the optimum value of \( \tilde{I}_w \) and \( l_w \) to minimize power losses of the line.

Let

\[
\tilde{I} = I_r + jI_i, \quad \tilde{I}_w = I_w \times \eta + jI_w \sqrt{1 - \eta^2}
\]

Where \( \eta \) the operating power factor of the wind turbine and \( \left| \tilde{I}_w \right| = I_w \). Then the loss along the line can be calculated as in (4) - (6).

\[
P_{\text{losses}} = 3r \int_0^l \tilde{I}_x \tilde{I}_x^* dx
\]

where \( \tilde{I}_x \) is the current flowing through the feeder at a distance \( x \) from the substation and is given by

\[
\tilde{I}_x = \begin{cases} 
I(l-x) - \tilde{I}_w & \text{for } x \leq l_w \\
\tilde{I}(l-x) & \text{for } x > l_w
\end{cases}
\]

And the total power losses in the feeder is given by

\[
P_{\text{losses}} = 3r \int_0^l \left[ (l-x)^2 I^2 - 2(l-x)(I_w \eta \times I_r + I_w \sqrt{1 - \eta^2} + I_w^2) dx + \int I^2(l-x)^2 dx \right] dI_w
\]

(6)

Where \( r \) is the per unit length resistance of the line and \( \left| \tilde{I} \right| = I \). Differentiating \( P_{\text{losses}} \) with respect to \( I_w \) and \( l_w \), then simultaneously solving the two differentiated equations, the optimal location and current can be determined as:

\[
l_{\text{op}}^w = \frac{2}{3} l
\]

(7)

Further analysis shows that the optimal power factor, \( \eta \), of the wind turbine needs to be the same as the power factor of the load in the system. Noting in (8) that \( \tilde{I} = I_r + jI_i \) is the current consumed per unit length of the line, the optimal location is still at 2/3 length of the line from the substation with optimal \( I_{\text{op}}^w \) being 2/3 of the total load current in the line. The resulting power losses in the system are given by

\[
P_{\text{losses}} = \frac{rI^2l^3}{9}
\]

(9)

The power losses originally in the system before wind power introduction are given by

\[
P_{\text{losses}} = rI^2l^3
\]

(10)

which results in a maximum loss reduction of 88.9%.

If, on the other hand, the wind power output has a different value than what is in (8), the optimal location varies depending on the capacity of the wind power, as in (9).

\[
l_{\text{op}}^w = l - \frac{l_w}{213l \eta + l_w \sqrt{1 - \eta^2}}
\]

(11)

These results, however, are based on average active and reactive power of the load and wind power. The case study in the following section investigates how far the 2/3 rule can be applied with variable and weakly correlated load and wind power data.

**CASE STUDY**

To assess the validity of the conclusions arrived by (7) - (11) and, moreover, to determine the amount of loss reduction that can be achieved in an actual system by using RPC the simple 4-bus radial feeder shown in Fig. 1 is converted into a 10-bus feeder with each section of the line having equal impedance of 1/9 \( Z_{\text{tot}} \). To represent a uniformly loaded line, hourly measured load data obtained from an actual distribution system substation is equally distributed on each bus. Similar hourly measured wind power data are used to represent the wind power generation. Both the wind power and load time series data are of one year length and are
shown in Fig. 5. The average power factor of the original load time series data, as shown Fig. 5, is 0.98. But another load time series data are generated from these data by scaling the load active and reactive power component, while keeping the apparent power constant. The newly generated load time series data have an average power factor of 0.88. This is done to investigate the effect of power factor of the load on the effectiveness of RPC.

The reactive power output from the wind turbine is controllable from 0.95 power factor lagging to 0.95 power factor leading at the full power output. This assumption is in-line with the majority of current grid code requirements on wind turbines [12]. At a lower power output, the wind turbines are assumed to supply reactive power as long as their thermal limit is not violated. For both load conditions, the wind power output is scaled so that its average output is 2/3 of the load active power.

A number of load flow calculations are carried out by iteratively placing the wind turbine from Bus 1 to Bus 10. For each position of the wind turbine, the following parameters are calculated:

- The percentage change in power losses of the feeder due to the introduction of wind power (generating only active power).
- The percentage change in power losses due to RPC compared to the case when the wind turbine is producing only active power.
- The average reactive power in magnitude produced or consumed by the wind turbine due to the RPC.

The results are presented in Fig. 6. Both plots in Fig. 6 show that the loss reduction due to RPC almost satisfies the 2/3 rule as it can be controlled to match the reactive power demand of the load in the system. Moreover, one can see that the loss and less reactive power is required to reduce the system loss as the wind turbine is placed further away from the substation. However, the loss reduction due to the active power of the wind turbine does not follow the 2/3 rule. Considering the low level of correlation that exists between load and wind power data i.e. ≈0.16, this is expected. In general, the wind turbine need to be sited closer to the substation than what is required by the 2/3 rule. Moreover, one need to observe how crucial the sitting problem is: the change in power losses has varied from a reduction of 23% (in Fig. 6 (a)) at Bus 4 to 36% increase at Bus 10 by mere changing of the connection point of the wind turbine.

Furthermore, the figure shows that when the average power factor of the load decreases from 0.98 to 0.88, the loss reduction due to active power injection of the wind power decreases, but not substantially. This is always expected as decrease in power factor relates to the decrease in active power demand in the system. This decreases the likelihood of the power demand of the system being met by local active power generation from the wind turbine. In contrast, for the same decrease in power factor, the loss reduction due to RPC has increased substantially, becoming almost as significant as the loss reduction obtained from wind turbine’s active power injection. This is because, although the active power from the wind turbine is the one that can highly affect the power flow in the cable it cannot be controlled to match the load demand in the system. However, reactive from the wind turbines can be controlled to match the reactive power demand in the system.

**CONCLUSION**

This paper investigates the effectiveness of reactive power compensation (RPC) from wind turbines for loss reduction in a distribution system. The effect of parameters such as the X/R ratio of the cable connecting the wind farm to the substation, the
power factor of the load in the system, the location of the wind turbine along the feeder, is investigated on the effectiveness of the methodology.

In the case study, when the average power factor of the load is 0.98, the maximum loss reduction achieved by using RPC is 2.7%. Considering this loss calculation does not include the loss increase in the converter due to the increase in the reactive power demand, this is too low to motivate any investment on RPC. However, with lower average power factor of 0.88, the loss reduction due to RPC can be as much as 20%. These results show that the power factor of the load in the system highly determines the applicability and actual implementation of RPC for loss reduction. Moreover, the results of the case study show that, with a modest decrease in the power factor of the load, the loss reduction achieved through RPC is comparable with that achieved through active power injection of the wind turbine. This is because, while the active power output from the wind turbine cannot be controlled to match the load in the system, the reactive power output from the wind turbine can be controlled to match the reactive power demand in the system. Thus, the reactive current injected from the wind turbine, though lower in magnitude compared to active power, can be effective in reducing the power losses in the system.

The next parameter that is found to determine the effectiveness of RPC for loss reduction is the location of the wind turbine in the distribution system. In this regard, if the wind turbine is to be sited from purely RPC consideration, the sitting of the wind turbine can be done by following the 2/3 rule. However, since wind turbines also generate active power which is weakly correlated with the active power demand in the system, the application of the rule does not minimize the system power losses. Hence, for the appropriate sitting of a wind turbine, a more thorough load flow analysis based on time series data need to be done.

Nevertheless, the X/R ratio of the cable is found to have minor effect on the effectiveness of the RPC in loss reduction.

REFERENCES